The macroeconomic implications of the Gen-AI economy

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Abstract

We study the potential impact of the generative artificial intelligence (Gen-AI) revolution on the US economy through the lens of a multi-sector model in which we explicitly model the role of Gen-AI services in customer base management. In our model with carefully calibrated input-output linkages and the size of the Gen-AI sector, we find large spillovers of the Gen-AI productivity gains into the overall economy. A 11% increase in productivity in the Gen-AI sector over a 10 year horizon implies a 7% increase in aggregate GDP, despite the AI sector representing only a 10% of the overall economy. That shock also implies a significant reallocation of labor away from the AI sector and into non-AI sectors. We decompose these effects into parts coming from the input-output structure and customer base management and find that they each contribute equally to the rise in GDP. In the absence of either channels, real GDP essentially does not respond to the increase in productivity in the AI sector.

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Accelerated computing and generative AI have hit the tipping point. Demand is surging worldwide across companies, industries and nations.

Jensen Huang, Nvidia CEO and founder

1 Introduction

In the wake of the 21st century, the world has witnessed an unprecedented surge in technological advancements, with Artificial Intelligence (AI) standing as one of the potentially most transformative innovations of our time. As AI systems continue to evolve and permeate various facets of economic activity, it becomes increasingly important to understand the implications of this technology for the macroeconomy. The integration of AI into industries spanning from healthcare to manufacturing, finance, and beyond gives rise to changes in the economic landscape that are critical to analyze and quantify. While the impact of AI through automation and substitution of tasks in a production process has received significant attention in the literature (Aghion et al. (2018), Acemoglu and Restrepo (2018a), Acemoglu et al. (2022)), much less is known about how AI impacts the economy through data collection, analysis and distribution for the purpose of sales and customer base management.

This paper aims to fill this gap by employing a quantitative multi-sector model, which explicitly incorporates the impact of AI on customer build up, acquisition and retention across sectors. This choice is motivated by several industry trends, contending that harnessing generative AI enhances the efficiency of customer service. For example, Brynjolfsson et al. (2023) finds that the use of an AI tool leads to an increase of almost 14% in the productivity of customer support agents in a Fortune 500 software company. A 2023 report published by the Boston Consulting Group (Bamberger et al., 2023) suggests that the adoption of generative AI could potentially lead to a substantial increase in productivity within customer service operations, ranging from 30% to 50%. Furthermore, recent surveys conducted by Mckinsey & Company (Chui et al., 2023) reveal that organizations are increasingly utilizing generative AI in areas such as marketing and sales, product and service development, and service operations. Notably, the survey indicates that 77% of respondents in business, legal, and professional services sectors have experimented with generative AI tools since their introduction. Additionally, research from the International Monetary Fund (Melina et al., 2024) highlights that approximately 30% of employment in professional occupations in the

UK exhibits a high degree of exposure to generative AI technologies. Finally, Felten et al. (2023) identifies key sectors most impacted by this technological advancement, including legal services, investment activities, accounting, software publishing, and computer systems design.

In order to capture these forces formally, we set up a 3-sector model with an explicit input-output structure and frictions in building customer base via marketing expenditures, along the lines of Drozd and Nosal (2012). In the model, the gen-AI-intensive sector produces marketing services, which are then used by all sectors to build their customer bases, which in turn determine the demand for their product. We model improvements in AI technology as a positive productivity shock in the gen-AI sector, which not only affects the cost of its services as an intermediate input into production, but the marketing cost as well. We calibrate the model's input-output structure using the 'use tables' of the BEA accounts. We map Sector 1, the gen-AI intensive sector, into NAICS 3-digit service industries whose occupations have currently at least 30% exposure to Gen-AI. Sector 2 is mapped into more traditional service industries and Sector 3 is mapped into manufacturing industries. We then parameterize the customer base frictions to match marketing expenditure to sales ratio of 7% and wholesale markups of 10%.

Through the lens of our calibrated model, we are able to study the impact of changes in the productivity of the gen-AI service sector on the allocation of inputs and aggregate economic activity. First, on business cycle frequency, a positive productivity shock in the gen-AI service sector leads to a shift in labor and capital away from gen-AI and towards manufacturing and other services. At the same time, it leads to an increase in aggregate output in all sectors, increased consumption and investment and a relatively modest impact on aggregate employment. Intuitively, the cross-sectoral spillovers are predominantly driven by the customer capital friction. Improvements in gen-AI technology make it cheaper to search for suppliers in all sectors, hence spurring aggregate output. In fact, in the version of the model without customer capital and just input-output linkages, aggregate output response is only 14% of the AI sector response on impact, while in the model with only customer capital the aggregate response is 90% of the AI sector response. Given that the gen-AI service sector size in our calibration is less than 10% of aggregate output, this shows a very powerful spillover effect.

Our main quantitative experiment simulates the effect of a permanent increase in gen-AI sector productivity, over the transition and across steady states. Specifically, we feed into the model the productivity increase in the gen-AI sector to deliver an increase of 7 p.p. in GDP over a 10-year horizon, motivated by industry estimates.[1] This gives a required increase in

[1]This exercise is motivated by a recent report by Goldman Sachs(Hatzius et al., 2023), which highlights

gen-AI productivity of almost 11% over 10 years, with most of the growth happening in the first 3 years, with a final steady state productivity increase of a bit more than 11%.

Along the transition, the economy responds to this shock by increasing the use of the gen-AI intermediate good by all sectors in the economy. At the same time, employment drops significantly in the gen-AI service sector, settling 5% below its steady state 10 years after the shock, while the remaining sectors exhibit an increase of about 2.5% above their respective steady states. Capital accumulation drops temporarily in the gen-AI sector, while increasing permanently in the non-AI sectors. Marketing expenditures and search effort increases in all sectors, reflecting more reliance on the gen-AI sector's output. Finally, real output goes up by around 7% in *all* sectors, even though the fundamental productivity gains only hit the relatively small gen-AI sector, which accounts for less than 10% of total output. This reflects large spillover effect of the gen-AI sector through increases in efficiency of search and customer build-up, which affects all sectors through customer base accumulation.

Across steady states, a permanent 11% increase in productivity in the Gen-AI sector leads to an approximately 9% increase in aggregate GDP. About 90% of the change in GDP happens after 16 years of the initial shock. Since our quantitative model incorporates an input-output structure and a customer capital component, we are able to shut down each of those in turn to investigate the main forces behind the change. By solving versions of our model that isolate each of these elements, we find that both the input-output structure and customer capital friction contribute about half of to the rise in GDP. Specifically, a 11.1% increase in productivity in the service sector with only the input-output component results in a GDP increase of about 4%. In the absence of both channels, real GDP rises by a mere 0.3%.

Crucially, the input-output structure and customer search elements play a vital role in the reallocation of resources following the AI shock. In the baseline model, labor declines in the gen-AI service sector by 4%, while increasing in the other two sectors by around 2%. However, this strong labor relocation is solely driven by the customer capital friction and essentially disappears in the model with just input-output linkages. As for capital relocation, the input-output component and the customer capital component work in opposite directions. Input-output linkages push capital to increase roughly equally in all sectors, while customer search pushes capital to relocate away from the gen-AI services towards the other sectors, just like labor. In the case of capital, input-output linkages are stronger and capital increases in the baseline model across all sectors, but by a smaller amount in the gen-AI sector relative to the other sectors. We find that customer capital creates significant spillovers across sectors of the improvement in productivity in the service sector. Absent customer capital friction,

the potential impact of generative AI on the world economy.

most of the output increases are observed in the service sector (18%, versus 5% and 4% in the other 2 sectors), while in the baseline model the gains are more evenly distributed (11%, 9% and 8%). As a result, the gain for aggregate GDP is much higher in the baseline model (9%) relative to a model with just input-output linkages (4%).

Our work relates to several strands of literature. First, our paper is connected to the literature on technology progress. Change (1990), Aghion and Howitt (1992), and Kogan et al. (2017) argue the important role of technological change in economic growth. Babina et al. (2024) empirically analyze AI-related technologies driven growth concentrates among larger firms through product innovation.

Second, our paper draws from the growing literature exploring the macroeconomic implications of AI. In the context of automation, Acemoglu and Restrepo (2018b) proposed task-based production technology and discussed the labor substitution effect. Acemoglu et al. (2022) show that AI affects the composition of occupations within AI-exposed firms. From the viewpoint of labor complement, Kanazawa et al. (2022) and Noy and Zhang (2023) show that an AI system improves workers' productivity and leads to a narrowing of the productivity gap between workers by benefiting the low-skilled more. Pizzinelli et al. (2023) examines the impact of AI on labor markets using cross-country variation. While existing literature on AI often investigates the implications on the labor market, our study attempts to assess the effect of AI via customer service efficiency using the customer capital structure.

Third, we contribute to the literature on the role of customer capital. Most of papers such as Kleshchelski and Vincent (2009), Drozd and Nosal (2012), Gourio and Rudanko (2014), Paciello et al. (2019), Roldan-Blanco and Gilbukh (2021), and Rudanko (2022) concentrate on the implications for firm price setting with long-term customer relationships in a product market with search friction. Morlacco and Zeke (2021) study the role of customer capital in the industry concentration and market power under the low-interest environment. The present paper develops the customer search framework of Drozd and Nosal (2012) to characterize the service industry and the focus on the economic dynamics through the special role of AI differentiates our work from the existing studies.

The paper is organized as follows. The model is outlined in the next section. Section 3 contains the optimality conditions of the different players in the economy. In Section 4, we explain how a shock in the Gen-AI sector affects other sectors and the macroeconomy. Section 5 contains the calibration while section 6 has the simulation exercises.

2 Model

Figure 1 provides a graphical representation of our multi-sector model with consumer capital. The model features intermediate producers, retailers, and households.

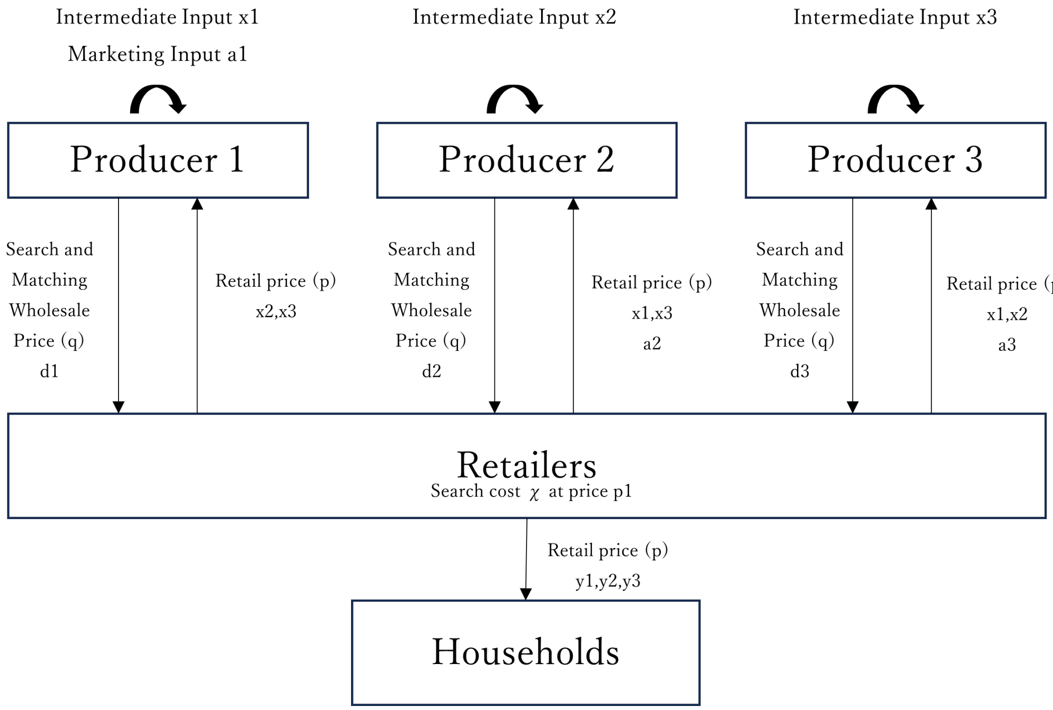


Figure 1: Overview of Model Structure

2.1 Intermediate Producers

For tractability reasons, we assume that intermediate producers are organized in three sectors. We classify US industries in these sector based on their potential exposure to AI. To this end, we rely on a recent report by Golman Sachs (Hatzius et al., 2023) and classify industries into 3 categories.

Type I: services highly susceptible to AI and help in marketing related activities (at least 30% of tasks automated). Examples include Legal services and data processing, internet publishing.

Type II: other service sectors like real estate with potential AI impact (between 20 % and 30 %). Educational instruction, arts, design, and sports fall within this sector.

Type III: the rest. Prime examples are manufacturing and construction.

Table 13 in the appendix contains the entire classification for US industries.

A unit measure of competitive producers are in each sector. Each producer is assumed to have access to a constant returns to scale production function $z_j F(k_j, l_j, \{x_{jm}\})$ that uses sector-specific capital k_j, labor l_j, and intermediate input x_{jm} produced by sector m and is subject to a sector-specific stochastic technology z_j following an exogenous AR(1) process:

$$\ln z_{jt} = (1 - \rho_z^j) \ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt}$$

where $\rho_z^j \in [0, 1]$ is a persistence parameter.

Since the production function is assumed to be constant returns to scale, we summarize the production process by an economy wide marginal cost v. Given factor prices w, r^K, p_j and productivity shock z, the marginal cost, equal to per unit cost, is given by:

$$v_{jt} \equiv \min_{k,l,x} \left\{ w_t l_{jt} + r^K k_{jt} + v_{jt} x_{jjt} + \sum_{m \neq j} p_{mt} x_{jmt} \mid z_{jt} F(k, l, x) = 1 \right\}$$

Note that producers purchase intermediate goods and marketing inputs from the retailers but not directly from wholesale producers.

To capture the outsized role of AI in the service-related industries, we impose that only goods produced by sector Type I (e.g. IT sector) can be used as marketing input a in the production process.[2]

2.1.1 List of customers and Market shares

To match with retailers, the good j producer i has access to an explicitly formulated marketing technology and accumulate a form of capital labeled marketing capital, m. Marketing capital is accumulated separately in each sector. We assume that each match with a retailer is long lasting and is subject to an exogenous destruction rate $\delta_H \in [0, 1]$, and thus the evolution of the endogenous list of customers H is described by the following law of motion:

$$H_{ijt} = (1 - \delta_H) H_{ijt-1} + \frac{m_{ij}}{\sum_j \bar{m}_j} h_t$$

where $\bar{m}_j$ denotes the average levels of marketing capital of the good j producer in the economy and h_t the measure of searching retailers. Note that $\bar{m}_j = m_j$ in an equilibrium. We assume here that in each match, one unit of the good can be traded per period. Thus,

[2]The Brandtech Group is an example of actual companies using AI to improve advertising. Per Financial Times, "The Brandtech Group was launched in 2015 aiming to make marketing services better, faster and cheaper using technology, including machine-generated content and AI."

sales of a given producer cannot exceed the size of the customer list H:

$$d_{ijt} \leq H_{ijt}$$

2.1.2 Marketing capital

The good j producer i accumulates marketing capital m_{ij} to attract searching retailers. Given last period's level of marketing capital m_{ijt-1} and the current level of marketing input a_{ijt}, current period marketing capital m_{ijt} is given by

$$m_{ijt} = (1 - \delta_m^j) m_{ijt-1} + a_{ijt} - \frac{\psi_j}{2} m_{ijt-1} \left(\frac{a_{ijt}}{m_{ijt-1}} - \delta_m^j \right)^2$$

where $\delta_m^j \in [0, 1]$ denotes the depreciation rate of marketing capital at industry j and $\psi_j \in [0, \infty)$ the market expansion friction parameter.

2.1.3 Profit Maximization

The good j producer i sells goods for the wholesale price q_j. These prices are determined by bargaining with the retailers. The instantaneous profit function Π_j of the producer is determined by the difference between the profit from sales and the total cost of marketing:

$$\Pi_{ijt} = \begin{cases} (q_{ijt} - v_{ijt}) d_{ijt} - v_{ijt} A_{jt} a_{ijt} & (j = 1) \\ (q_{ijt} - v_{ijt}) d_{ijt} - p_{1t} A_{jt} a_{ijt} & (j \neq 1) \end{cases}$$

where A denotes the marketing investment productivity.

The maximization problem is given by

$$\max_{a_{ijt}, m_{ijt}, d_{ijt}, H_{ijt}} \mathbb{E}_t \left[\sum_{k=0} \Omega_{t,t+k} \Pi_{ijt+k} \right]$$

$$\text{s.t. } m_{ijt} = (1 - \delta_m^j) m_{ijt-1} + a_{ijt} - \frac{\psi_j}{2} m_{ijt-1} \left(\frac{a_{ijt}}{m_{ijt-1}} - \delta_m^j \right)^2$$

$$H_{ijt} = (1 - \delta_H) H_{ijt-1} + \frac{m_{ijt}}{\sum_j \bar{m}_{ijt}} h_t$$

$$d_{ijt} \leq H_{ijt}$$

where $\Omega_{t,t+k}$ denotes the discount factor defined by $\beta^k u_c(c_{t+k}, l_{t+k}) / u_c(c_t, l_t)$ derived from the household's problem.

2.2 Retailers

Atomless retailers who purchase goods from each sector and resell them in a local competitive market to households. We assume that new retailers who enter into the market must incur an initial search cost $\chi \bar{p}_1$ (i.e. χ units of good 1 priced by the average retail price of sector 1) in order to find a producer with whom they can match and trade.

In each period, there is a mass of retailers already matched with the producers H_{ij} and a mass of new entrants h (searching retailers). A new entrant, upon paying the initial search cost χp_1, meets with probability π_{ij} a producer i from the sector j. The entrant takes this probability as given, but in equilibrium it is determined by the marketing capital levels accumulated by the producers, according to

$$\pi_{ijt} = \frac{m_{ijt}}{\sum_j \bar{m}_{jt}}$$

2.2.1 Bargaining and Wholesale Prices

We assume that each retailer bargains with the producer over the total future surplus from a given match. This surplus is split in consistency with Nash bargaining solution with continual renegotiation. The value of the wholesale producer i at industry j, W_{ij}, and the value for the retailer matched with a producer i at industry j, J_{ij}, are defined by

$$\begin{aligned} W_{ijt} &= \max\{0, q_{ijt} - v_{ijt}\} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1} W_{ijt+1}] \\ J_{ijt} &= \max\{0, p_{ijt} - q_{ijt}\} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1} J_{ijt+1}] \end{aligned}$$

Given bargaining power $\theta \in [0,1]$, the Nash bargaining problem is set up by

$$q^*_{ijt} = \underset{q}{\text{argmax}}\, J^{\theta}_{ijt} W^{1-\theta}_{ijt}$$

Under continual renegotiation, the price schedule resulting from Nash Bargaining allocates θ fraction of the total instantaneous trade surplus to the producer and fraction $1-\theta$ to the retailer:

$$q_{ijt} = \theta p_{ijt} + (1-\theta) v_{ijt}$$

2.2.2 Free Entry and Exit condition

Free entry and exit into the retail sector governs the measures of searching retailers. It relates the expected surplus for the retailer from matching with each sector to the search cost incurred to identify a match opportunity:

$$\sum_{i,j} \pi_{ijt} J_{ijt} \leq \chi p_{1t}$$

The condition holds with equality whenever $h > 0$. The search cost χ is assumed uniformly bounded away from zero.

2.3 Households

Households are assumed to be a unit measure of identical, and to infinitely live. In each period, they choose the level of consumption c, investment in physical capital i, labor supply l, purchases of sectoral goods y_j, and purchases one-period bonds b_{t+1} to maximize the expected discounted lifetime utility

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

where u satisfies the standard assumptions and $\beta \in (0, 1)$. The preferences over sectoral goods is determined via the aggregator G:

$$G(\{y_j\}) = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where $\gamma > 0$ denotes the elasticity of substitution and $\omega_j \in [0, 1]$ the share of expenditure on good j, satisfying $\sum_j \omega_j = 1$.

Households combine sectoral goods y_j through the above aggregator into a composite good which they use for consumption and investment, according to the aggregation constraint

$$c_t + i_t = G(\{y_j\})$$

Physical capital follows the standard law of motion with adjustment cost:

$$k_t = (1 - \delta) k_{t-1} + i_t - \phi(i_t, k_{t-1})$$

where $\delta \in (0, 1)$ denotes the depreciation of physical capital, ϕ adjustment cost function.

The budget constraint is given by

$$\sum_j p_{jt} y_{jt} + b_{t+1} = R_{t-1} b_t + w_t l_t + r_t^K k_{t-1} + \Pi_t$$

where p_j denotes the real retail price of good j, w the real wage, R the real (gross) risk free rate, r^K the real return on capital, Π the real profit from firms.

2.4 Market Clearing

We consider the symmetric equilibrium so that the producer i in industry j can be treaded as identical (i.e. $\bar{\cdot}_j = \cdot_j = \cdot_{ij}$). Therefore, we abstract the subscript i hereafter. The aggregate resource constraint is given by

$$z_{jt}F(k_{jt}, l_{jt}, \{x_{jmt}\}) = \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_m x_{mjt} & (j \neq 1) \end{cases}$$

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m \neq j} x_{mjt} + \sum_{m \neq j} a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_{m \neq j} x_{mjt} & (j \neq 1) \end{cases}$$

Labor and Capital markets clear when

$$\sum_{jt} l_{jt} = l_t, \quad \sum_{jt} k_{jt} = k_{t-1}$$

where we assume the perfect labor and capital mobility across sectors.

2.5 Setup for quantitative analysis

For quantitative analysis, we assume that households' utility function takes the form

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \xi \frac{l_t^{1+\eta}}{1+\eta}$$

where ξ denotes the relative value of labor which determines the steady state value of l. We assume the production technology for each sector is given by

$$F_{jt} = F(k_{jt}, l_{jt}, \{x_{jmt}\}) = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j}$$

Capital Stock Adjustment cost takes the form

$$\phi(i_t, k_{t-1}) = \frac{\phi}{2} k_{t-1} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2,$$

so that its derivatives are

$$\begin{aligned}
\phi_1(i_t, k_{t-1}) =& \phi\left(\frac{i_t}{k_{t-1}} - \delta\right) \\
\phi_2(i_t, k_{t-1}) =& -\phi\left(\frac{i_t}{k_{t-1}} - \delta\right)\frac{i_t}{k_{t-1}}
\end{aligned}$$

3 Optimality conditions

This section flushes out the optimality conditions for the different actors in our model.

3.1 Households

From cost minimization problem, we obtain

$$y_{jt} = \omega_j (p_{jt})^{-\gamma} G_t, \quad 1 = P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \quad c_t + i_t = \sum_j p_{jt} y_{jt}$$

Optimality requires the following FOCs:

$$\begin{aligned}
\frac{u_2(c_t, l_t)}{u_1(c_t, l_t)} &= -w_t \\
u_1(c_t, l_t) &= \beta R_t \mathbb{E}_t[u_1(c_{t+1}, l_{t+1})] \\
\frac{u_1(c_t, l_t)}{1 - \phi_1(i_t, k_{t-1})} &= \beta \mathbb{E}_t\left[\frac{u_1(c_{t+1}, l_{t+1})}{1 - \phi_1(i_{t+1}, k_t)}\left\{1 - \delta + r^K_{t+1}(1 - \phi_1(i_{t+1}, k_t)) - \phi_2(i_{t+1}, k_t)\right\}\right]
\end{aligned}$$

3.2 Producers

From cost minimization problem:

$$\min r^K_t k_{jt} + w_t l_{jt} + \sum_m p_{jmt} x_{jmt} \quad \text{s.t.} \quad Y_{jt} = z_{jt} k_{jt}^{\alpha^j_k} l_{jt}^{\alpha^j_l} \prod_m x_{jmt}^{\alpha^j_m}, \quad \alpha^j_k + \alpha^j_l + \sum_m \alpha^j_m = 1$$

the marginal cost v_{jt} under the above production technology is given by[3]

$$v_{jt} = \frac{1}{z_{jt}} \frac{(r^K_t)^{\alpha^j_k} (w_t)^{\alpha^j_l} v_{jt}^{\alpha^j_j} \prod_{m \neq j} p_{mt}^{\alpha^j_m}}{(\alpha^j_k)^{\alpha^j_k} (\alpha^j_l)^{\alpha^j_l} \prod_m (\alpha^j_m)^{\alpha^j_m}} = \left(\frac{1}{z_{jt}\varrho_j} (r^K_t)^{\alpha^j_k} (w_t)^{\alpha^j_l} \prod_{m \neq j} p_{mt}^{\alpha^j_m}\right)^{\frac{1}{1-\alpha^j_j}}$$

[3]See Appendix for the detailed derivation

where $\varrho_j = (\alpha_k^j)^{\alpha_k^j}(\alpha_l^j)^{\alpha_l^j}\prod_m(\alpha_m^j)^{\alpha_m^j}$.
The implied factor demands are given by

$$\frac{k_{jt}}{l_{jt}} = \frac{a_k^j}{a_l^j}\frac{w_t}{r_t^K}, \quad \frac{x_{jmt}}{l_{jt}} = \begin{cases} \frac{\alpha_j^j}{\alpha_l^j}\frac{w_t}{v_{jt}} & (m=j) \\ \frac{\alpha_m^j}{\alpha_l^j}\frac{w_t}{p_{mt}} & (m \neq j) \end{cases}$$

Note that

$$k_{jt} = \frac{\alpha_k^j \tau_{jt}}{\varrho_j r_t^K} F_{jt} \tag{1}$$

$$l_{jt} = \frac{\alpha_l^j \tau_{jt}}{\varrho_j w_t} F_{jt} \tag{2}$$

$$x_{jmt} = \begin{cases} \frac{\alpha_j^j \tau_{jt}}{\varrho_j v_{jt}} F_{jt} & (m=j) \\ \frac{\alpha_m^j \tau_{jt}}{\varrho_j p_{mt}} F_{jt} & (m \neq j) \end{cases} \tag{3}$$

where

$$\tau_{jt} = (r_t^K)^{\alpha_k^j}(w_t)^{\alpha_l^j}(v_{jt})^{\alpha_j^j}\prod_{m\neq j}(p_{mt})^{\alpha_m^j}$$

As for the profit maximization, define the Lagrangian multipliers by λ_{jt}, μ_{jt}, ν_{jt}.

$$\begin{aligned}\mathbb{E}_t\Bigg[\sum_{k=0}\Omega_{t,t+k}\Bigg\{\Pi_{jt+k} &+ \lambda_{jt}\left((1-\delta_m^j)m_{jt-1} + a_{jt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2 - m_{jt}\right) \\ &+ \mu_{jt}\left((1-\delta_H)H_{jt-1} + \frac{m_{jt}}{\sum_j \bar{m}_{jt}}h_t - H_{jt}\right) \\ &+ \nu_{jt}(H_{jt} - d_{jt})\Bigg\}\Bigg]\end{aligned}$$

where

$$\Pi_{jt} = \begin{cases} (q_{jt} - v_{jt})d_{jt} - v_{jt}A_{jt}a_{jt} & (j=1) \\ (q_{jt} - v_{jt})d_{jt} - p_{1t}A_{jt}a_{jt} & (j \neq 1) \end{cases}$$

FOCs yield

$$
\begin{aligned}
[a_{jt}]: &\quad \lambda_{jt} = \begin{cases} \dfrac{A_{jt}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} v_{1t} & (j=1) \\ \dfrac{A_{jt}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} p_{1t} & (j \neq 1) \end{cases} \\
[d_{jt}]: &\quad \nu_{jt} = (q_{jt} - v_{jt}) \\
[H_{jt}]: &\quad \mu_{jt} = \nu_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}\mu_{jt+1}] \\
[m_{jt}]: &\quad \lambda_{jt} = \frac{1}{\sum_j \bar{m}_{jt}} h_t \mu_{jt} + \mathbb{E}_t\left[\lambda_{jt+1}\Omega_{t,t+1}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]
\end{aligned}
$$

Note that $\mu_{jt} = W_{jt}$.

4 A Gen-AI shock in Sector I

Before diving into the quantitative predictions generated by our framework, it is worth discussing what an improvement in technology in Sector I (a Gen-AI shock) does to the economy. From the problem of a producer in sector Type 1, we know that a productivity shock reduces its marginal cost $v_{1,t}$. This results in a reduction of the price of the intermediate good in sector I ($p_{1,t}$). This drop makes marketing activities cheap for all sectors. Importantly, the shock generates a reduction in the entry cost for new retailer and the reduction of its marketing cost. In relative terms, the other intermediate goods, capital, and labor become more expensive for a firm in sector I, which leads to a higher use of its own input and the reduction of the other inputs.

Figure 2 shows the dynamics of several variables following the shock implied by the calibrated version of our model. A few characteristics are worth mentioning. The price of the intermediate good type I drops with the improvement in productivity. Simultaneously, the aggregate economy expands as seen by the increases in consumption and investment. Crucially, this expansion comes with a muted response in aggregated labor (first panel). The demand for intermediate goods by Sector II and III increases across the board. The shock triggers a reallocation of labor and capital away from sector I into the other sectors.

An important observation is that AI shock leads to an increase in marketing capital in all sectors ($m_{j,t}$). Unsurprisingly, the impact is stronger and front-loaded in Sector Type I. A technological improvement in the other sectors leads to an increase in marketing capital mostly in the sector where the shock starts. Interestingly, measured Solow residual in sector

I is smaller than the actual productivity shock. To understand this results, let's look at the log-linear version of the Solow residual

(A: w/ Search j=1) and (B: w/ Search j≠1 and w/o Search)

$$\text{Solow } \hat{\text{Residual}}_{jt} = \begin{cases} \hat{z}_{jt} + \frac{1}{z_j F_j - \sum_m x_{mj} - \chi h}\left(\sum_{m \neq j} x_{mj}(\hat{x}_{jjt} - \hat{x}_{mjt}) + \chi h(\hat{F}_{jt} - \hat{h}_t)\right) & \text{(A:)} \\ \hat{z}_{jt} + \frac{1}{z_j F_j - \sum_m x_{mj}} \sum_{m \neq j} x_{mj}(\hat{x}_{jjt} - \hat{x}_{mjt}) & \text{(B:)} \end{cases}$$

One can see that factor reallocation and the cost incurred by retailers looking for new matches lead to deviations between sectoral productivity z_{1t} and the Solow residual in that sector. In particular, if the productivity shock leads to a strong increase in the demand for input Type I by the other sectors and/or a strong increase in the number of new searching retailers, the Solow residual Type I will be lower than the actual productivity change.

Finally, note that a 1% technology improvement in sector I leads to an increase in aggregate GDP of 45 bps. This is in spite of that sector corresponding to only 10% percent of the aggregate GDP, which indicates that our model displays strong spillovers.

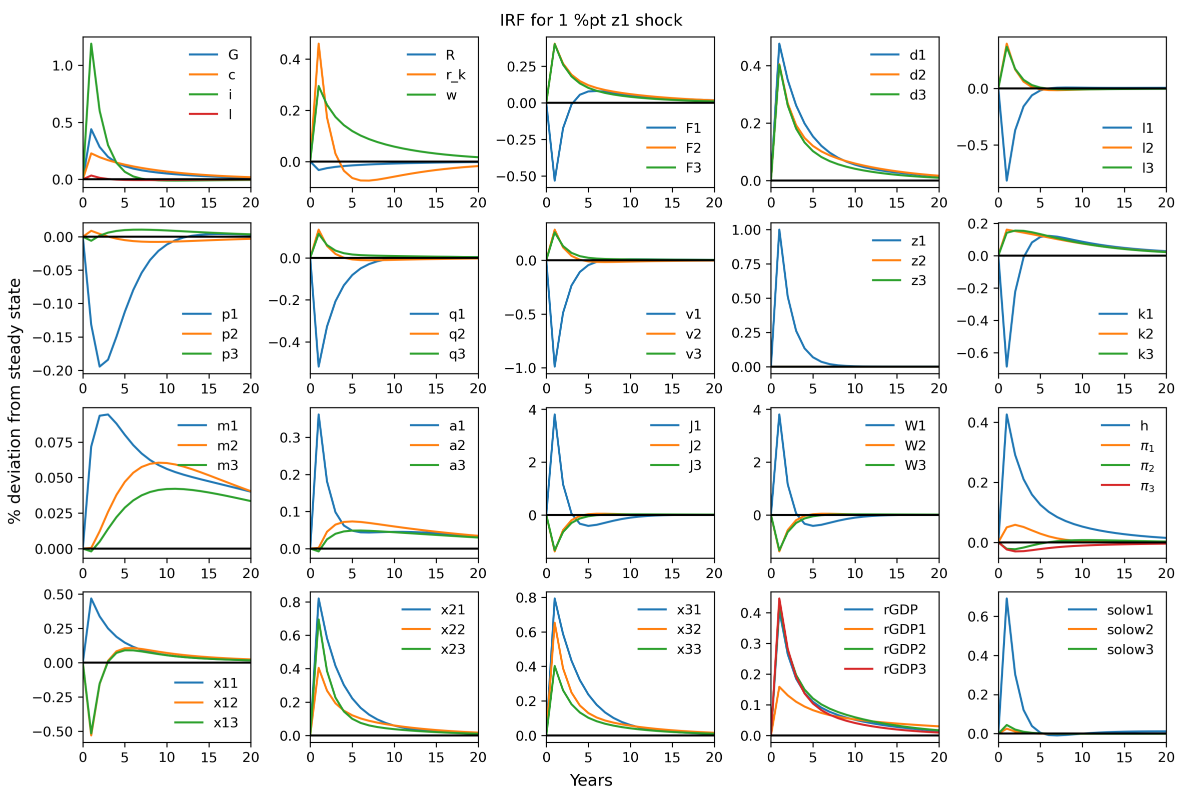


Figure 2: IRFs after a Gen-AI shock

x_{jm} is the use of input m in sector j.

5 Calibration Parameters

In this section, we explain our calibration approach. A period in the model corresponds to a year in the data.

5.1 Parameter values calibrated independently

Consider first the parameters that can be selected independently from all other parameters by targeting a single moment from the data. In accordance with previous research, we select standard values for the discount factor $\beta = 0.9$, relative risk aversion $\sigma = 1.0$, inverse of Frisch labor supply elasticity $\eta = 2.0$. We refer to the business cycle literature, such as Smets and Wouters (2007), for the values of physical capital adjustment cost parameter ϕ. For the values of physical capital depreciation δ and elasticity of substitution γ. we set the standard value. Finally, the customer destruction rate δ_H is arbitrarily set to 1, which means that contractual relationships last only 1 period.

Table 1: Independently calibrated parameters

Parameter	Symbol	Value	Source/Target
Discount factor	β	0.9	Standard
Relative Risk Aversion	σ	1.0	log-utility
Inverse of Frisch labor supply elasticity	η	2.0	Standard
Elasticity of Substitution	γ	1.1	-
Bargaining power	θ	0.5	Drozd and Nosal (2012)
Physical Capital Depreciation	δ	0.1	Standard
Customer list destruction rate	δ_H	1.0	-

We compute the value of factor shares α from the "use tables" of the input-output accounts constructed by the BEA.[4] The use table shows how commodities are utilized by different sectors both as intermediate inputs and final goods. We calculate the payment values from industries categorized as type i to those categorized as type j, considering the intermediate input shares α_j^i, the labor input share represented by compensation to employees α_L^i, and the capital input share reflected in gross operating surplus α_k^i. These values are normalized by the total sum of intermediate inputs, labor income (compensation to employees), and capital income (gross operating surplus). Then compute the averages of these normalized values over the period spanning from 2005 to 2018, in accordance with (Chui et al., 2023).

[4] They have recently released the 2023 version of IO table up to 2017 and haven't yet published 2000-2016 series. As a result, we are currently using the older version, which suggests that we will eventually need to update.

Table 2: Input share α_m^j based on the classification in Table 13

m \ j	Type1	Type2	Type3
Type1	0.208	0.104	0.075
Type2	0.074	0.185	0.068
Type3	0.107	0.094	0.389
Labor	0.386	0.228	0.288
Capital	0.225	0.389	0.180

For sectoral consumption, we use sectoral quarterly consumption from the National Income and Product Accounts (NIPA). To match these sectoral consumption from NIPA classification to the IO account classification, we use the PCE Bridge Table provided by the BEA.[5] [6] To allocate the NIPA components spending into the IO classification spending amount, we re-classify Purchasers' Value by Commodity Code and sum up the Purchasers' Value along our classification (Types I-III) and calculate the ratios. We use the averages of these values over 2007 to 2018.

Table 3: Spending share ω_j based on the classification in Table 13

	Type1	Type2	Type3
ω_j	0.015	0.537	0.448

5.2 Parameter values calibrated jointly with targeted statistics.

The remaining parameters are selected to align the model's predictions with certain empirical moments. The labor disutility parameter ξ, the search cost χ, and the marketing capital depreciation δ_m^j are included in the first group. They are chosen to match in the steady state: the unity labor for the normalization, the real GDP-weighted average gross wholesale markup[7] as 10% following Drozd and Nosal (2012), and the marketing expenditure to sale ratio as 7% following Drozd and Nosal (2012). Note that in the model we define the marketing expenditure to sale ratio as $\mathcal{M}_1 = (v_1 a_1)/(q_1 d_1)$ for $j = 1$ and $\mathcal{M}_j = (p_1 a_j)/(q_j d_j)$ for all other values of j.

[5] Web Page Link: BEA (clickable)

[6] They have recently released the 2023 version of IO table and haven't yet published the updated PCE Bridge Table. As a result, we are currently using the older version, which suggests that we will eventually need to update. Also, the detailed PCE Bridge Table is available only for 2007 and 2012. So we use the 73 commodities composition table to apply for each year respectively.

[7] $(\text{rGDP}_1\mathcal{U}_1 + \text{rGDP}_2\mathcal{U}_2 + \text{rGDP}_3\mathcal{U}_3)/\text{rGDP}$, where $\mathcal{U}_j$ is sectoral markup.

Table 4: Jointly calibrated parameters 1

Parameter	Symbol	Value	Source/Target
Parameter of labor disutility	ξ	0.6477	$l_{ss} = 1$
Search cost	χ	0.1172	10% gross wholesale markup
Marketing Capital Depreciation	δ_m^1	0.1988	Marketing expenditure to Sales ratio: 0.07
Marketing Capital Depreciation	δ_m^2	0.2607	Marketing expenditure to Sales ratio: 0.07
Marketing Capital Depreciation	δ_m^3	0.2567	Marketing expenditure to Sales ratio: 0.07

The second group includes the productivity persistence ρ_j, standard deviation of productivity σ_j, and parameter of marketing capital adjustment cost ψ_j. The volatility ratio of the producer price index (PPI) to the price index of personal consumption (PCE) at sector j, as well as the persistence and standard deviation of the solow residual at sector j, are targeted by the simulated method of moments to match the empirical moments. We use the BEA supply table to create the weighted average PPI, which we then use to create the sectoral PPI. The industry's supply to the outside of the industry is used as weight in accordance with the BLS[8] industry PPI weighting method. Note that we use the equally-weighted average index on sub-indices when the corresponding index is unavailable.[9] Using the PCE Bridge Table provided by the BEA, we calculate the weighted average PCE for sectoral PCE, using the purchaser's value as the weight. First, we use the HP-filter for log price indices with $\lambda = 100$ to calculate the sectoral price volatility. Next, we compute the cycle components' standard deviation. The standard deviation of the corresponding variables' deviation from steady states is used to compute the model-generated moments.

Table 5: Price volatility based on the classification in Table 13

	Type1	Type2	Type3
σ_{PPI}	0.005	0.040	0.032
σ_{PCE}	0.005	0.016	0.012
$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.042	3.488	2.588

The data for the Solow residual's persistence and standard deviation come from the BEA-BLS Integrated Industry-level Production Accounts (KLEMS)[10][11]. We create the weighted average TFP using the value added ratio as weights in order to construct the annual sectoral productivity. Next, we use the cubic-detrended log series to estimate the AR(1) model,

[8]https://www.bls.gov/ppi/faqs/questions-and-answers.htm

[9]We treat the weights of indices that are unavailable until the middle of the period as zero and compute the index using the available indices only.

[10]Web Page Link: BLS (clickable)

[11]File Link: BLS (clickable)

yielding the persistence parameter ρ_i and the volatility parameter σ_i[12]. Fernald's utility-adjusted TFP is utilized for alternative productivity process targets.[13] He provides % annual change in natural logs for annual series.

Table 6: Parameters of productivity process based on the classification in Table 13

	ρ_i			σ_i		
	Type1	Type2	Type3	Type1	Type2	Type3
KLEMS	0.551	0.563	0.450	0.012	0.008	0.008
Fernald utility-adjusted TFP	0.862	-	-	0.013	-	-

The corresponding data in the model is calculated using rGDP_{jt}/F_{jt}, where we define

$$\text{rGDP}_{jt} = \begin{cases} p_{10}(y_{1t} + \sum_j a_{jt}) & (j = 1) \\ p_{j0}y_{jt} & (j \neq 1) \end{cases}$$

The standard deviation of the corresponding variables' deviation from steady states is used to compute the model-generated moments.

Table 7: Jointly calibrated parameters 2

Parameter	Symbol	Value	Target		Model	
Physical Capital Adjustment cost	ϕ	0.8756	$\sigma_i/\sigma_{\text{GDP}}$	3.142	$\sigma_i/\sigma_{\text{rGDP}}$	2.3581
Marketing Capital Adjustment cost	ψ_1	35.0093	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.042	$\sigma_{q_1}/\sigma_{p_1}$	1.1278
Marketing Capital Adjustment cost	ψ_2	30.0447	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	3.488	$\sigma_{q_2}/\sigma_{p_2}$	2.9367
Marketing Capital Adjustment cost	ψ_3	34.9813	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	2.588	$\sigma_{q_3}/\sigma_{p_3}$	2.8027
Persistence of productivity in Sector 1	ρ_1	0.5110	KLEMS	0.551	rGDP_1/F_1	0.5017
Persistence of productivity in Sector 2	ρ_2	0.6131	KLEMS	0.563	rGDP_2/F_2	0.6223
Persistence of productivity in Sector 3	ρ_3	0.4149	KLEMS	0.450	rGDP_3/F_3	0.4410
Standard deviation of productivity in Sector 1	σ_1	0.0070	KLEMS	0.012	rGDP_1/F_1	0.0159
Standard deviation of productivity in Sector 2	σ_2	0.0070	KLEMS	0.008	rGDP_2/F_2	0.0067
Standard deviation of productivity in Sector 3	σ_3	0.0070	KLEMS	0.008	rGDP_3/F_3	0.0065

[12] Alternatively, we implement (i) Hamilton Filter, (ii) HP filter, (iii) Linear Trend for removing a trend. The Hamilton Filter and HP filter give lower persistent parameters than a cubic trend. See Appendix G.

[13] Web Page Link (clickable) data_quarterly_2023.03.07.xlsx

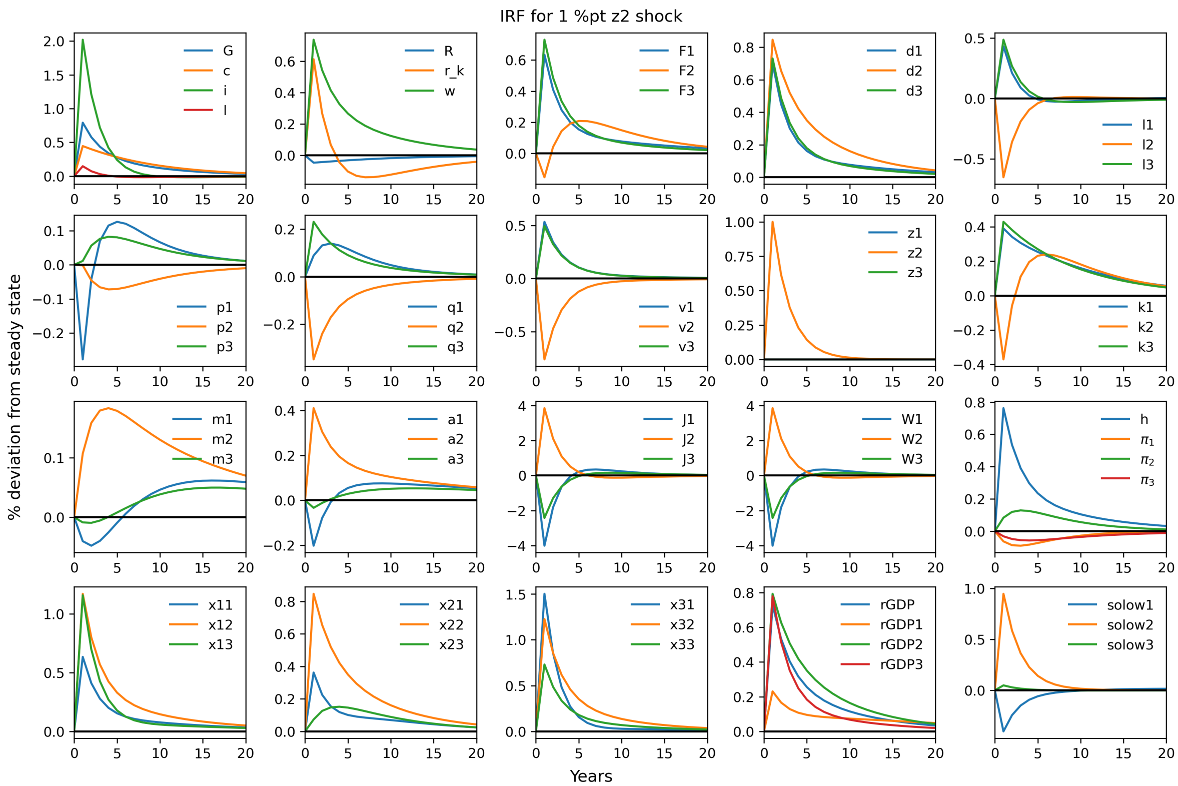
Table 8: Jointly calibrated parameters 2 with Fernald's TFP

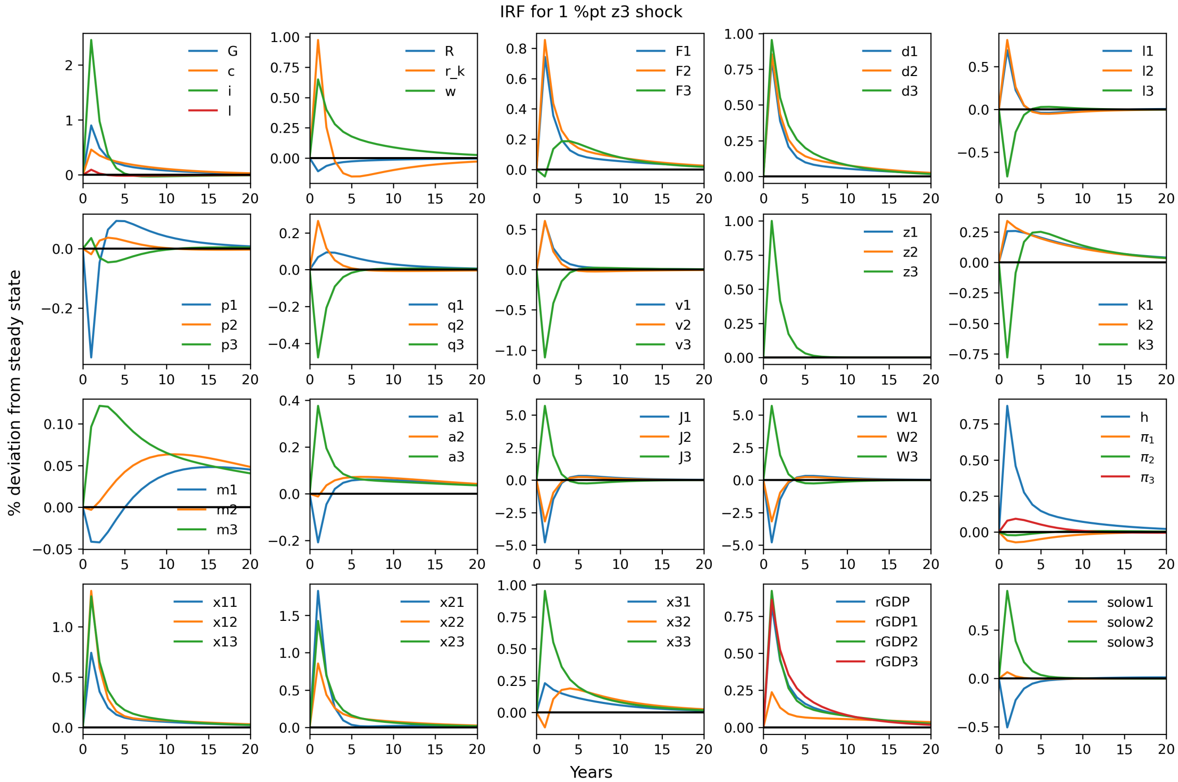
Parameter	Symbol	Value	Target		Model	
Physical Capital Adjustment cost	ϕ	0.8731	$\sigma_i/\sigma_{\text{GDP}}$	3.142	$\sigma_i/\sigma_{\text{rGDP}}$	2.3444
Marketing Capital Adjustment cost	ψ_1	35.0083	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.042	$\sigma_{q_1}/\sigma_{p_1}$	1.1309
Marketing Capital Adjustment cost	ψ_2	30.0040	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	3.488	$\sigma_{q_2}/\sigma_{p_2}$	2.8387
Marketing Capital Adjustment cost	ψ_3	35.0056	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	2.588	$\sigma_{q_3}/\sigma_{p_3}$	2.6654
Persistence of productivity in Sector 1	ρ_1	0.5013	Fernald	0.862	rGDP_1/F_1	0.4852
Persistence of productivity in Sector 2	ρ_2	0.6230	KLEMS	0.563	rGDP_2/F_2	0.6387
Persistence of productivity in Sector 3	ρ_3	0.4174	KLEMS	0.450	rGDP_3/F_3	0.4257
Standard deviation of productivity in Sector 1	σ_1	0.0070	KLEMS	0.013	rGDP_1/F_1	0.0157
Standard deviation of productivity in Sector 2	σ_2	0.0070	KLEMS	0.008	rGDP_2/F_2	0.0067
Standard deviation of productivity in Sector 3	σ_3	0.0070	KLEMS	0.008	rGDP_3/F_3	0.0065

6 Quantitative Results

We conduct three quantitative exercises. Impulse responses to a 1 standard-deviation shock to productivity in the service sector Type 1. Business cycle analysis. Finally, we analyze the implications of two scenarios: a) An increase in productivity similar to that experienced in the U.K. during the first industrial revolution; and b) The path of productivity required to generate an increase of 7 p.p. in GDP as predicted by Goldman Sachs.

6.1 IRF of 1%pt z_i shock





6.2 Model Generated Moments

Table 9 shows business cycle moments generated by our benchmark model – column Baseline (z_1, z_2, z_3). We see that the model delivers reasonable moments. For example, the volatility of GDP is 1.12%. Consumption is less volatile than output while investment is more volatile. Moreover, the model delivers persistent series and positive co-movement between aggregate variables. The next three columns displays the model's business cycles when only one shock is active at a time. A noteworthy feature of the model is when only the productivity shock in sector Type I is on. Specifically, this variant delivers a negative correlation between labor and other macro variables such as GDP or consumption.

Table 9: Business Cycle Moments

	Baseline				Fernald	
	z_1, z_2, z_3	z_1	z_2	z_3	z_1, z_2, z_3	z_1
ρ_{rGDP}	0.668	0.719	0.782	0.629	0.673	0.711
ρ_C	0.842	0.875	0.894	0.832	0.845	0.871
ρ_I	0.469	0.526	0.615	0.420	0.473	0.516
ρ_L	0.442	0.507	0.526	0.360	0.448	0.498
σ_{rGDP} (%)	1.118	0.409	0.812	0.740	1.125	0.404
σ_C (%)	0.889	0.326	0.689	0.576	0.897	0.321
σ_I (%)	2.717	0.971	1.780	1.877	2.723	0.967
σ_L (%)	0.144	0.028	0.122	0.074	0.144	0.028
Corr$(rGDP, C)$	0.949	0.949	0.962	0.940	0.949	0.948
Corr$(rGDP, I)$	0.917	0.910	0.912	0.917	0.917	0.911
Corr$(rGDP, L)$	0.691	0.593	0.704	0.663	0.689	0.599
Corr(C, I)	0.745	0.733	0.765	0.727	0.744	0.732
Corr(C, L)	0.455	0.310	0.482	0.369	0.454	0.314
Corr(I, L)	0.886	0.871	0.932	0.906	0.886	0.874

6.3 Simulation: 7% GDP increase in 10 years

In a recent study, Goldman and Sachs (2023) makes the case that world GDP could go up by as much as 10% in the next decade. Figure 3 presents the dynamic progression of multiple variables as the economy moves towards a state where the GDP is 7 percentage points higher than the steady state, a decade into the transition.[14] Our simulations indicate that achieving this goal necessitates a significant and lasting boost of approximately 11% in the productivity of the sector Type 1 z_1. Notably, the majority of this productivity increase occurs within the first 3 years of the transition, demonstrating a front-loaded effect. As the second upper panel shows, investment is the main driver of the boom in GDP.

The lower panels in Figure 3 help us to understand how higher z_1 productivity affects the demand for inputs across the 3 sectors. In the short run, the sector Type I reduces significantly its usage of capital and labor while raising its demand for its own output. In response, the other sectors take advantage of cheaper capital and labor increasing their demand of these inputs. In the longer run, the demand for labor in the sector Type I is down by 4 p.p. from the pre-shock level. Labor migrates to the other sectors resulting in a muted response at the aggregate level. Contrary to labor, capital in the AI sectors experiences increase in about 4 p.p. Capital increases strongly in the other sectors. In the aggregate capital increases by about 9 %.

[14]See Appendix H.

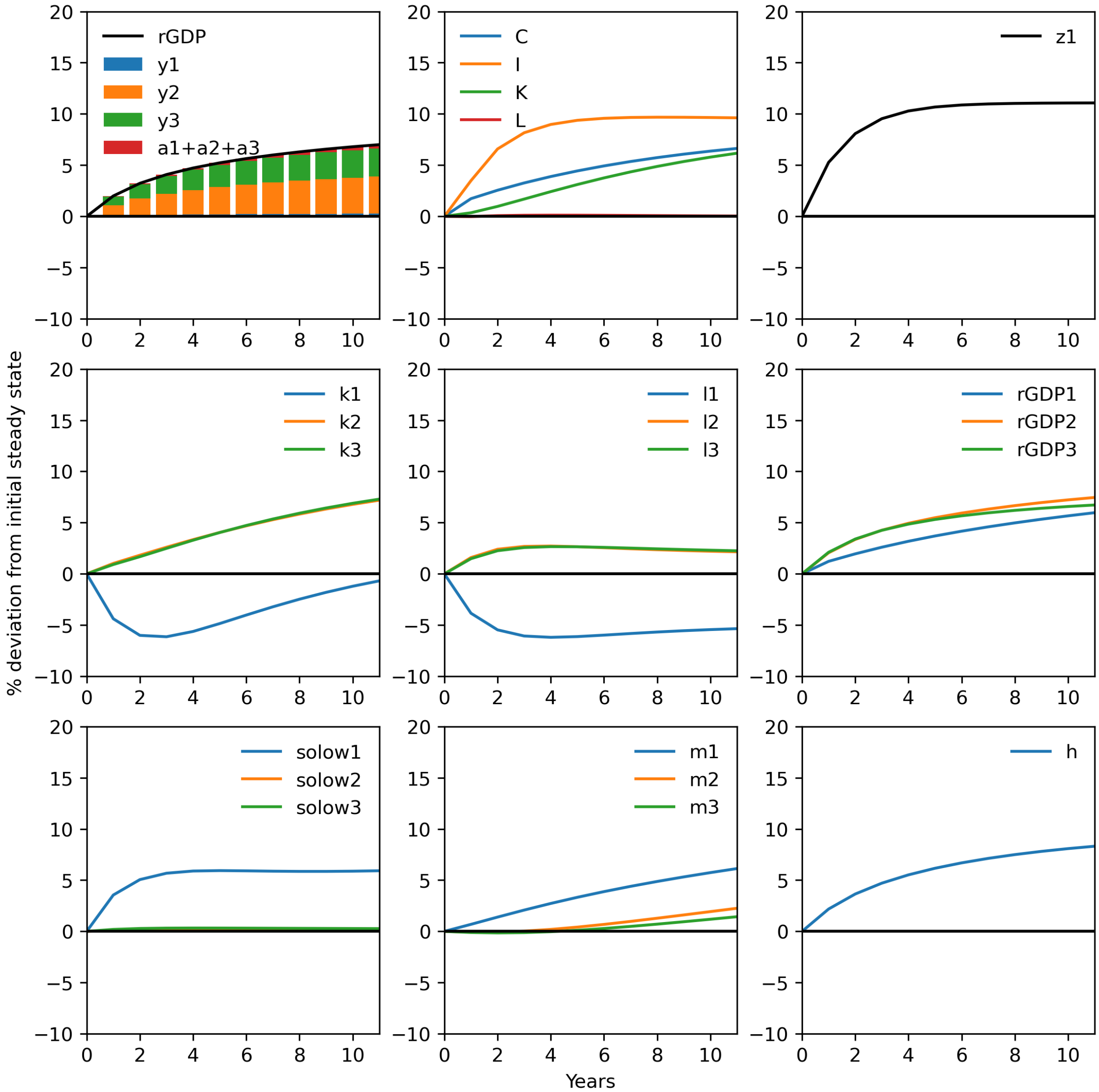


Figure 3: Transitional paths toward 7% increase in GDP

In the medium run, we see a marked shift of labor away from sector I (-5%) toward the other two sectors, each increases by roughly 2%. A similar reallocation is seen in capital but with a weaker effect on sector Type I. Real GDP in all sectors go up by around 5%. The shape of variables like capital sectoral GDP, and aggregate consumption indicate that 10 years after the shock the economy is still transitioning to the new steady state. In practice, in terms of real GDP, our simulation shows that it takes 16 years to reach 90% of its new steady state after the shock.

We observe that the transition to the higher GDP state involves no quantitatively significant change in aggregate labor. To understand this result, note that the return on capital in steady is fixed and given by $r^k = 1/\beta - 1 + \beta$, which is independent of the productivity level. In contrast, aggregate wages do depend on relative prices and hence on productivity in the AI sector. These two forces combined results in higher capital in steady state, it is cheaper, and no change in labor because it is more expensive.

Bouscasse et al. (2021) estimates significant uncertainty around the productivity growth in the U.K. during the first industrial revolution. Based on their results, we add 2 scenarios to our baseline exercise. We impose that GDP growth can be 50% higher or 50% lower than the Goldman Sach's estimate. Figure 4 displays the resulting dynamic paths.

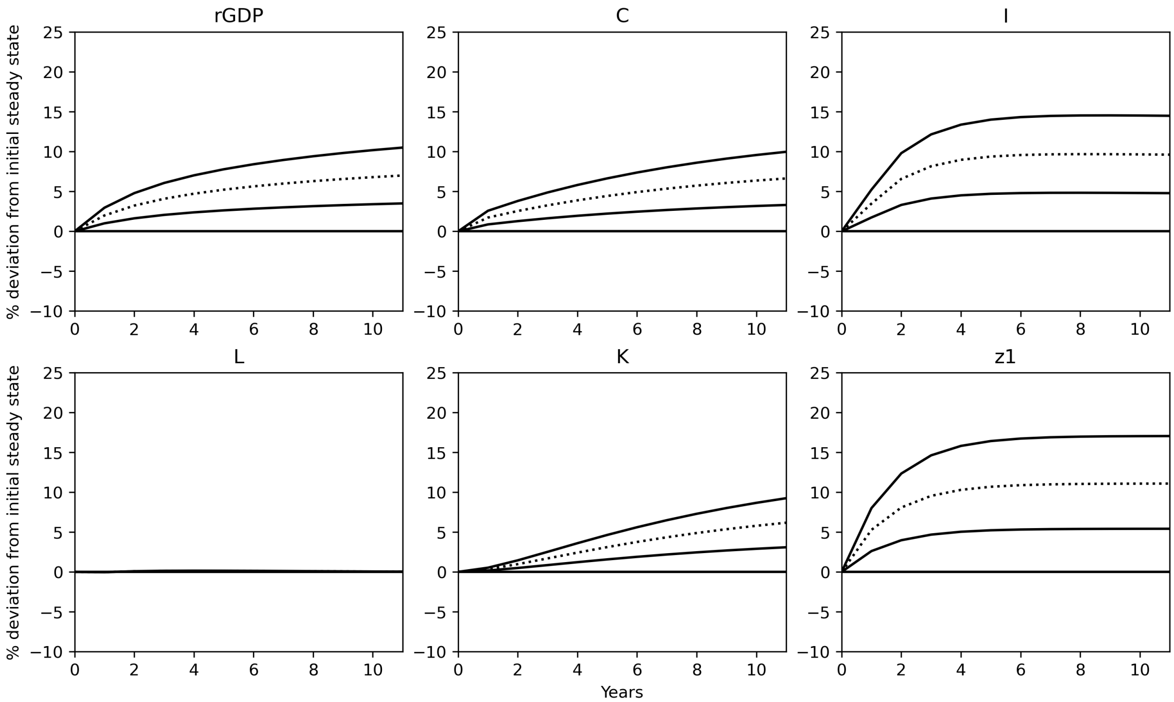


Figure 4: Simulation with 1.5 times higher / 0.5 times lower path than baseline

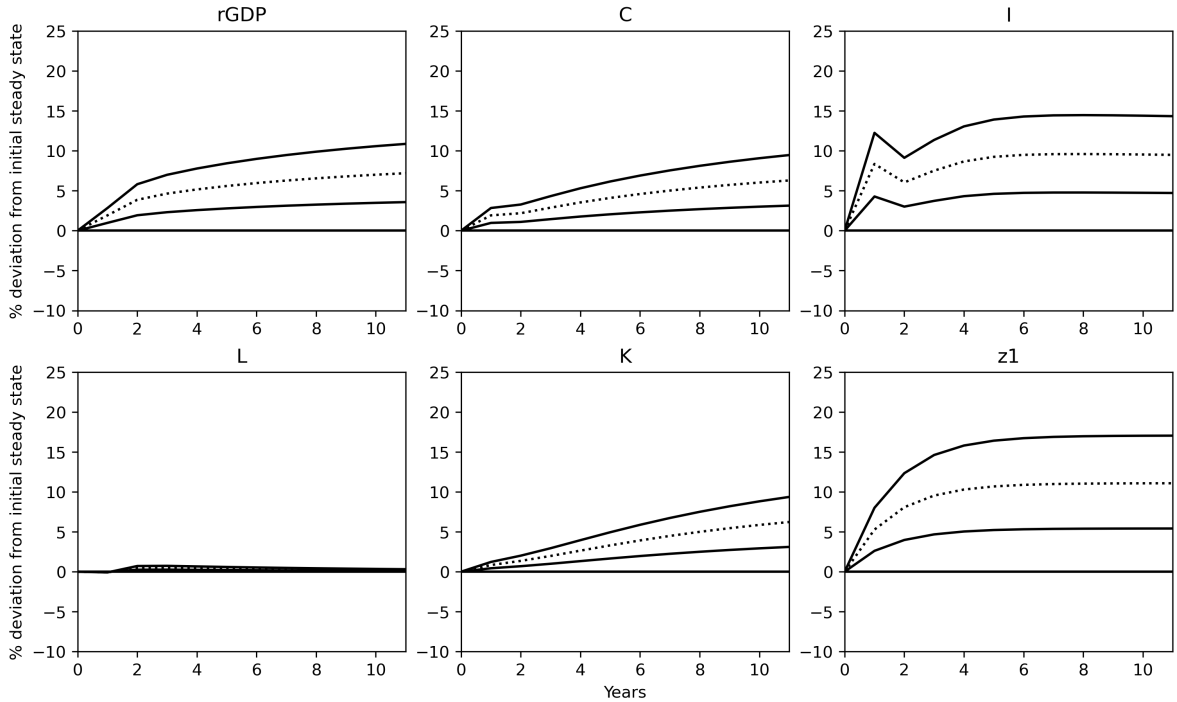


Figure 5: Simulation with 1.5 times higher / 0.5 times lower path than baseline with $\psi_j = 0$

A pertinent inquiry concerns the comparison of the 11% productivity boost in Sector I with historical epochs of marked productivity enhancements. Oxford Economics, in a recent report, highlighted the productivity leaps observed across various nations and eras. Notably, U.S. productivity witnessed a 20% increase from 1917 to 1927, a period that succeeded the advent of groundbreaking technologies such as electrification, the internal combustion engine, and the telephone in the late 19th century. Viewing from the lens of historical precedents, the anticipated productivity surge in Sector I appears plausible. Nonetheless, it's critical to acknowledge that the technological innovations of the second industrial revolution took considerable time to manifest in tangible productivity gains. This historical gradualism stands in stark contrast to the expected rapid realization of productivity gains in our current projections, which assumes a more immediate impact.

6.4 Simulation: 6% z1 increase in 10 years

Bouscasse et al. (2021) estimate that productivity grew at 3% per decade between 1600 and 1760. The number is 6% for the period 1770 and 1870. Figure 2 displays the model's predictions following a shock that increases productivity in the AI sector by 6% in a decade.

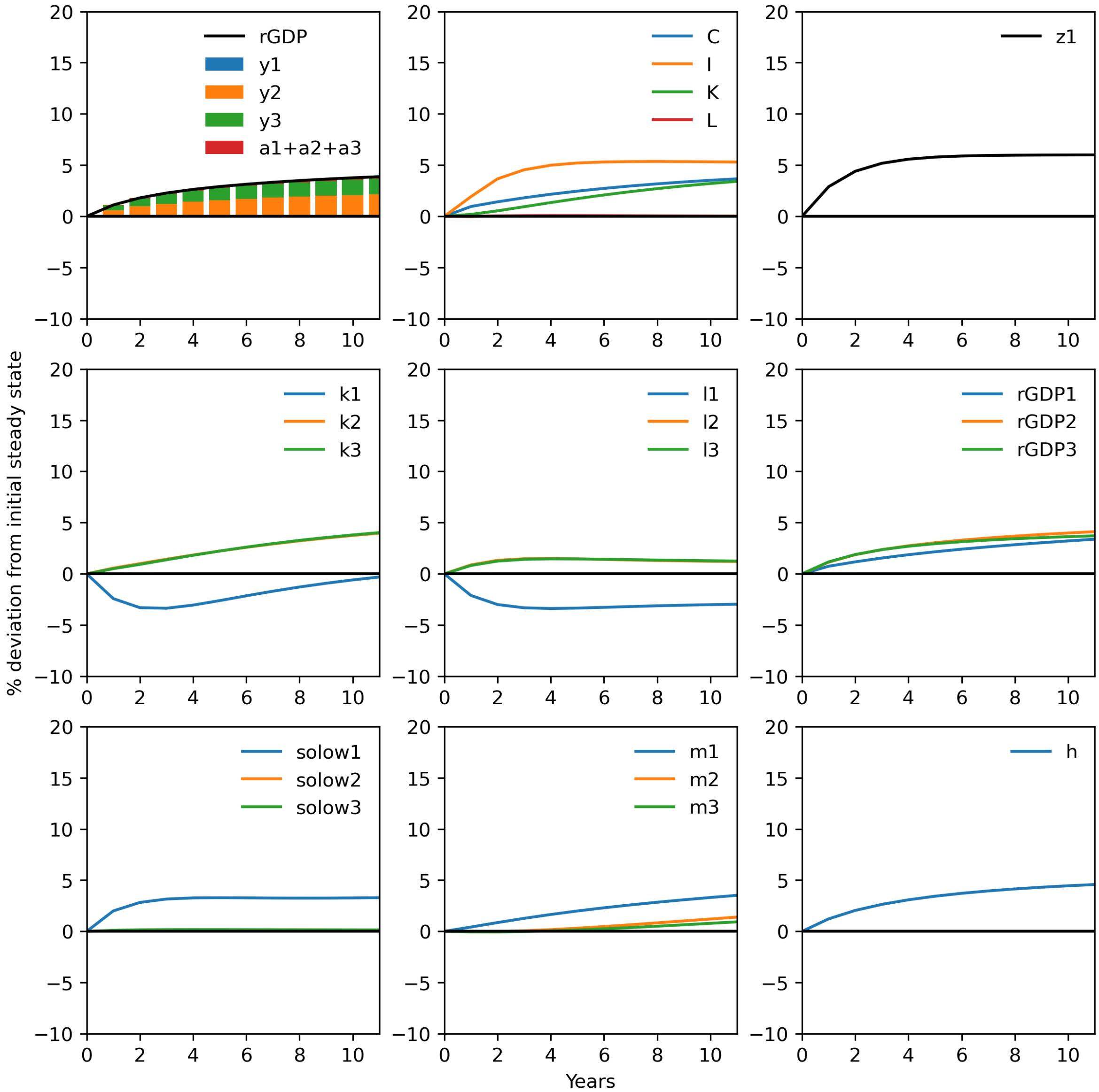


Figure 6: Transitional paths toward 6% increase in z1

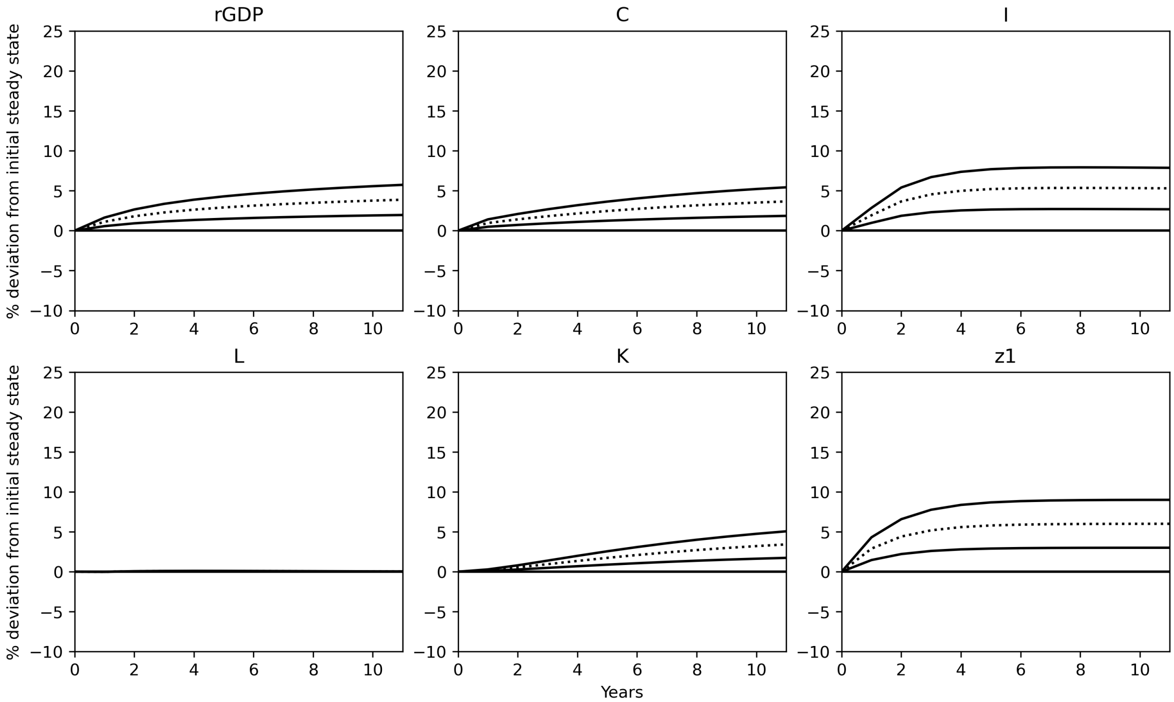


Figure 7: Simulation with 1.5 times higher / 0.5 times lower shock than baseline

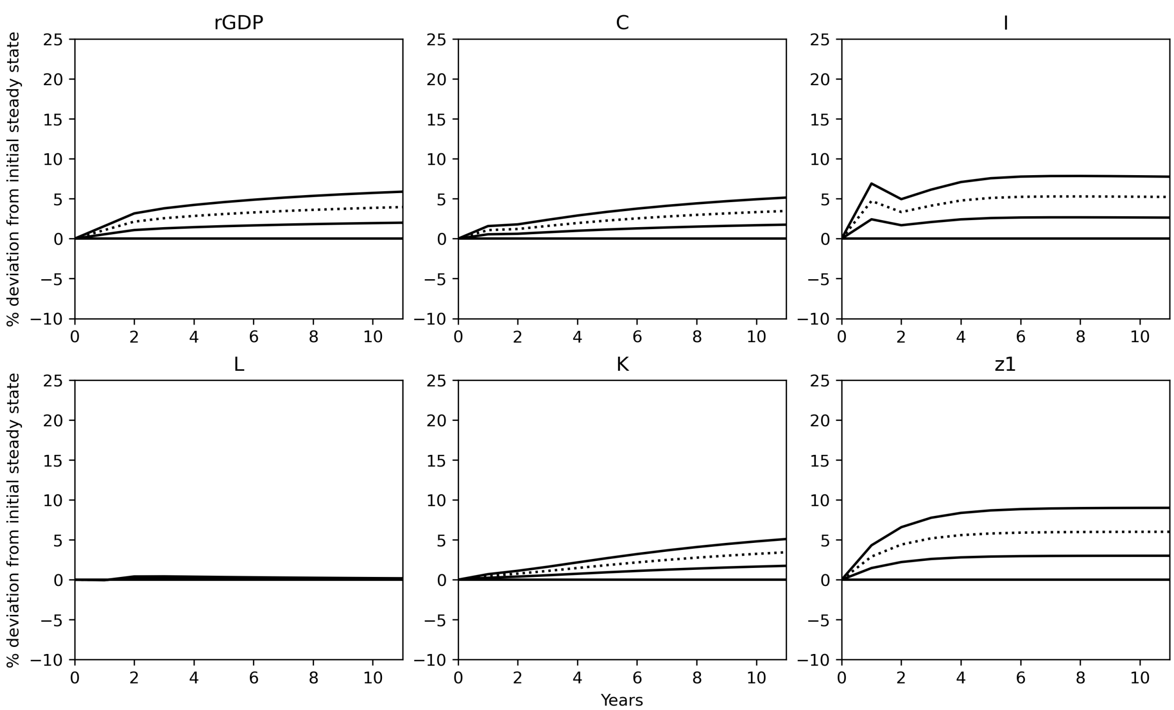


Figure 8: Simulation with 1.5 times higher / 0.5 times lower path than baseline with $\psi_j = 0$

6.5 The role of customer capital and production network in Steady State

Given the exogenous parameters we calibrated in Table 1, Table 2, Table 3, and Table 4, to understand the role of customer capital and production network, we compare the four types of models: (i) Baseline (ii) No Production Network model (iii) No Customer Search model (iv) No Production Network nor Customer Search model. To compute steady states without production network, we set intermediate goods input parameter $\alpha_j^m = 0$ and set α_k^j and α_l^j to be $\alpha_k^j = \alpha_l^j = 1$. For aggregate solow residuals, we define the aggregate real GDP $\text{rGDP}_t = \sum_j \text{rGDP}_{jt}$ and the aggregate intermediate input $X_t = \sum_j \left(\frac{\text{rGDP}_{jt}}{\text{rGDP}_t} \sum_m x_{jmt} \right)$ and then assume the following relationship:

$$\text{rGDP}_t = \text{Solow}_t K_{t-1}^{\alpha_k} L_t^{\alpha_l} X_t^{\alpha_x}, \quad \alpha_i \in [0,1]$$

and then we estimate input share parameters by non-negative least square using stochastic simulated data

$$\underset{\{\alpha_i\}}{\text{argmin}} \, ||\text{r}\hat{\text{GDP}}_t - (\alpha_0 + \alpha_1 \hat{K}_{t-1} + \alpha_2 \hat{L}_t + \alpha_3 \hat{X}_t)||_2^2, \quad \alpha_i \in [0,1]$$

Table 10 shows the steady state values in each model. Table 11 shows the percent deviation from the steady state with $z_j = 1$ after 6 p.p increase in z_1.

Table 10: Steady State value with $z_1 = z_2 = z_3 = 1$

Variable	Baseline	No Network	No Search	No Network nor Search	Variable	Baseline	No Network	No Search	No Network nor Search
c	0.0231	1.4334	0.0366	2.1114	w	0.0150	0.9191	0.0241	1.3462
i	0.0065	0.4268	0.0113	0.6887	r_k	0.2111	0.2111	0.2111	0.2111
l	1.0000	1.0000	1.0000	1.0000	k	0.0652	4.2680	0.1125	6.8868
l_1	0.2889	0.2028	0.1549	0.0192	k_1	0.0119	0.5147	0.0103	0.0713
l_2	0.2845	0.3365	0.3408	0.4216	k_2	0.0344	2.4998	0.0663	4.5868
l_3	0.4266	0.4607	0.5043	0.5592	k_3	0.0189	1.2535	0.0359	2.2288
G	0.0296	1.8602	0.0478	2.8001	p	1.0000	1.0000	1.0000	1.0000
y_1	0.0007	0.0225	0.0011	0.0311	p_1	0.6398	1.2179	0.6640	1.3140
y_2	0.0157	1.1915	0.0264	1.8987	p_2	1.0139	0.8519	0.9767	0.8089
y_3	0.0133	0.6770	0.0204	0.9518	p_3	0.9988	1.2079	1.0433	1.2853
v_1	0.5254	1.0325	0.6640	1.3140	q_1	0.5826	1.1252	-	-
v_2	0.8467	0.7025	0.9767	0.8089	q_2	0.9303	0.7772	-	-
v_3	0.8334	1.0163	1.0433	1.2853	q_3	0.9161	1.1121	-	-
x_{11}	0.0044	-	0.0030	-	d_1	0.0157	0.2655	-	-
x_{12}	0.0008	-	0.0007	-	d_2	0.0180	1.1915	-	-
x_{13}	0.0012	-	0.0010	-	d_3	0.0162	0.6770	-	-
x_{21}	0.0030	-	0.0056	-	a_1	0.0012	0.0203	-	-
x_{22}	0.0041	-	0.0068	-	a_2	0.0018	0.0532	-	-
x_{23}	0.0018	-	0.0032	-	a_3	0.0016	0.0433	-	-
x_{31}	0.0026	-	0.0048	-	m_1	0.0061	0.0561	-	-
x_{32}	0.0015	-	0.0029	-	m_2	0.0070	0.2518	-	-
x_{33}	0.0103	-	0.0157	-	m_3	0.0063	0.1431	-	-
F_1	0.0213	0.2858	0.0145	0.0311	W_1	0.0572	0.0927	-	-
F_2	0.0220	1.1915	0.0368	1.8987	W_2	0.0836	0.0747	-	-
F_3	0.0266	0.6770	0.0404	0.9518	W_3	0.0827	0.0958	-	-
$rGDP$	0.0326	2.0024	0.0478	2.8001	J_1	0.0572	0.0927	-	-
$rGDP_1$	0.0035	0.1696	0.0007	0.0409	J_2	0.0836	0.0747	-	-
$rGDP_2$	0.0159	1.0151	0.0257	1.5359	J_3	0.0827	0.0958	-	-
$rGDP_3$	0.0133	0.8178	0.0213	1.2233	$\mathcal{U}_1$	1.1089	1.0898	-	-
$Solow$	1.8703	0.6713	2.0196	0.6050	$\mathcal{U}_2$	1.0987	1.1063	-	-
$Solow_1$	0.1621	0.5933	0.0514	1.3140	$\mathcal{U}_3$	1.0992	1.0943	-	-
$Solow_2$	0.7204	0.8519	0.6988	0.8089	$\mathcal{M}_1$	0.0700	0.0700	-	-
$Solow_3$	0.4992	1.2079	0.5281	1.2853	$\mathcal{M}_2$	0.0700	0.0700	-	-
h	0.0499	2.1340	-	-	$\mathcal{M}_3$	0.0700	0.0700	-	-
Calibration									
ξ	0.1172	0.0687	0.6581	0.6376	δ_m^1	0.1988	0.3610	-	-
χ	0.6477	0.6412	-	-	δ_m^2	0.2607	0.2114	-	-
-	-	-	-	-	δ_m^3	0.2567	0.3025	-	-
Calibration for Aggregate Solow									
ρ_1	0.5110	0.5107	0.5405	0.5381	σ_1	0.0070	0.0070	0.0070	0.0070
ρ_2	0.6131	0.6249	0.6154	0.6563	σ_2	0.0070	0.0070	0.0070	0.0070
ρ_3	0.4149	0.4191	0.4419	0.4707	σ_3	0.0070	0.0070	0.0070	0.0070
ψ_1	35.0093	35.0306	-	-	ϕ	0.8756	1.0027	0.8756	0.9233
ψ_2	30.0447	29.9938	-	-	-	-	-	-	-
ψ_3	34.9813	35.0039	-	-	-	-	-	-	-
α_K	0.2375	0.7531	0.2408	0.7941	-	-	-	-	-
α_L	1.0000	1.0000	1.0000	1.0000	-	-	-	-	-
α_X	0.7522	-	0.8113	-	-	-	-	-	-

Table 11: z1 shock of 7% GDP increase in 10 years: % deviation from initial steady state

Variable	Baseline	No Network	No Search	No Network nor Search	Variable	Baseline	No Network	No Search	No Network nor Search
c	8.3021	3.6718	4.3100	0.3193	w	8.3668	3.6486	4.3088	0.3211
i	9.0483	4.4107	4.3136	0.3146	r_k	0.0000	0.0000	0.0000	0.0000
l	0.0299	-0.0112	-0.0006	0.0009	k	9.0483	4.4107	4.3136	0.3146
l_1	-4.0484	-7.0026	0.0997	1.0328	k_1	3.9797	-3.6095	4.4128	1.3572
l_2	1.6609	1.8360	0.0152	-0.0148	k_2	10.1667	5.5516	4.3247	0.3063
l_3	1.7040	1.7171	-0.0421	-0.0227	k_3	10.2134	5.4284	4.2648	0.2984
G	8.4664	3.8414	4.3109	0.3182	p	0.0000	0.0000	0.0000	0.0000
y_1	20.0229	13.6889	17.9156	12.3582	p_1	-8.7929	-7.9065	-10.5462	-9.7910
y_2	8.9540	4.1630	4.5092	0.1875	p_2	-0.4069	-0.2808	-0.1725	0.1185
y_3	7.5018	3.1288	3.6233	0.1007	p_3	0.8154	0.6279	0.6030	0.1975
v_1	-8.8608	-7.9121	-10.5462	-9.7910	q_1	-8.8235	-7.9090	-	-
v_2	1.1739	1.3331	-0.1725	0.1185	q_2	0.3125	0.4486	-	-
v_3	2.6467	2.2298	0.6030	0.1975	q_3	1.6484	1.3599	-	-
x_{11}	14.0889	-	16.7226	-	d_1	14.0547	4.6692	-	-
x_{12}	4.4045	-	4.5932	-	d_2	8.8884	4.1630	-	-
x_{13}	3.1388	-	3.7870	-	d_3	7.3715	3.1288	-	-
x_{21}	20.7875	-	16.6241	-	a_1	14.5292	4.7113	-	-
x_{22}	8.8884	-	4.5050	-	a_2	9.3415	4.2049	-	-
x_{23}	9.2757	-	3.6994	-	a_3	7.8183	3.1703	-	-
x_{31}	20.8386	-	16.5572	-	m_1	14.5292	4.7113	-	-
x_{32}	10.6636	-	4.4450	-	m_2	9.3415	4.2049	-	-
x_{33}	7.3715	-	3.6399	-	m_3	7.8183	3.1703	-	-
F_1	2.7102	-5.7673	5.0813	1.1521	W_1	-8.4815	-7.8751	-	-
F_2	8.8884	4.1630	4.5050	0.1875	W_2	-8.4134	-7.8694	-	-
F_3	7.3715	3.1288	3.6399	0.1007	W_3	-8.4134	-7.8694	-	-
$rGDP$	8.6313	3.8528	4.3235	0.3272	J_1	-8.4815	-7.8751	-	-
$rGDP_1$	11.4870	5.4873	17.9156	12.3582	J_2	-8.4134	-7.8694	-	-
$rGDP_2$	8.9540	4.1630	4.5092	0.1875	J_3	-8.4134	-7.8694	-	-
$rGDP_3$	7.5018	3.1288	3.6233	0.1007	$\mathcal{U}_1$	0.0409	0.0033	-	-
$Solow$	-1.6640	0.5427	-2.3770	0.0764	$\mathcal{U}_2$	-0.8514	-0.8728	-	-
$Solow_1$	8.5452	11.9433	12.2136	11.0784	$\mathcal{U}_3$	-0.9726	-0.8510	-	-
$Solow_2$	0.0602	0.0000	0.0040	0.0000	$\mathcal{M}_1$	0.3751	0.0369	-	-
$Solow_3$	0.1214	0.0000	-0.0160	0.0000	$\mathcal{M}_2$	-8.6988	-8.2809	-	-
h	10.0168	3.8979	-	-	$\mathcal{M}_3$	-9.8987	-9.1054	-	-

Note: In all the models, z1 reaches 11.1% increase after transition in line with 7% GDP increase in 10 years under the baseline model.

7 Conclusions

This paper uses a quantitative multi-sector model, which explicitly incorporates the impact of AI on customer build up, acquisition and retention across sectors, in order to explore the impact of AI on aggregate economic activity. Our findings provide a quantification of the so far unexplored channel by which gen-AI can improve customer acquisition and management. We find large spillover effects of productivity improvements in AI technology into all sectors in the economy, especially those for which customer base management and marketing activities are an important part of the production and sales process.

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Appendix

A Model Summary

A.1 Equations

There are 62 (=10+15+12+7+8+6+4) equations

A.1.1 Household (10(=3×1[y]+7) equations)

$$\xi \frac{l_t^{\eta}}{c_t^{-\sigma}} = w_t \tag{4}$$

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t[c_{t+1}^{-\sigma}] \tag{5}$$

$$\frac{c_t^{-\sigma}}{1-\phi\left(\frac{i_t}{k_{t-1}}-\delta\right)} = \beta \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)}\left\{1-\delta+r_{t+1}^K\left(1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\right)\right.\right.$$
$$\left.\left. + \phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\frac{i_{t+1}}{k_t}\right\}\right] \tag{6}$$

$$\Omega_{t,t+1} = \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tag{7}$$

$$c_t + i_t = G_t \tag{8}$$

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\phi}{2}k_{t-1}\left(\frac{i_t}{k_{t-1}}-\delta\right)^2 \tag{9}$$

$$P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = 1 \tag{10}$$

$$y_{jt} = \omega_j p_{jt}^{-\gamma} G_t \tag{11}$$

A.1.2 Producers (15(=3×2[$v, k/l$]+9×1[x/l]) equations)

$$v_{jt} = \left(\frac{1}{z_{jt}\varrho_j}(r_t^K)^{\alpha_k^j}(w_t)^{\alpha_l^j}\prod_{m\neq j} p_{mt}^{\alpha_m^j}\right)^{\frac{1}{1-\alpha_j^j}} \tag{12}$$

$$\frac{k_{jt}}{l_{jt}} = \frac{\alpha_k^j}{\alpha_l^j}\frac{w_t}{r_t^K} \tag{13}$$

$$\frac{x_{jmt}}{l_{jt}} = \begin{cases} \frac{\alpha_j^j}{\alpha_l^j}\frac{w_t}{v_{jt}} & (m = j) \\ \frac{\alpha_m^j}{a_l^j}\frac{w_t}{p_{mt}} & (m \neq j) \end{cases} \tag{14}$$

A.1.3 Customer Market: Producers (12(=3×4) equations)

$$\frac{h_t W_{jt}}{\sum_j m_{jt}} = \begin{cases} \frac{A_{jt}v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}v_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j=1) \\ \frac{A_{jt}p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}p_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j\neq 1) \end{cases} \tag{15}$$

$$d_{jt} = (1-\delta_H)d_{jt-1} + \frac{m_{jt}}{\sum_j m_{jt}}h_t \tag{16}$$

$$m_{jt} = (1-\delta_m^j)m_{jt-1} + a_{jt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2 \tag{17}$$

$$W_{jt} = q_{jt} - v_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}W_{jt+1}] \tag{18}$$

A.1.4 Customer Market: Retailers (7(=3×2[J, q]+1) equations)

$$J_{jt} = p_{jt} - q_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}J_{jt+1}] \tag{19}$$

$$q_{jt} = \theta p_{jt} + (1-\theta)v_{jt} \tag{20}$$

$$\sum_j \frac{m_{jt}}{\sum_j m_{jt}} J_{jt} = \chi p_{1t} \tag{21}$$

A.1.5 Market Cleaning (8(=3×2[F, d]+2) equations)

$$z_{jt}F_{jt} = = \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j=1) \\ y_{jt} + \sum_m x_{mjt} & (j\neq 1) \end{cases} \tag{22}$$

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m\neq j} x_{mjt} + \sum_{m\neq j} a_{mt} + \chi h_t & (j=1) \\ y_{jt} + \sum_{m\neq j} x_{mjt} & (j\neq 1) \end{cases} \tag{23}$$

$$\sum_j l_{jt} = l_t \tag{24}$$

$$\sum_j k_{jt} = k_{t-1} \tag{25}$$

A.1.6 Exogenous process (6(=3×2) equations)

$$\ln z_{jt} = (1-\rho_z^j)\ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \tag{26}$$

$$\ln A_{jt} = \rho_A \ln A_{jt-1} + \varepsilon_{jt}^A \tag{27}$$

A.1.7 Auxiliary variables (4(=3×1[F]+1) equations)

$$F_{jt} = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \tag{28}$$

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{29}$$

A.2 Variables

There are 62 (=15+18+15+4+6+4) variables

A.2.1 Household

$$\left\{c, l, w, i, k, R, r^K, P, \Omega\right\} + 3 \times \{y_j, p_j\} = 15$$

A.2.2 Producers

$$3 \times \{v_j, k_j, l_j\} + 9 \times \{x_{ij}\} = 18$$

A.2.3 Customer capital: Producers

$$3 \times \{d_j, a_j, m_j, q_j, W_j\} = 15$$

A.2.4 Customer Market: Retailers

$$\{h\} + 3 \times \{J_j\} = 4$$

A.2.5 Exogenous

$$3 \times \{z_j, A_j\} = 6$$

A.2.6 Auxiliary variables

$$\{G\} + 3 \times \{F_j\} = 4$$

B Deterministic Steady State

B.1 Summary of Steady State

B.1.1 Household

$$
\begin{aligned}
c &= G - i \\
l &= 1 \\
R &= \frac{1}{\beta} \\
r^K &= \frac{1}{\beta} - 1 + \delta \\
i &= \delta k \\
k &= \text{equation}(67) \\
w &= \text{equation}(66) \\
P &= 1 \\
\Omega &= \beta \\
y_j &= \omega_j p_j^{-\gamma} G
\end{aligned}
$$

B.1.2 Producers

$$
\begin{aligned}
v_j &= \frac{\theta}{\mathcal{U}_j - (1-\theta)} p_j \\
k_j &= \frac{\alpha_k^j \tau_j}{\varrho_j r^K} F_j \\
l_j &= \frac{\alpha_l^j \tau_j}{\varrho_j w} F_j \\
x_{jm} &= \begin{cases} \frac{\alpha_j^j \tau_j}{\varrho_j v_j} F_j & (m = j) \\ \frac{\alpha_m^j \tau_j}{\varrho_j p_m} F_j & (m \neq j) \end{cases}
\end{aligned}
$$

B.1.3 Customer capital: Producers

$$
\begin{aligned}
d_j &= \text{equation}(67) \\
a_j &= \delta_m m_j \\
m_j &= \frac{\delta_H(\mathcal{U}_1 - 1)}{(1-\beta(1-\delta_m^j)(1-\beta(1-\delta_H)} d_j
\end{aligned}
$$

$$
\begin{aligned}
q_j &= \mathcal{U}_j v_j \\
W_j &= \frac{\mathcal{U}_j - 1}{1 - \beta(1 - \delta_H)} v_j
\end{aligned}
$$

B.1.4 Customer Market: Retailers

$$
\begin{aligned}
h &= \delta_H \sum_j d_j \\
J_j &= \frac{1-\theta}{\theta} W_j \\
p_1 &= \left(\omega_1 + \omega_2 \left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1} \right)^{1-\gamma} + \omega_3 \left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1} \right)^{1-\gamma} \right)^{\frac{1}{\gamma - 1}} \\
p_2 &= \frac{\mathcal{U}_2 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1} p_1 \\
p_3 &= \frac{\mathcal{U}_3 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1} p_1
\end{aligned}
$$

B.1.5 Exogenous

$$
\begin{aligned}
z_j &= z_j^* \\
A_j &= 1
\end{aligned}
$$

B.1.6 Auxiliary variables

$$
\begin{aligned}
\mathcal{U}_2 &= \text{equation}(66) \\
\mathcal{U}_3 &= \text{equation}(66) \\
F_j &= \text{equation}(67) \\
G &= \text{equation}(67)
\end{aligned}
$$

B.1.7 Endogenous Parameters

$$
\begin{aligned}
\xi l^\eta &= w c^{-\sigma} \\
\chi &= \frac{1}{p_1} \sum_j \frac{m_j}{\sum_j m_j} J_j
\end{aligned}
$$

C log-linearization

Define $\hat{\cdot}_t = (\cdot_t - \cdot)/\cdot$

C.1 Household

$$\eta \hat{l}_t + \sigma \hat{c}_t = \hat{w}_t \tag{30}$$

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma}\hat{R}_t \tag{31}$$

$$-\sigma \hat{c}_t + \phi\delta(\hat{i}_t - \hat{k}_{t-1}) = \mathbb{E}_t\Big[-\sigma \hat{c}_{t+1} + \beta\phi\delta(\hat{i}_{t+1} - \hat{k}_t) + (1-\beta(1-\delta))\hat{r}^K_{t+1}\Big] \tag{32}$$

$$\hat{\Omega}_{t,t+1} = -\sigma(\hat{c}_{t+1} - \hat{c}_t) \tag{33}$$

$$\frac{c}{G}\hat{c}_t + \frac{i}{G}\hat{i}_t = \hat{G}_t \tag{34}$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta \hat{i}_t \tag{35}$$

$$\hat{P}_t = \sum_j \omega_j p_j^{1-\gamma}\hat{p}_{jt} = 0 \tag{36}$$

$$\hat{y}_{jt} = -\gamma \hat{p}_{jt} + \hat{G}_t \tag{37}$$

C.2 Producers

$$(1-\alpha_j^j)\hat{v}_{jt} = -\hat{z}_{jt} + \alpha_k^j \hat{r}_t^K + \alpha_l^j \hat{w}_t + \sum_{m\neq j}\alpha_m^j \hat{p}_{mt} \tag{38}$$

$$\hat{k}_{jt} - \hat{l}_{jt} = \hat{w}_t - \hat{r}_t^K \tag{39}$$

$$\hat{x}_{jmt} - \hat{l}_{jt} = \begin{cases} \hat{w}_t - \hat{v}_{jt} & (m=j) \\ \hat{w}_t - \hat{p}_{mt} & (m\neq j) \end{cases} \tag{40}$$

C.3 Customer Market: Producers

$$(1-\beta(1-\delta_m^j))\left(\hat{h}_t + \hat{W}_{jt} - \sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right) =$$
$$\begin{cases} \hat{A}_{jt} + \hat{v}_{1t} + \psi_j\delta_m^j(\hat{a}_{jt} - \hat{m}_{t-1}) \\ -\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1} + \hat{v}_{1t+1} + \hat{\Omega}_{t,t+1}) + \psi_j\delta_m^j(\hat{a}_{jt+1} - \hat{m}_{jt})\Big] & (j=1) \\ \hat{A}_{jt} + \hat{p}_{1t} + \psi_j\delta_m^j(\hat{a}_{jt} - \hat{m}_{t-1}) \\ -\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1} + \hat{p}_{1t+1} + \hat{\Omega}_{t,t+1}) + \psi_j\delta_m^j(\hat{a}_{jt+1} - \hat{m}_{jt})\Big] & (j\neq 1) \end{cases} \tag{41}$$

$$\hat{d}_{jt} = (1-\delta_H)\hat{d}_{jt-1} - \delta_H \sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt} + \delta_H(\hat{m}_{jt} + \hat{h}_t) \tag{42}$$

$$\hat{m}_{jt} = (1-\delta_m^j)\hat{m}_{jt-1} + \delta_m^j \hat{a}_{jt} \tag{43}$$

$$\hat{W}_{jt} = \frac{1}{W_j}(q_j\hat{q}_{jt} - v_j\hat{v}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{W}_{jt+1}\Big] \tag{44}$$

C.4 Customer Market: Retailers

$$\hat{J}_{jt} = \frac{1}{J_j}(p_j\hat{p}_{jt} - q_j\hat{q}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{J}_{jt+1}\Big] \tag{45}$$

$$q_j\hat{q}_{jt} = \theta p_j\hat{p}_{jt} + (1-\theta)v_j\hat{v}_{jt} \tag{46}$$

$$\sum_j m_j J_j(\hat{m}_{jt} + \hat{J}_{jt}) = \chi \sum_j p_1 m_j(\hat{p_{1t}} + \hat{m}_{jt}) \tag{47}$$

C.5 Market Cleaning

$$\hat{z}_{jt} + \hat{F}_{jt} = \begin{cases} \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} + \sum_m \frac{a_m}{F_j}\hat{a}_{mt} + \frac{\chi h}{F_j}\hat{h}_t & (j=1) \\ \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} & (j \neq 1) \end{cases} \tag{48}$$

$$\hat{d}_{jt} = \begin{cases} \frac{y_j}{d_j}\hat{y}_{jt} + \sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt} + \sum_{m\neq j} \frac{a_m}{d_j}\hat{a}_{mt} + \frac{\chi h}{d_j}\hat{h}_t & (j=1) \\ \frac{y_j}{d_j}\hat{y}_{jt} + \sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt} & (j \neq 1) \end{cases} \tag{49}$$

$$\sum_j \frac{l_j}{l}\hat{l}_{jt} = \hat{l}_t \tag{50}$$

$$\sum_j \frac{k_j}{k}\hat{k}_{jt} = \hat{k}_{t-1} \tag{51}$$

C.6 Exogenous process

$$\hat{z}_{jt} = \rho_z^j \hat{z}_{jt-1} + \varepsilon_{jt} \tag{52}$$

$$\hat{A}_{jt} = \rho_A \hat{A}_{jt-1} + \varepsilon_{jt}^A \tag{53}$$

C.7 Auxiliary variables

$$\hat{F}_{jt} = \alpha_k^j \hat{k}_{jt} + \alpha_l^j \hat{l}_{jt} + \sum_m \alpha_m^j \hat{x}_{jmt} \tag{54}$$

$$\hat{G}_t = \sum_j \omega_j p_j^{1-\gamma}\hat{y}_{jt} \tag{55}$$

D Marginal Cost of Cobb Douglas Function

Consider

$$\min \sum_j p_{jt}x_{jt} \quad \text{s.t.} \quad Y_t = z_t \prod_j x_{jt}^{\alpha_j} = z_t \varrho \prod_j \left(\frac{x_{jt}}{\alpha_j}\right)^{\alpha_j}, \quad \sum_j \alpha_j = 1, \quad \varrho = \prod_j \alpha_j^{\alpha_j}$$

FOCs yields

$$p_{jt} = \lambda_t \left[z_t \varrho \prod_j \left(\frac{x_{jt}}{\alpha_j}\right)^{\alpha_j} \right] \left(\frac{x_{jt}}{\alpha_j}\right)^{-1} \iff \frac{\lambda_t Y_t}{p_{jt}} = \frac{x_{jt}}{\alpha_j}$$

Insert in production function

$$Y_t = z_t \varrho \prod_j \left(\frac{\lambda_t Y_t}{p_{jt}}\right)^{\alpha_j} = z_t \varrho \frac{\lambda_t Y_t}{\prod_j p_{jt}^{\alpha_j}} \iff \lambda_t = \frac{\prod_j p_{jt}^{\alpha_j}}{z_t \varrho} = \frac{p_t}{z_t \varrho}$$

where $p_t \equiv \prod_j p_{jt}^{\alpha_j}$
Insert in FOCs

$$p_{jt} = \frac{p_t}{z_t \varrho} Y_t \left(\frac{x_{jt}}{\alpha_j}\right)^{-1} \iff p_{jt}x_{jt} = \alpha_j \frac{p_t}{z_t \varrho} Y_t$$

Taking sum over j

$$\sum_j p_{jt}x_{jt} = \frac{p_t}{z_t \varrho} Y_t$$

Therefore, the cost function is given by

$$C(\{p\}, Y) = \frac{p_t}{z_t \varrho} Y_t$$

Eventually the marginal cost is given by

$$MC(Y) = \frac{p_t}{z_t \varrho} = \frac{1}{z_t} \frac{\prod_j p_{jt}^{\alpha_j}}{\prod_j \alpha_j^{\alpha_j}}$$

Note that the implied factor demand is given by

$$x_{jt} = \frac{\alpha_j}{\prod_j \alpha_j^{\alpha_j}} \frac{\prod_j p_{jt}^{\alpha_j}}{p_{jt}} \frac{Y_t}{z_t}$$

E Derivation: Deterministic Steady State

Normalize

$$P = 1$$

From equation (7),

$$\Omega = \beta$$

From Euler Equations (5) and (6)

$$R = \frac{1}{\beta}, \quad r^K = \frac{1}{\beta} - 1 + \delta$$

From equations (26) and (27)

$$z_j = z_j^*, \quad A_j = 1$$

From law of motion for capital, equation (9),

$$i = \delta k \tag{56}$$

Assume gross wholesale markup as

$$\frac{q_j}{v_j} \equiv \mathcal{U}_j$$

Note that we treat $\mathcal{U}_1$ as calibration targets and determine χ as the endogenous parameter. From Nash Bargaining, equation (20), it yields retail prices

$$p_j = \frac{\mathcal{U}_j - 1 + \theta}{\theta} v_j \tag{57}$$

From the value for retailer, equation (19) and Nash Bargaining, equation (20)

$$J_j = \frac{p_j - q_j}{1 - \beta(1 - \delta_H)} = \frac{1 - \theta}{\theta} \frac{q_j - v_j}{1 - \beta(1 - \delta_H)} = \frac{1 - \theta}{\theta} W_j$$

From the law of motion for marketing capital equation (17) and make use of the fact that the adjustment cost is 0 in the steady state

$$a_j = \delta_m^j m_j \tag{58}$$

From equation (16), we obtain

$$\delta_H d_j = \frac{m_j}{\sum_j m_j} h \Longrightarrow \delta_H \sum_j d_j = h \tag{59}$$

Also, we obtain

$$\frac{d_i}{d_j} = \frac{m_i}{m_j}$$

Using equation (15)

$$\frac{hW_j}{\sum_j m_j} = \begin{cases} (1-\beta(1-\delta_m^j))v_1 & (j=1) \\ (1-\beta(1-\delta_m^j))p_1 & (j \neq 1) \end{cases} \tag{60}$$

From equation (18)

$$W_j = \frac{\mathcal{U}_j - 1}{1-\beta(1-\delta_H)} v_j \tag{61}$$

Combining equations (60) and (61) for $j = 1$

$$h = \frac{(1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))}{\mathcal{U}_1 - 1} \sum_j m_j \tag{62}$$

and using the above

$$\frac{hW_j}{\sum_j m_j} = \frac{\mathcal{U}_j - 1}{\mathcal{U}_1 - 1}(1-\beta(1-\delta_m^1))v_j$$

Comparison with equation (60) gives

$$v_j = \frac{\mathcal{U}_1 - 1}{\mathcal{U}_j - 1} \frac{1-\beta(1-\delta_m^j)}{1-\beta(1-\delta_m^1)} p_1, \quad (j \neq 1) \tag{63}$$

Using equation (57) and (63)

$$p_j = \frac{\mathcal{U}_j - 1 + \theta}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_j - 1} \frac{1 - \beta(1 - \delta_m^j)}{1 - \beta(1 - \delta_m^1)} p_1, \quad (j \neq 1) \tag{64}$$

Substitute into the aggregate price function, equation (10), we obtain the form of p_1 by $\mathcal{U}_j$

$$\begin{aligned}
1 &= \omega_1 (p_1)^{1-\gamma} + \omega_2 (p_2)^{1-\gamma} + \omega_3 (p_3)^{1-\gamma} \\
&= \Bigg(\omega_1 + \omega_2 \left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1} \frac{1 - \beta(1 - \delta_m^2)}{1 - \beta(1 - \delta_m^1)} \right)^{1-\gamma} \\
&\qquad + \omega_3 \left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1} \frac{1 - \beta(1 - \delta_m^3)}{1 - \beta(1 - \delta_m^1)} \right)^{1-\gamma} \Bigg) p_1^{1-\gamma} \\
\iff p_1 &= \Bigg(\omega_1 + \omega_2 \left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1} \frac{1 - \beta(1 - \delta_m^2)}{1 - \beta(1 - \delta_m^1)} \right)^{1-\gamma} \\
&\qquad + \omega_3 \left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta} \frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1} \frac{1 - \beta(1 - \delta_m^3)}{1 - \beta(1 - \delta_m^1)} \right)^{1-\gamma} \Bigg)^{\frac{1}{\gamma-1}}
\end{aligned} \tag{65}$$

From the marginal cost of producers and the relationship between the retail price and the marginal cost

$$\begin{aligned}
v_j^{1-\alpha_j^j} &= \frac{1}{\varrho_j} (r^K)^{\alpha_k^j} (w)^{\alpha_l^j} \prod_{m \neq j} p_m^{\alpha_m^j} \\
\iff \left(\frac{\theta}{\mathcal{U}_j - 1 + \theta} p_j \right)^{1-\alpha_j^j} &= \frac{1}{z_j^* \varrho_j} (r^K)^{\alpha_k^j} (w)^{\alpha_l^j} \prod_{m \neq j} p_m^{\alpha_m^j}
\end{aligned}$$

Thus we obtain

$$\begin{aligned}
(1-\alpha_1^1)(\ln\theta - \ln(\mathcal{U}_1 - 1 + \theta) + \ln p_1) &= -\ln z_1^* - \ln \varrho_1 + \alpha_k^1 \ln r^k + \alpha_l^1 \ln w + \alpha_2^1 \ln p_2 + \alpha_3^1 \ln p_3 \\
(1-\alpha_2^2)(\ln\theta - \ln(\mathcal{U}_2 - 1 + \theta) + \ln p_2) &= -\ln z_2^* - \ln \varrho_2 + \alpha_k^2 \ln r^k + \alpha_l^2 \ln w + \alpha_1^2 \ln p_1 + \alpha_3^2 \ln p_3 \\
(1-\alpha_3^3)(\ln\theta - \ln(\mathcal{U}_3 - 1 + \theta) + \ln p_3) &= -\ln z_3^* - \ln \varrho_3 + \alpha_k^3 \ln r^k + \alpha_l^3 \ln w + \alpha_1^3 \ln p_1 + \alpha_2^3 \ln p_2
\end{aligned} \tag{66}$$

Given $\{\mathcal{U}_j\}, \{\delta_m^j\}, \{\omega_j\}, \theta, \beta$, we can compute $\{p_j\}$ using equations (64) and (65). The equilibrium values of $\{\mathcal{U}_2, \mathcal{U}_3, w\}$ solve the above system. $\{v_j, q_j\}$ are given by

$$v_j = \frac{\theta}{\mathcal{U}_j - (1-\theta)} p_j$$

$$q_j = \mathcal{U}_j v_j$$

From equations (22), (58), (59), and (62)

$$F_j = \begin{cases} y_j + \sum_m x_{mj} + \sum_j \delta_m^j m_j + \chi h & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases}$$
$$= \begin{cases} y_j + \sum_m x_{mj} + \Delta \sum_j \delta_m^j d_j + \chi \delta_H \sum_j d_j & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases}$$
$$= \begin{cases} y_j + \sum_m x_{mj} + \sum_j \left(\Delta \delta_m^j + \frac{\delta_H}{p_1} J_j\right) d_j & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases}$$

where we use

$$\delta_H d_j = \frac{m_j}{\sum_j m_j} h \Longrightarrow \delta_H d_j = \frac{m_j}{\sum_j m_j} \frac{(1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))}{\mathcal{U}_1 - 1} \sum_j m_j \Longrightarrow \Delta d_j = m_j$$
$$\frac{m_j}{\sum_j m_j} = \frac{d_j}{\sum_j d_j} \Longrightarrow \sum_j \frac{d_j}{\sum_j d_j} J_j = \chi p_1 \Longrightarrow \frac{\delta_H}{p_1} \sum_j J_j d_j = \chi \delta_H \sum_j d_j$$
$$\Delta = \frac{\delta_H(\mathcal{U}_1 - 1)}{(1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))}$$

Also from equations (23), (58), (59), and (62)

$$d_j = \begin{cases} y_j + \sum_{m\neq j} x_{mj} + (\delta_m^2 m_2 + \delta_m^3 m_3) + \chi h & (j=1) \\ y_j + \sum_{m \neq j} x_{mj} & (j \neq 1) \end{cases}$$
$$= \begin{cases} y_j + \sum_{m\neq j} x_{mj} + \Delta(\delta_m^2 d_2 + \delta_m^3 d_3) + \sum_j \frac{\delta_H}{p_1} J_j d_j & (j=1) \\ y_j + \sum_{m \neq j} x_{mj} & (j \neq 1) \end{cases}$$

Combining the factor markets cleaning and the factor demands

$$\sum_j l_j = l, \quad \sum_j k_j = k$$

Therefore

$$x_{11} + (\Delta \delta_m^1 + 1) d_1 = z_1^* F_1$$
$$x_{22} + d_2 = z_2^* F_2$$

$$
\begin{aligned}
x_{33} + d_3 &= z_3^* F_3 \\
y_1 + x_{21} + x_{31} + \Delta\delta_m^2 d_2 + \Delta\delta_m^3 d_3 + \frac{\delta_H}{p_1} J_1 d_1 + \frac{\delta_H}{p_1} J_2 d_2 + \frac{\delta_H}{p_1} J_3 d_3 &= d_1 \\
y_2 + x_{12} + x_{32} &= d_2 \\
y_3 + x_{13} + x_{23} &= d_3 \\
l_1 + l_2 + l_3 &= l \\
k_1 + k_2 + k_3 &= k
\end{aligned}
$$

Note that

$$
\begin{aligned}
y_j &= \omega_j p_j^{-\gamma} \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} = \omega_j p_j^{-\gamma} G \\
k_j &= \frac{\alpha_k^j \tau_j}{\varrho_j r^K} F_j \\
l_j &= \frac{\alpha_l^j \tau_j}{\varrho_j w} F_j \\
x_{jm} &= \begin{cases} \frac{\alpha_j^j \tau_j}{\varrho_j v_j} F_j & (m = j) \\ \frac{\alpha_m^j \tau_j}{\varrho_j p_m} F_j & (m \neq j) \end{cases}
\end{aligned}
$$

Combining the above equations and normalizing $l = 1$ gives

$$
\begin{pmatrix}
0 & \frac{\alpha_1^1 \tau_1}{\varrho_1 v_1} - z_1^* & 0 & 0 & \Delta\delta_m^1 + 1 & 0 & 0 & 0 \\
0 & 0 & \frac{\alpha_2^2 \tau_2}{\varrho_2 v_2} - z_2^* & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{\alpha_3^3 \tau_3}{\varrho_3 v_3} - z_3^* & 0 & 0 & 1 & 0 \\
\omega_1 p_1^{-\gamma} & 0 & \frac{\alpha_1^2 \tau_2}{\varrho_2 p_1} & \frac{\alpha_1^3 \tau_3}{\varrho_3 p_1} & \frac{\delta_H}{p_1} J_1 - 1 & \Delta\delta_m^2 + \frac{\delta_H}{p_1} J_2 & \Delta\delta_m^3 + \frac{\delta_H}{p_1} J_3 & 0 \\
\omega_2 p_2^{-\gamma} & \frac{\alpha_2^1 \tau_1}{\varrho_1 p_2} & 0 & \frac{\alpha_2^3 \tau_3}{\varrho_3 p_2} & 0 & -1 & 0 & 0 \\
\omega_3 p_3^{-\gamma} & \frac{\alpha_3^1 \tau_1}{\varrho_1 p_3} & \frac{\alpha_3^2 \tau_2}{\varrho_2 p_3} & 0 & 0 & 0 & -1 & 0 \\
0 & \frac{\alpha_l^1 \tau_1}{\varrho_1 w} & \frac{\alpha_l^2 \tau_2}{\varrho_2 w} & \frac{\alpha_l^3 \tau_3}{\varrho_3 w} & 0 & 0 & 0 & 0 \\
0 & \frac{\alpha_k^1 \tau_1}{\varrho_1 r^K} & \frac{\alpha_k^2 \tau_2}{\varrho_2 r^K} & \frac{\alpha_k^3 \tau_3}{\varrho_3 r^K} & 0 & 0 & 0 & -1
\end{pmatrix}
\begin{pmatrix} G \\ F_1 \\ F_2 \\ F_3 \\ d_1 \\ d_2 \\ d_3 \\ k \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}
\tag{67}
$$

A solution for the linear system above yields (if exists)

$$
\{G, F_1, F_2, F_3, d_1, d_2, d_3, k\}
$$

and then we obtain

$$c = G - \delta k$$

which determine the endogenous parameter ξ through the equation (4)

$$\xi l^{\eta} = w c^{-\sigma}$$

Using equations (1), (2), (3) and (11), we can obtain

$$\begin{aligned}
k_j &= \frac{\alpha_k^j \tau_j}{\varrho_j r^K} F_j \\
l_j &= \frac{\alpha_l^j \tau_j}{\varrho_j w} F_j \\
x_{jm} &= \begin{cases} \frac{\alpha_j^j \tau_j}{\varrho_j v_j} F_j & (m = j) \\ \frac{\alpha_m^j \tau_j}{\varrho_j p_m} F_j & (m \neq j) \end{cases} \\
y_j &= \omega_j (p_j)^{-\gamma} G
\end{aligned}$$

Using equations (59) and (62), we can derive

$$\begin{aligned}
m_j &= \frac{\delta_H (\mathcal{U}_1 - 1)}{(1 - \beta(1 - \delta_m^1))(1 - \beta(1 - \delta_H))} d_j \\
a_j &= \delta_m^j m_j \\
h &= \delta_H \sum_j d_j
\end{aligned}$$

The free entry and exit condition, equation (21), gives an endogenous parameter of the search cost χ

$$\chi = \frac{1}{p_1} \sum_j \frac{d_j}{\sum_j d_j} J_j$$

The marketing expenditure to sale ratio is given by

$$\begin{aligned}
\mathcal{M}_j &= \begin{cases} \frac{v_1 a_1}{q_1 d_1} & (j = 1) \\ \frac{p_1 a_j}{q_j d_j} & (j \neq 1) \end{cases} \\
&= \begin{cases} \frac{v_1}{q_1} \frac{\delta_m^1 m_1}{d_1} & (j = 1) \\ \frac{p_1}{q_j} \frac{\delta_m^j m_j}{d_j} & (j \neq 1) \end{cases}
\end{aligned}$$

$$= \begin{cases} \frac{\delta_m^1 \delta_H (\mathcal{U}_1 - 1)}{\mathcal{U}_1 (1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))} & (j=1) \\ \frac{\delta_m^j \delta_H (\mathcal{U}_j - 1)}{\mathcal{U}_j (1-\beta(1-\delta_m^j))(1-\beta(1-\delta_H))} & (j \neq 1) \end{cases}$$

F Derivation: log-linearization

F.1 Household

F.1.1 Consumption Labor choice

$$\xi \frac{l_t^\eta}{c_t^{-\sigma}} = w_t$$
$$\ln \xi + \eta \ln l_t = \ln w_t - \sigma \ln c_t$$
$$\eta \hat{l}_t + \sigma \hat{c}_t = \hat{w}_t$$

F.1.2 EE for Bond

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t[c_{t+1}^{-\sigma}]$$
$$-\sigma c^{-\sigma} \hat{c}_t = \beta R c^{-\sigma} \Big(\hat{R}_t - \sigma \mathbb{E}_t[\hat{c}_{t+1}] \Big)$$
$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma} \hat{R}_t$$

F.1.3 EE for capital

$$\frac{c_t^{-\sigma}}{1 - \phi_1(i_t, k_{t-1})} = \beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{1 - \phi_1(i_{t+1}, k_t)} \left\{ 1 - \delta + r_{t+1}^K (1 - \phi_1(i_{t+1}, k_t)) - \phi_2(i_{t+1}, k_t) \right\} \right]$$

Recall

$$\phi_1(i_t, k_{t-1}) = \phi \left(\frac{i_t}{k_{t-1}} - \delta \right), \quad \phi_2(i_t, k_{t-1}) = -\phi \left(\frac{i_t}{k_{t-1}} - \delta \right) \frac{i_t}{k_{t-1}}$$

Then

$$\frac{c_t^{-\sigma}}{1 - \phi \left(\frac{i_t}{k_{t-1}} - \delta \right)} = \beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{1 - \phi \left(\frac{i_{t+1}}{k_t} - \delta \right)} \left\{ 1 - \delta + \phi \left(\frac{i_{t+1}}{k_t} - \delta \right) \frac{i_{t+1}}{k_t} \right\} + r_{t+1}^K c_{t+1}^{-\sigma} \right]$$
$$c^{-\sigma} \Big(-\sigma \hat{c}_t + \phi \delta (\hat{i}_t - \hat{k}_{t-1}) \Big) = \beta \mathbb{E}_t \Big[c^{-\sigma} \Big(-\sigma (1-\delta) \hat{c}_{t+1} + \phi \delta (\hat{i}_{t+1} - \hat{k}_t) + r^K \hat{r}_{t+1}^K - \sigma r^K \hat{c}_{t+1} \Big) \Big]$$
$$c^{-\sigma} \Big(-\sigma \hat{c}_t + \phi \delta (\hat{i}_t - \hat{k}_{t-1}) \Big) = \beta \mathbb{E}_t \Big[-\sigma (1 - \delta + r^K) \hat{c}_{t+1} + \phi \delta (\hat{i}_{t+1} - \hat{k}_t) + r^K \hat{r}_{t+1}^K \Big]$$

$$-\sigma\hat{c}_t + \phi\delta(\hat{i}_t - \hat{k}_{t-1}) = \mathbb{E}_t\Big[-\sigma\hat{c}_{t+1} + \beta\phi\delta(\hat{i}_{t+1} - \hat{k}_t) + (1-\beta(1-\delta))\hat{r}^K_{t+1}\Big]$$

F.1.4 SDF

$$\begin{aligned}
\Omega_{t,t+1} &= \beta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \\
\Omega\hat{\Omega}_{t,t+1} &= \beta\sigma(-\hat{c}_{t+1} + \hat{c}_t) \\
\hat{\Omega}_{t,t+1} &= -\sigma(\hat{c}_{t+1} - \hat{c}_t)
\end{aligned}$$

F.1.5 Goods expenditure

$$\begin{aligned}
c_t + i_t &= \left(\sum_j \omega_j^{\frac{1}{\gamma}}(y_{jt})^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} = G_t \\
c\hat{c}_t + i\hat{i}_t &= G\hat{G}_t \\
\frac{c}{G}\hat{c}_t + \frac{i}{G}\hat{i}_t &= \hat{G}_t
\end{aligned}$$

F.1.6 LOM for capital

$$\begin{aligned}
k_t &= (1-\delta)k_{t-1} + i_t - \frac{\phi}{2}k_{t-1}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2 \\
k\hat{k}_t &= (1-\delta)k\hat{k}_{t-1} + i\hat{i}_t \\
\hat{k}_t &= (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_t
\end{aligned}$$

F.1.7 Aggregate Price

$$\begin{aligned}
P_t &= \left[\sum_j \omega_j p_{jt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = 1 \\
P\hat{P}_t &= \frac{1}{1-\gamma}\left[\sum_j \omega_j p_j^{1-\gamma}\right]^{\frac{1}{1-\gamma}-1}(1-\gamma)\omega_j p_j^{1-\gamma}\hat{p}_{jt} \\
\hat{P}_t &= \sum_j \frac{\omega_j p_j^{1-\gamma}}{\sum_j \omega_j p_j^{1-\gamma}}\hat{p}_{jt} \\
\hat{P}_t &= \sum_j \omega_j p_j^{1-\gamma}\hat{p}_{jt} = 0
\end{aligned}$$

F.1.8 Demand Schedule

$$
\begin{aligned}
y_{jt} &= \omega_j p_{jt}^{-\gamma} \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} = \omega_j p_{jt}^{-\gamma} G_t \\
\ln y_{jt} &= \ln \omega_j - \gamma \ln p_{jt} + \ln G_t \\
\hat{y}_{jt} &= -\gamma \hat{p}_{jt} + \hat{G}_t
\end{aligned}
$$

F.2 Producers

F.2.1 Marginal cost

$$
\begin{aligned}
v_{jt}^{1-\alpha_j^j} &= \frac{1}{z_{jt}\varrho_j} (r_t^K)^{\alpha_k^j} (w_t)^{\alpha_l^j} \prod_{m \neq j} p_{mt}^{\alpha_m^j} \\
(1-\alpha_j^j) \ln v_{jt} &= -\ln z_{jt} - \ln \varrho_j + \alpha_k^j \ln r_t^K + \alpha_l^j \ln w_t \sum_{m \neq j} \alpha_m^j \ln p_{mt} \\
(1-\alpha_j^j) \hat{v}_{jt} &= -\hat{z}_{jt} + \alpha_k^j \hat{r}_t^K + \alpha_l^j \hat{w}_t + \sum_{m \neq j} \alpha_m^j \hat{p}_{mt}
\end{aligned}
$$

F.2.2 Capital Labor choice

$$
\begin{aligned}
\frac{k_{jt}}{l_{jt}} &= \frac{\alpha_k^j}{\alpha_l^j} \frac{w_t}{r_t^K} \\
\ln k_{jt} - \ln l_{jt} &= \ln \alpha_k^j - \ln \alpha_l^j + \ln w_t - \ln r_t^K \\
\hat{k}_{jt} - \hat{l}_{jt} &= \hat{w}_t - \hat{r}_t^K
\end{aligned}
$$

F.2.3 Intermediate Labor choice

$$
\begin{aligned}
\frac{x_{jmt}}{l_{jt}} &= \begin{cases} \frac{\alpha_j^j}{\alpha_l^j} \frac{w_t}{v_{jt}} & (m = j) \\ \frac{\alpha_m^j}{\alpha_l^j} \frac{w_t}{p_{mt}} & (m \neq j) \end{cases} \\
\ln x_{jmt} - \ln l_{jt} &= \begin{cases} \ln \alpha_j^j - \ln \alpha_l^j + \ln w_t - \ln v_{jt} & (m = j) \\ \ln \alpha_m^j - \ln \alpha_l^j + \ln w_t - \ln p_{mt} & (m \neq j) \end{cases} \\
\hat{x}_{jmt} - \hat{l}_{jt} &= \begin{cases} \hat{w}_t - \hat{v}_{jt} & (m = j) \\ \hat{w}_t - \hat{p}_{mt} & (m \neq j) \end{cases}
\end{aligned}
$$

F.3 Customer Market: Producers

F.3.1 Optimal Marketing Capital

$$\frac{h_t W_{jt}}{\sum_j m_{jt}} = \begin{cases} \frac{A_{jt}v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}v_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j=1) \\ \frac{A_{jt}p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}p_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j\neq 1) \end{cases}$$

For LHS

$$\frac{h_t W_{jt}}{\sum_j m_{jt}} \Longrightarrow \frac{hW_j}{\sum_j m_j}\left(\hat{h}_t + \hat{W}_{jt} - \sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right)$$

For RHS

$$\frac{A_{jt}v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} \Longrightarrow v_1\left(\hat{A}_{jt} + \hat{v}_{1t} + \psi_j\delta_m^j(\hat{a}_{jt} - \hat{m}_{t-1})\right)$$

$$\frac{A_{jt}p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} \Longrightarrow p_1\left(\hat{A}_{jt} + \hat{p}_{1t} + \psi_j\delta_m^j(\hat{a}_{jt} - \hat{m}_{t-1})\right)$$

and

$$\mathbb{E}_t\left[\frac{A_{jt+1}v_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]$$

$$\Longrightarrow v_1\beta\mathbb{E}_t\left[(1-\delta_m^j)(\hat{A}_{jt+1} + \hat{v}_{1t+1} + \hat{\Omega}_{t,t+1}) + \psi_j\delta_m^j(\hat{a}_{jt+1} - \hat{m}_{jt})\right]$$

$$\mathbb{E}_t\left[\frac{A_{jt+1}p_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]$$

$$\Longrightarrow p_1\beta\mathbb{E}_t\left[(1-\delta_m^j)(\hat{A}_{jt+1} + \hat{p}_{1t+1} + \hat{\Omega}_{t,t+1}) + \psi_j\delta_m^j(\hat{a}_{jt+1} - \hat{m}_{jt})\right]$$

Note that

$$\frac{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)} \Longrightarrow (\psi_j(\delta_m^j)^2 + (1-\delta_m^j)\psi_j\delta_m^j)(\hat{a}_{jt+1} - \hat{m}_{jt})$$

Thus

$$
\begin{aligned}
&(1-\beta(1-\delta_m^j))\left(\hat{h}_t+\hat{W}_{jt}-\sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right)= \\
&\qquad\begin{cases}
\hat{A}_{jt}+\hat{v}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{v}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j=1) \\
\hat{A}_{jt}+\hat{p}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{p}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j\neq 1)
\end{cases}
\end{aligned}
$$

F.3.2 LOM for Client List

$$
\begin{aligned}
d_{jt}\sum_j m_{jt} &= (1-\delta_H)d_{jt-1}\sum_j m_{jt}+m_{jt}h_t \\
\left(d_j\hat{d}_{jt}\sum_j m_j\right)+\left(d_j\sum_j m_j\hat{m}_{jt}\right) &= \left((1-\delta_H)d_j\hat{d}_{jt-1}\sum_j m_j\right)+\left((1-\delta_H)d_j\sum_j m_j\hat{m}_{jt}\right) \\
&\quad + m_j h(\hat{m}_{jt}+\hat{h}_t) \\
\hat{d}_{jt}\sum_j m_j+\sum_j m_j\hat{m}_{jt} &= (1-\delta_H)\hat{d}_{jt-1}\sum_j m_j+(1-\delta_H)\sum_j m_j\hat{m}_{jt} \\
&\quad + \delta_H\sum_j m_j(\hat{m}_{jt}+\hat{h}_t) \\
\hat{d}_{jt} &= (1-\delta_H)\hat{d}_{jt-1}-\delta_H\sum_j\frac{m_j}{\sum_j m_j}\hat{m}_{jt}+\delta_H(\hat{m}_{jt}+\hat{h}_t)
\end{aligned}
$$

F.3.3 LOM for Marketing Capital

$$
\begin{aligned}
m_{jt} &= (1-\delta_m^j)m_{jt-1}+a_{jt}-\frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)^2 \\
m_j\hat{m}_{jt} &= (1-\delta_m^j)m_j\hat{m}_{jt-1}+a_j\hat{a}_{jt} \\
\hat{m}_{jt} &= (1-\delta_m^j)\hat{m}_{jt-1}+\delta_m^j\hat{a}_{jt}
\end{aligned}
$$

F.3.4 Wholesaler Value

$$
\begin{aligned}
W_{jt} &= q_{jt}-v_{jt}+(1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}W_{jt+1}] \\
W_j\hat{W}_{jt} &= q_j\hat{q}_{jt}-v_j\hat{v}_{jt}+(1-\delta_H)\beta W_j\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1}+\hat{W}_{jt+1}\Big]
\end{aligned}
$$

$$\hat{W}_{jt} = \frac{1}{W_j}(q_j\hat{q}_{jt} - v_j\hat{v}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{W}_{jt+1}\Big]$$

F.4 Customer Market: Retailers

F.4.1 Retailer Value

$$\begin{aligned}
J_{jt} &= p_{jt} - q_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}J_{jt+1}] \\
J_j\hat{J}_{jt} &= p_j\hat{p}_{jt} - q_j\hat{q}_{jt} + (1-\delta_H)\Omega J_j\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{J}_{jt+1}\Big] \\
\hat{J}_{jt} &= \frac{1}{J_j}(p_j\hat{p}_{jt} - q_j\hat{q}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{J}_{jt+1}\Big]
\end{aligned}$$

F.4.2 Nash Bargaining

$$\begin{aligned}
q_{jt} &= \theta p_{jt} + (1-\theta)v_{jt} \\
q_j\hat{q}_{jt} &= \theta p_j\hat{p}_{jt} + (1-\theta)v_j\hat{v}_{jt}
\end{aligned}$$

F.4.3 Free entry condition

$$\begin{aligned}
\sum_j \frac{m_{jt}}{\sum_j m_{jt}} J_{jt} &= \chi p_{1t} \\
\sum_j m_{jt}J_{jt} &= \chi p_{1t}\sum_j m_{jt} \\
\sum_j m_j J_j(\hat{m}_{jt} + \hat{J}_{jt}) &= \chi\sum_j p_1 m_j(\hat{p_{1t}} + \hat{m}_{jt})
\end{aligned}$$

F.5 Market Cleaning

F.5.1 Good Market

$$\begin{aligned}
z_{jt}F_{jt} = &= \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j=1) \\ y_{jt} + \sum_m x_{mjt} & (j \neq 1) \end{cases} \\
F_j(\hat{z}_{jt} + \hat{F}_{jt}) &= \begin{cases} y_j\hat{y}_{jt} + \sum_m x_{mj}\hat{x}_{mjt} + \sum_m a_m\hat{a}_{mt} + \chi h\hat{h}_t & (j=1) \\ y_j\hat{y}_{jt} + \sum_m x_{mj}\hat{x}_{mjt} & (j \neq 1) \end{cases} \\
\hat{z}_{jt} + \hat{F}_{jt} &= \begin{cases} \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} + \sum_m \frac{a_m}{F_j}\hat{a}_{mt} + \frac{\chi h}{F_j}\hat{h}_t & (j=1) \\ \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} & (j \neq 1) \end{cases}
\end{aligned}$$

F.5.2 Retail Market

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m \neq j} x_{mjt} + \sum_{m \neq j} a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_{m \neq j} x_{mjt} & (j \neq 1) \end{cases}$$

$$d_j \hat{d}_{jt} = \begin{cases} y_j \hat{y}_{jt} + \sum_{m \neq j} x_{mj} \hat{x}_{mjt} + \sum_{m \neq j} a_m \hat{a}_{mt} + \chi h \hat{h}_t & (j = 1) \\ y_j \hat{y}_{jt} + \sum_{m \neq j} x_{mj} \hat{x}_{mjt} & (j \neq 1) \end{cases}$$

$$\hat{d}_{jt} = \begin{cases} \frac{y_j}{d_j} \hat{y}_{jt} + \sum_{m \neq j} \frac{x_{mj}}{d_j} \hat{x}_{mjt} + \sum_{m \neq j} \frac{a_m}{d_j} \hat{a}_{mt} + \frac{\chi h}{d_j} \hat{h}_t & (j = 1) \\ \frac{y_j}{d_j} \hat{y}_{jt} + \sum_{m \neq j} \frac{x_{mj}}{d_j} \hat{x}_{mjt} & (j \neq 1) \end{cases}$$

F.5.3 Labor

$$\sum_j l_{jt} = l_t$$
$$\sum_j l_j \hat{l}_{jt} = l \hat{l}_t$$

F.5.4 Capital

$$\sum_j k_{jt} = k_{t-1}$$
$$\sum_j k_j \hat{k}_{jt} = k \hat{k}_{t-1}$$

F.6 Exogenous process

F.6.1 Productivity

$$\begin{aligned} \ln z_{jt} &= (1 - \rho_z^j) \ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \\ \ln \frac{z_j + \Delta z_{jt}}{z_j} &= (1 - \rho_z^j) \ln \frac{z_j^*}{z_j} + \rho_z^j \ln \frac{z_j + \Delta z_{jt-1}}{z_j} + \varepsilon_{jt} \\ \ln(1 + \hat{z}_{jt}) &= \rho_z^j \ln(1 + \hat{z}_{jt-1}) + \varepsilon_{jt} \\ \hat{z}_{jt} &= \rho_z^j \hat{z}_{jt-1} + \varepsilon_{jt} \end{aligned}$$

F.6.2 Investment Productivity

$$\ln A_{jt} = \rho_A \ln A_{jt-1} + \varepsilon_{jt}^A$$

$$\ln \frac{A_j + \Delta A_{jt}}{A_j} = \rho_A \ln \frac{A_j + \Delta A_{jt-1}}{A_j} + \varepsilon^A_{jt}$$
$$\ln\left(1 + \hat{A}_{jt}\right) = \rho_A \ln\left(1 + \hat{A}_{jt-1}\right) + \varepsilon^A_{jt}$$
$$\hat{A}_{jt} = \rho_A \hat{A}_{jt-1} + \varepsilon^A_{jt}$$

F.7 Auxiliary variables

F.7.1 F

$$F_{jt} = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j}$$
$$\ln F_{jt} = \alpha_k^j \ln k_{jt} + \alpha_l^j \ln l_{jt} + \sum_m \alpha_m^j \ln x_{jmt}$$
$$\hat{F}_{jt} = \alpha_k^j \hat{k}_{jt} + \alpha_l^j \hat{l}_{jt} + \sum_m \alpha_m^j \hat{x}_{jmt}$$

F.7.2 G

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_{jt}\right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$
$$G\hat{G}_t = G \sum_j \frac{\gamma}{\gamma-1} \left(\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_{jt}\right)^{\frac{\gamma-1}{\gamma}} \right)^{-1} \omega_j^{\frac{1}{\gamma}} \frac{\gamma-1}{\gamma} \left(y_j\right)^{\frac{\gamma-1}{\gamma}} \hat{y}_{jt}$$
$$G\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}}} \hat{y}_{jt}$$
$$\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}}} \hat{y}_{jt}$$
$$\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}} \left(\omega_j p_j^{-\gamma} \left(\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \right)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}} \left(y_j\right)^{\frac{\gamma-1}{\gamma}}} \hat{y}_{jt}$$
$$\hat{G}_t = \sum_j \omega_j p_j^{1-\gamma} \hat{y}_{jt}$$

G Detrend

Table 12: Parameters of productivity process (data period up to 2020)

	ρ_i			σ_i		
	Type1	Type2	Type3	Type1	Type2	Type3
KLEMS						
Hamilton filter ($h = 2, p = 4$)	0.498	0.552	0.476	0.0157	0.0101	0.0101
HP filter ($\lambda = 100$)	0.384	0.505	0.344	0.0107	0.0075	0.0081
Linear trend	0.876	0.831	0.534	0.0138	0.0091	0.0090
Cubic trend	0.551	0.563	0.450	0.0118	0.0077	0.0082
Fernald utility-adjusted TFP						
Hamilton filter ($h = 2, p = 4$)	0.492	-	-	0.0160	-	-
HP filter ($\lambda = 100$)	0.460	-	-	0.0108	-	-
Linear trend	0.920	-	-	0.0141	-	-
Cubic trend	0.862	-	-	0.0130	-	-

Figure 9: Hamilton filter 1987-2020

Figure 10: HP filter 1987-2020

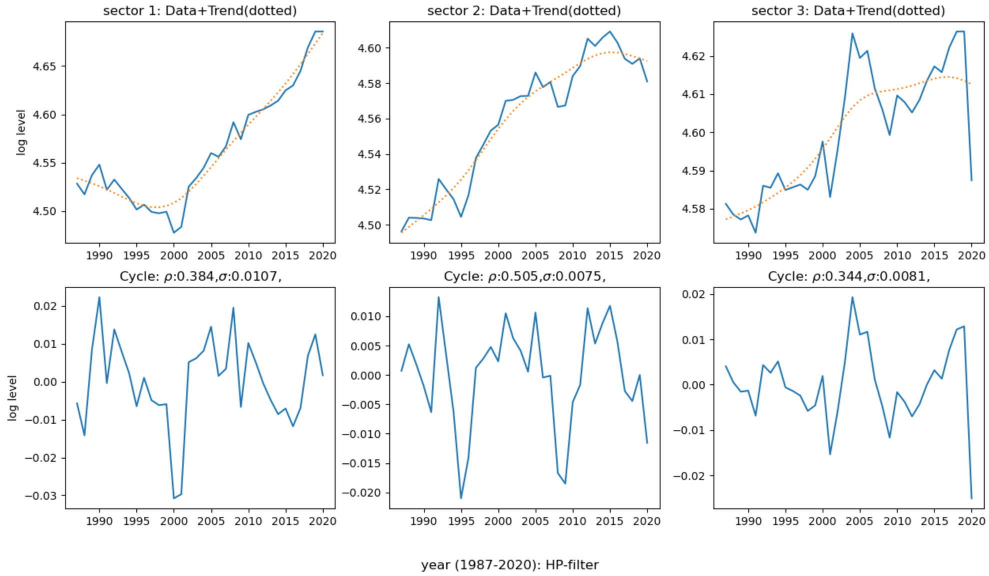


Figure 11: Linear Trend 1987-2020

Figure 12: Cubic Trend 1987-2020

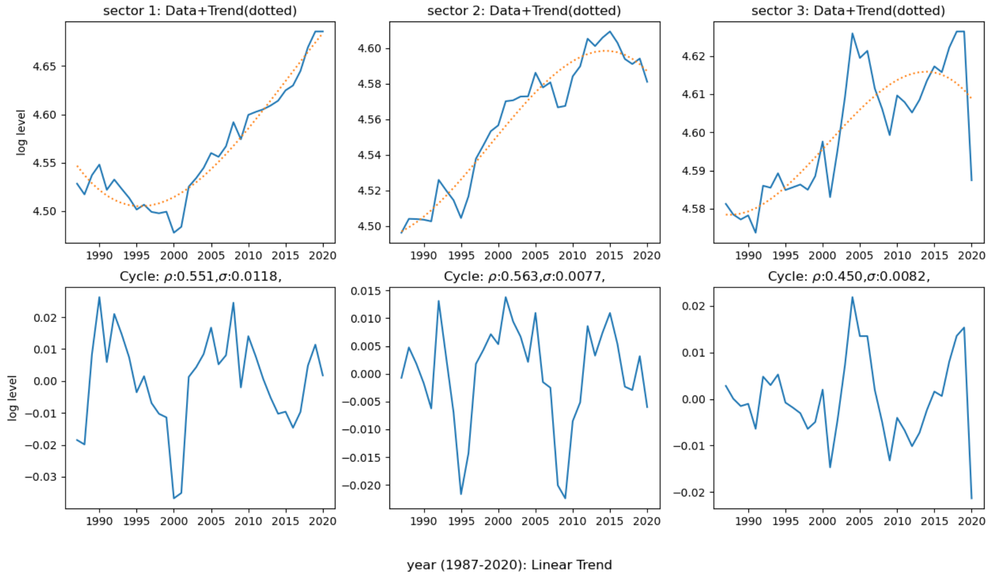


Figure 13: Fernald TFP 1947-2020

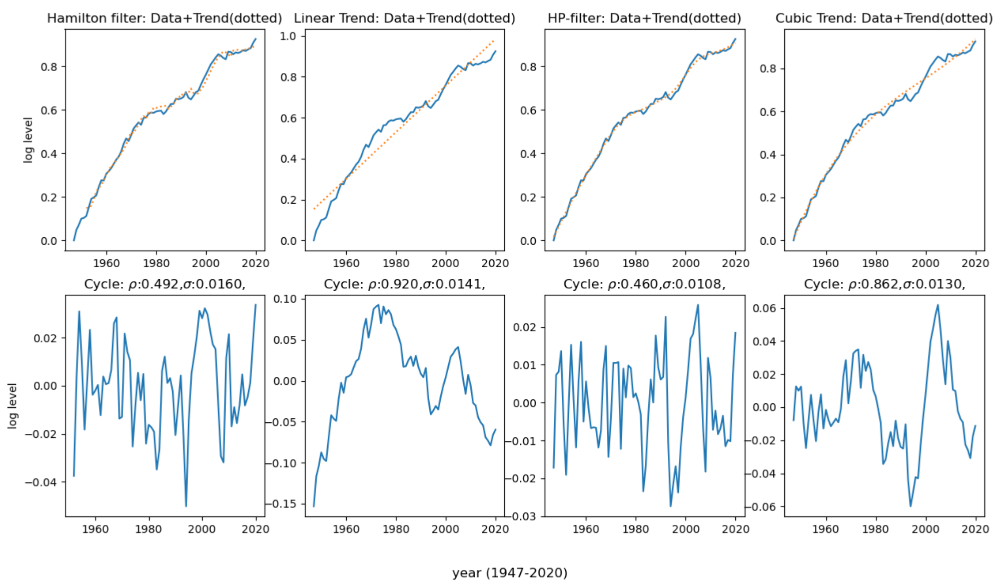
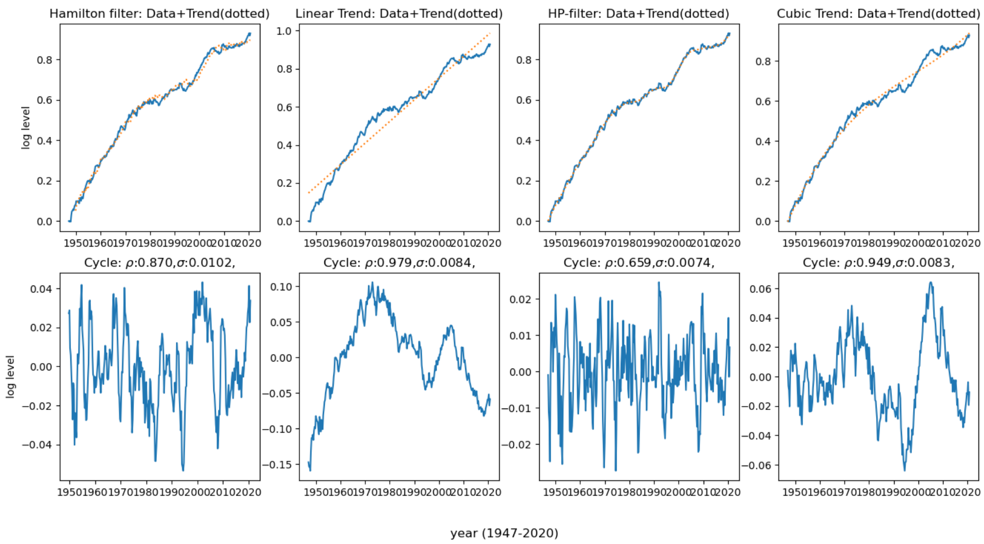


Figure 14: Fernald TFP 1947-2020 Quarterly



H Simulation

To understand the implication of the AI revolution on the economy, we simulate the 7 p.p. increase in GDP over a 10-year horizon through an increase in the AI-related sector productivity. Concretely, we simulate the dynamic progression such that the economy moves towards a new steady state where the GDP is 7 percentage points higher than the current steady state after a decade during the transition. To derive the corresponding new steady value of productivity, z_1^*, we implement numerical optimization that satisfies our target, 7 p.p. increase in GDP after a decade over the transition path.

I Industry Classification

Table 13: Industry Classification based on AI-exposure

NAICS 3 digit	NAICS code (IO table)	Industry Description	Type
111,112	111CA	Crop & animal production (Farms)	3
113-115	113FF	Forestry, fishing, and related activities	3
211	211	Oil and gas extraction	3
212	212	Mining, except oil and gas	3
213	213	Support activities for mining	3
22	22	Utilities	3
23	23	Construction	3
311,312	311FT	Food and beverage and tobacco products	3
313,314	313TT	Textile mills and textile product mills	3
315,316	315AL	Apparel and leather and applied products	3
322	322	Paper products	3
323	323	Printing and related support activities	3
324	324	Petroleum and coal products	3
325	325	Chemical products	3
326	326	Plastics and rubber products	3
321	321	Wood products	3
327	327	Nonmetallic mineral products	3
331	331	Primary metal products	3
332	332	Fabricated metal products	3
333	333	Machinery	3
334	334	Computer and electronic products	3
335	335	Electrical equipment, appliances, and components	3
3361-3363	3361MV	Motor vehicles, bodies and trailers, and parts	3
3364-3369	3364OT	Other transportation equipment	3
337	337	Furniture and related products	3
339	339	Miscellaneous manufacturing	3
42	42	Wholesale trade	2
44,45	441	Retail trade	2
481	481	Air transportation	3
482	482	Rail transportation	3
483	483	Water transportation	3
484	484	Truck transportation	3
485	485	Transit and ground passenger transportation	3
486	486	Pipeline transportation	3
487,488,492	487OS	Other transportation and support activities	3
493	493	Warehousing and storage	3
511	511	Publishing industries, except internet (includes software)	1
512	512	Motion picture and sound recording industries	1
515,517	513	Broadcasting and telecommunications	1
518,519	514	Data processing, internet publishing, and other information services	1
521,522	521CI	Federal reserve banks, credit intermediation, and related activities	2
523	523	Securities, commodity contracts, and other financial investments and related activities	2
524	524	Insurance carriers and related activities	2
525	525	Funds, trusts, and other financial vehicles	2
531	HS	Real estate	2
531	ORE	Real estate	2
532,533	532RL	Rental and leasing services and lessors of nonfinancial and intangible assets	2
5411	5411	Legal services	2
5415	5415	Computer systems design and related services	1
5412-5414,5416-5419	5412OP	Miscellaneous professional, scientific, and technical services	1
55	55	Management of companies and enterprises	1
561	561	Administrative and support services	1
562	562	Waste management and remediation services	3
61	61	Educational services	2
621	621	Ambulatory health care services	3
622,623	622	Hospitals and nursing and residential care facilities	3
622,623	623	Hospitals and nursing and residential care facilities	3
624	624	Social assistance	3
711,712	711AS	Performing arts, spectator sports, museums, and related activities	3
713	713	Amusements, gambling, and recreation industries	3
721	721	Accommodation	3
722	722	Food services and drinking places	3
81	81	Other services, except government	3

J PCE Bridge Table

Table 14: An excerpt of PCE Bridge Table of 2020 (Millions of dollars)

PCE Category	Commodity Code (NAICS 3 digit)	Producers' Value	Transportation Costs	Trade Margins Wholesale	Trade Margins Retail	Purchasers' Value
New motor vehicles	3361MV	215,969	3,261	6,158	67,520	292,908
Net purchases of used motor vehicles	Used	106,168	1,626	3,471	56,643	167,908
Motor vehicles parts and accessories	327	100	17	15	69	201
Motor vehicles parts and accessories	331	17	1	2	11	31
Motor vehicles parts and accessories	332	634	21	311	436	1,401
Motor vehicles parts and accessories	333	0	0	0	0	1
Motor vehicles parts and accessories	334	37	0	20	25	83
Motor vehicles parts and accessories	335	2,258	26	330	1,555	4,169
Motor vehicles parts and accessories	3361MV	20,277	254	2,951	13,965	37,447
Motor vehicles parts and accessories	339	266	37	86	183	572
Motor vehicles parts and accessories	325	269	5	27	149	450
Motor vehicles parts and accessories	326	17,539	379	4,327	11,996	34,241
Motor vehicles parts and accessories	Used	-264	12	1,280	824	1,853
Furniture and furnishings	321	4,138	292	757	6,481	11,667
Furniture and furnishings	327	3,439	534	1,488	1,795	7,256
Furniture and furnishings	331	256	8	29	481	774
Furniture and furnishings	332	804	26	327	1,513	2,670
Furniture and furnishings	334	1,248	35	340	2,015	3,638
Furniture and furnishings	335	2,585	358	782	4,212	7,936
Furniture and furnishings	337	59,555	8,295	17,896	52,092	137,837
Furniture and furnishings	339	6,505	565	2,403	6,472	15,945
Furniture and furnishings	313TT	9,618	543	5,887	19,374	35,422
Furniture and furnishings	315AL	12	1	5	20	38
Furniture and furnishings	323	265	13	30	427	735
Furniture and furnishings	326	499	10	145	908	1,562
Furniture and furnishings	532RL	15,819	0	0	0	15,819
Furniture and furnishings	Used	595	985	459	2,048	4,088
Household Appliances	331	4	0	0	3	7
Household Appliances	333	2,659	86	1,324	2,024	6,093
Household Appliances	335	27,662	1,137	8,280	20,309	57,389
Household Appliances	326	5	0	2	11	18
Household Appliances	Used	-132	14	3	6	-110
Glassware, Tableware, and Household Utensils	321	514	36	85	240	875
Glassware, Tableware, and Household Utensils	327	3,576	573	2,085	2,082	8,316
Glassware, Tableware, and Household Utensils	331	258	8	29	120	416
Glassware, Tableware, and Household Utensils	332	4,496	145	1,783	2,115	8,539
Glassware, Tableware, and Household Utensils	339	243	3	168	113	527
Glassware, Tableware, and Household Utensils	326	17,155	345	2,108	8,007	27,615
Glassware, Tableware, and Household Utensils	Used	-1,827	120	56	41	-1,610
Tools and Equipment for House and Garden	321	5	0	1	9	14
Tools and Equipment for House and Garden	327	630	175	80	349	1,234
Tools and Equipment for House and Garden	339	385	53	237	737	1,411
Tools and Equipment for House and Garden	313TT	12	1	5	31	49
Tools and Equipment for House and Garden	325	2,493	71	423	2,177	5,164
Tools and Equipment for House and Garden	326	200	4	44	524	771
Tools and Equipment for House and Garden	532RL	1,778	0	0	0	1,778
Tools and Equipment for House and Garden	Used	52	70	22	37	180
Video, Audio, Photographic, and Information Processing Equipment and Media	333	1,042	34	2,959	1,351	5,385
Video, Audio, Photographic, and Information Processing Equipment and Media	334	82,835	920	22,803	37,644	144,202
Video, Audio, Photographic, and Information Processing Equipment and Media	335	1,631	19	493	1,143	3,286
Video, Audio, Photographic, and Information Processing Equipment and Media	339	10	1	1	52	65
Video, Audio, Photographic, and Information Processing Equipment and Media	315AL	3	0	1	3	7
Video, Audio, Photographic, and Information Processing Equipment and Media	511	70,780	721	17,005	15,177	103,684
Video, Audio, Photographic, and Information Processing Equipment and Media	512	7,198	53	1,660	8,106	17,018
Video, Audio, Photographic, and Information Processing Equipment and Media	514	10,039	0	0	0	10,039
Video, Audio, Photographic, and Information Processing Equipment and Media	Used	-58	124	42	35	144
Sporting Equipment, Supplies, Guns, and Ammunition	331	142	5	16	133	296
Sporting Equipment, Supplies, Guns, and Ammunition	332	9,202	294	3,383	8,523	21,402
Sporting Equipment, Supplies, Guns, and Ammunition	3364OT	4,432	49	278	2,493	7,252
Sporting Equipment, Supplies, Guns, and Ammunition	339	20,819	2,673	7,501	21,159	52,153
Sporting Equipment, Supplies, Guns, and Ammunition	313TT	2,112	134	866	1,988	5,100
Sporting Equipment, Supplies, Guns, and Ammunition	315AL	142	7	61	134	345
Sporting Equipment, Supplies, Guns, and Ammunition	325	2,133	64	303	2,007	4,506
Sporting Equipment, Supplies, Guns, and Ammunition	Used	-65	0	0	0	-65
Sports and Recreational Vehicles	332	10	0	6	7	23
Sports and Recreational Vehicles	333	44	1	9	29	84
Sports and Recreational Vehicles	335	0	0	0	0	1
Sports and Recreational Vehicles	3361MV	19,146	226	658	7,340	27,370
Sports and Recreational Vehicles	3364OT	20,017	214	2,784	11,951	34,966
Sports and Recreational Vehicles	326	1,471	32	365	1,009	2,878
Sports and Recreational Vehicles	Used	763	6,475	975	1,545	9,758
⋮	⋮	⋮	⋮	⋮	⋮	⋮

K Model Summary: No Customer Search

Household

$$\xi \frac{l_t^\eta}{c_t^{-\sigma}} = w_t \tag{68}$$

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t[c_{t+1}^{-\sigma}] \tag{69}$$

$$\frac{c_t^{-\sigma}}{1-\phi\left(\frac{i_t}{k_{t-1}}-\delta\right)} = \beta \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)}\left\{1-\delta+r_{t+1}^K\left(1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\right)+\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\frac{i_{t+1}}{k_t}\right\}\right] \tag{70}$$

$$\Omega_{t,t+1} = \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tag{71}$$

$$c_t + i_t = G_t \tag{72}$$

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\phi}{2}k_{t-1}\left(\frac{i_t}{k_{t-1}}-\delta\right)^2 \tag{73}$$

$$P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = 1 \tag{74}$$

$$y_{jt} = \omega_j p_{jt}^{-\gamma} G_t \tag{75}$$

Producers

$$p_{jt} = v_{jt} \tag{76}$$

$$p_{jt} = \left(\frac{1}{z_{jt}\varrho_j}(r_t^K)^{\alpha_k^j}(w_t)^{\alpha_l^j}\prod_{m\neq j} p_{mt}^{\alpha_m^j}\right)^{\frac{1}{1-\alpha_j^j}} \tag{77}$$

$$\frac{k_{jt}}{l_{jt}} = \frac{\alpha_k^j}{\alpha_l^j}\frac{w_t}{r_t^K} \tag{78}$$

$$\frac{x_{jmt}}{l_{jt}} = \frac{\alpha_m^j}{a_l^j}\frac{w_t}{p_{mt}} \tag{79}$$

Market Cleaning

$$z_{jt}F_{jt} = y_{jt} + \sum_m x_{mjt} \tag{80}$$

$$\sum_j l_{jt} = l_t \tag{81}$$

$$\sum_j k_{jt} = k_{t-1} \tag{82}$$

Exogenous process

$$\ln z_{jt} = (1-\rho_z^j)\ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \tag{83}$$

Auxiliary variables

$$F_{jt} = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \tag{84}$$

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{85}$$