EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 845 / DECEMBER 2007

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by Marie Hoerova

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1 I am grateful for comments and advice from Bruno Biais, David Easley, Huberto Ennis, Miquel Faig, Florian Heider, Todd Keister, B. Ravikumar, Assaf Razin, Karl Shell, Yi Wen, and seminar participants at the European University Institute, the European Central Bank, Erasmus University Rotterdam, Tilburg University, Cambridge University, the European Bank for Reconstruction and Development, the Bank of Canada, Cornell University, the Cornell/Pennsylvannia State University Macro Workshop, the 2005 Midwest Macro Meetings, the 2005 SAET Conference, and the 2005 EEA Congress. All remaining errors are my own. Financial support from the Center for Analytic Economics at Cornell University and the Thorne Fund is gratefully acknowledged. The views expressed in this paper do not necessarily reflect those of the European Central Bank or the Eurosystem.

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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

I analyze the role that asset markets play in the performance and stability of the run-prone banking sector. Banks insure consumers against privately observed liquidity shocks. Asset market investments insure consumers against losses from bank runs. If the probability of a run is small, then banks specialize fully into the provision of liquidity insurance: They provide a higher degree of liquidity insurance when compared to the economy with banks alone. If the probability of a run is high, consumers prefer to invest solely through the asset market. Insurance against runs provided by the market investment reduces consumers' incentives to run. Increased provision of liquidity insurance by banks has the opposite effect. I derive conditions under which the latter effect dominates and the probability of a run is higher than with banks alone.

JEL classification: E44, G21

Keywords: Bank runs; Asset markets; Liquidity; Financial stability; Mechanism design

Non-Technical Summary

Financial arrangements based on liquid liabilities and illiquid assets can be prone to panic-based runs. Adverse shocks to fundamentals often precede runs. Once a spiralling liquidity crisis is in place, even solvent institutions may fail. A disruption of the efficient allocation of savings into investment then ensues. Runs are not necessarily a thing of the past, as illustrated by a run on the Abacus Federal Savings Bank in the United States (April 2003) and the recent financial market turmoil triggered by the crisis in the sub-prime mortgage market.

The mechanisms behind panic-based runs are well-known since Diamond and Dybvig (1983). They show that bank deposits can provide insurance against privately observed liquidity shocks. However, the provision of liquidity insurance comes at the risk of losses due to a panic-based bank run. The reason is that if all depositors at once demand their withdrawals, the bank will not be able to serve all of them and will fail. In the short run, there is a mismatch between banks' assets, which are illiquid, and banks' liabilities, which are liquid.

This classic model is based on a bank in which the consumer deposits all of his assets in exchange for a bank demand deposit contract. This raises the question: Would it be better to expand the banking contract to a banking-financial contract in which a fraction of deposits can be invested in an asset market portfolio? I argue that market portfolio investment can act as an insurance in the event of a run.

I find that banks have more flexibility in their decisions about investments and deposit returns in an economy with an asset market. This is for two reasons. First, banks can adjust their portfolio in the interim period by trading in the asset market. Second, depositors who lose their bank deposits in the event of a run are insured through their market investment. This has the following implications for the performance and stability of the run-prone banking sector.

Banks choose to provide a higher degree of liquidity insurance when compared to the economy with banks alone for small run probabilities. They fully specialize into liquidity provision, which constitutes their comparative advantage. Also, banks economize on liquid asset holdings.

The stability of banks in the economy with an asset market is determined by two forces. Insurance against runs provided by the market investment reduces consumers' incentives to run. Increased provision of liquidity insurance by banks has the opposite effect. I derive conditions under which the latter effect dominates and the probability of a run is higher than with banks alone. Thus, there is a tradeoff between diversification and the risk of runs.

Despite higher instability under the banking-financial contract, diversification using both bank deposits and market investments is welfare-improving when consumers are sufficiently risk-averse. A key to implementing this welfare-improving contract as an equilibrium in an economy with banks and asset markets is to ensure that consumers use bank withdrawals to satisfy their liquidity needs and not for trading purposes. I discuss various implementation mechanisms.

1 Introduction

Financial arrangements based on liquid liabilities and illiquid assets can be prone to panic-based runs. Adverse shocks to fundamentals often precede runs. Once a spiralling liquidity crisis is in place, even solvent institutions may fail. A disruption of the efficient allocation of savings into investment then ensues. Some of the banking crises which occurred in the past were, at least in part, panic-based.[1] A recent financial market turmoil triggered by the crisis in the sub-prime mortgage market illustrates that maturity mismatch can lead to a confidence crisis also outside the traditional banking sector. It has been, in fact, compared to an old style bank run.[2]

The mechanisms behind panic-based runs are well-known since Diamond and Dybvig (1983). They show that bank deposits can provide insurance against privately observed liquidity shocks. However, the provision of liquidity insurance comes at the risk of losses due to a panic-based bank run. The reason is that if all depositors at once demand their withdrawals, the bank will not be able to serve all of them and will fail. In the short run, there is a mismatch between banks' assets, which are illiquid, and banks' liabilities, which are liquid. This classic model is based on a bank in which the consumer deposits *all* of his assets in exchange for a bank demand deposit contract. This raises the question: Would it be better to expand the banking contract to a banking-financial contract in which a fraction of deposits can be invested in an asset market portfolio? An investment in a market portfolio cannot provide liquidity insurance since shocks are observed privately and therefore it is impossible to write type-contingent insurance contracts. I argue, however, that market portfolio investment can act as insurance in the event of a run. It ensures positive consumption regardless of whether or not a consumer is able to recover his bank deposit in the event of a run. The possibility to insure against runs affects the degree of liquidity insurance provided by bank deposits, the probability of a run on banks, and welfare.

Competitive financial intermediaries design a banking-financial mechanism (contract) and offer it to consumers in exchange for their endowments. The endowment

[1] A run on the Abacus Federal Savings Bank in the United States (April 2003) shows that runs are not necessarily a thing of the past. The run was triggered by rumors of embezzlement. Those turned out to be false. The crisis spread through the bank's six branches in New York and Philadelphia. For more details, see Federal Reserve Bank of Richmond Region Focus (2005). For a summary of the empirical evidence concerning whether runs are panic-based or fundamentals-based see Allen and Gale (2007).

[2] This parallel was drawn by both the Governor of the Bank of England Mervyn King and the Bundesbank president Axel Weber. See, e.g., Financial Times, ft.com, September 2, 2007.

can be split between a bank deposit contract and an asset market portfolio. The former is used for insurance against private-information liquidity shocks and may be subject to losses from a panic-based run (I refer to the bank deposit part of the banking-financial contract as a "bank"). The latter is not used for liquidity insurance and is not subject to runs.

I compare outcomes in the banking-financial economy to outcomes in the economy with banks alone (my benchmark). The main results can be summarized as follows. First, I show that for high risks of panic-based runs, consumers prefer to pass up on liquidity insurance and place all savings in the asset market portfolio. Bank deposits susceptible to runs simply become too risky. Deposits immune to runs are not offered since they provide no liquidity insurance and are inferior to the market allocation.

Second, I show that if the probability of a run is small, then banks fully "specialize" into the provision of liquidity insurance, i.e. banks provide as much liquidity insurance as the incentive-compatibility constraint allows them to. This implies that when faced with the same probability of a run, bank deposits provide a higher degree of liquidity insurance in the banking-financial economy when compared to the economy with banks alone. The intuition is as follows: The higher the degree of liquidity insurance provided, the higher is the number of consumers not served by the bank in the event of a run, *ceteris paribus*. In the economy with banks alone, those consumers that are not served end up with zero consumption level. Therefore in the optimum, banks need to "minimize" the number of consumers they leave behind in a run. By contrast, in the banking-financial economy, intermediaries are less concerned about the number of consumers not served in a run since those consumers are insured through the market investment. Banks can increase liquidity provision.

Third, I find that under some conditions the probability of run in the banking-financial economy is higher than in the economy with banks alone. The probability of a run is determined by the contract and fundamentals. There are two effects working in opposite directions. A higher degree of liquidity insurance provided by banks in the banking-financial economy increases the incentives of consumers to run since smoother intertemporal consumption profile reduces the benefits of postponing a withdrawal. Insurance against runs provided by the market portfolio works towards reducing the incentives to run. For a range of parameters, consumers prefer smoother consumption profile to higher banking stability. This is because increasing consumption smoothing has a first-order effect on welfare.

Despite higher instability under the banking-financial contract, diversification between the bank deposits and the market investment is welfare-improving when consumers are sufficiently risk-averse. A key to implementing this welfare-improving contract as an equilibrium in an economy with banks and asset markets is to ensure that consumers use bank withdrawals to satisfy their liquidity needs and *not* for trading purposes. Otherwise, the so-called "Jacklin's critique" would apply (Jacklin (1987)): If liquidity insurance, i.e. a relatively high short-term return on withdrawals, leads consumers to arbitrage between banks and markets, then banks must lower this return until arbitrage is no longer profitable. One way to prevent arbitrage yet preserve liquidity insurance is to impose a fee which discourages using bank withdrawals for investing in the asset market *ex post*.

This paper is at the intersection of two literatures, the literature on panic-based bank runs and the growing literature analyzing the interaction between banks and markets. The former does not consider the possibility to invest in markets and the latter does not analyze panic-based runs.

The role of banks that I focus on is the provision of liquidity and maturity transformation.[3,4] Following Cooper and Ross (1998) and Peck and Shell (2003), I model bank runs as chance events: whether or not the bank run happens depends on the realization of an extrinsic variable, a sunspot. The probability of a run is a fixed, exogenous parameter of the economy. Bank runs emerge as equilibrium phenomena: Consumers choose to accept contracts which admit runs provided that the probability of a run is small.[5] This is because to eliminate runs, it is necessary to eliminate liquidity insurance and that leads to lower welfare. A common criticism of this approach is that the probability of a run is exogenous to the contract offered and to fundamentals. In an extension of the baseline model, I consider the case when the possibility of a sunspot-driven run is linked to the banking contract and fundamentals, following Ennis and Keister (2003). This enables a comparison between the probability of a run in the banking-financial economy and the economy with banks alone.[6]

The analysis in this paper complements the line of literature that focuses on

[3] Although banks perform a number of other important functions in the economy (lending, payments etc.), those are not considered here for the purpose of tractability.

[4] The literature on the liquidity provision is too extensive to provide a full review here. Allen and Gale (2007) provide a guide through the existing theoretical and empirical literature on financial crises. See also von Thadden (1999) for a survey of theories of the intertemporal allocation of funds through demand deposits and markets.

[5] Cooper and Ross (1998) show that this is the case for demand deposits and Peck and Shell (2003) consider a broad set of contracts.

[6] An alternative approach would be to consider purely fundamentals-driven runs (Goldstein and Pauzner (2005)). For more on the determination of the probability of a run see Section 5.

mechanisms that can eliminate bank runs in the economy with banks alone. Credible suspension of convertibility or deposit insurance schemes were shown to be welfare-improving measures that address the problem of runs. Papers analyzing suspension schemes and the role of sequential service include, e.g., Wallace (1988, 1990) and Green and Lin (2003). The importance of credibility is highlighted in Ennis and Keister (2007). Optimal design of these measures is crucial for their effectiveness in preventing runs.[7] Moreover, deposit insurance can contribute to instability of banks by encouraging moral hazard of bank managers (see Cooper and Ross (2002) for a theoretical argument and Demirgüç-Kunt and Detragiache (2002) for empirical evidence). This paper considers consumers' portfolio diversification as an alternative welfare-improving mechanism.

This work is also related to research on the interaction between banks and markets. Diamond (1997) examines the effects of financial development in a model with restricted participation in the asset market. Restricted participation leads to market illiquidity in the economy with markets alone. Adding banks to such an economy enhances the supply of liquidity to the asset market. In a recent paper, Allen and Gale (2004) build a model of a complex financial system and use it for evaluating government intervention and regulation of liquidity provision. They find that as long as markets are complete, there is no scope for welfare-improving government intervention to prevent financial crises.

The remainder of the paper is organized as follows. In Section 2, I describe the model. In Section 3, I characterize asset market equilibrium and I formulate the problem of financial intermediaries in the banking-financial economy. In Section 4, I present and discuss the results assuming that the probability of a run is fixed. In Section 5, I extend the baseline model and allow the probability of a run to vary depending on the contract offered. I compare the probability of a run in the banking and banking-financial economy. In Section 6, I discuss implementation of the optimal contract. I conclude in Section 7. All proofs are in Appendix.

2 The Model

There are three time periods, $t = 0, 1$, and 2, and a single homogeneous good. Consumption and asset returns are measured in terms of the good. There is no aggregate

[7] For example, a run on the Northern Rock bank in the United Kingdom in September 2007 occured because deposits were not covered fully and the pay out could take up to several month.

uncertainty about fundamentals of the economy.

2.1 Consumers

There is a $[0, 1]$ continuum of consumers, each lives for three periods. Every consumer has an endowment of 1 unit of the good in period 0.

Ex ante (as of period 0), consumers are identical. In period 1 consumers receive an idiosyncratic liquidity shock which causes some of them to become "early" consumers (they only value period-one consumption, $u = u(c_1)$) and some "late" consumers (who only value period-two consumption; if they receive good early they can costlessly store it, $u = u(c_1 + c_2)$). A consumer's realized type is private information. Probability of being an early type is $\lambda > 0$ (thus, probability of being a late type is $(1 - \lambda)$). As is standard in the literature, I invoke the "law of large numbers" convention and assume that the cross-sectional distribution of types is the same as the probability distribution λ. Hence, λ is also the fraction of consumers in population who face an urgent liquidity need in period 1.

Individuals maximize expected utility of consumption. Let $u(c_t)$ denote utility of a consumer in period t. The function $u(c)$ is twice continuously differentiable, strictly increasing, and strictly concave. Moreover, $u(0) = 0$ and the coefficient of the relative risk aversion ρ_R is greater than 1 for $c \in [1, R]$.[8]

2.2 Asset Structure

There are two real assets in the economy: a short-term (liquid) asset and a long-term (illiquid) asset. A short-term asset is represented by the costless storage technology: it offers a return equal to 1 after one period. A long-term asset yields a return equal to $R > 1$ after two periods. Note that the long-term asset is more productive than the short-term asset over the long-run. If the long-term asset needs to be liquidated in period 1, it can be sold in the asset market at the prevailing market price P_1. In an economy without the asset market, I assume that there exists a liquidation technology such that liquidation yields a fixed return L. I let L be equal to the corresponding P_1 when solving the model to facilitate the comparison of outcomes in economies with and without the asset market.

[8] The following utility function satisfies these properties, for example: $u(c) = \frac{(c+b)^{1-\rho} - b^{1-\rho}}{1-\rho}$, $b > 0$, $\rho > 1 + b$. Note that I cannot use the constant-relative-risk-aversion utility function $u(c) = \frac{c^{1-\rho_R}}{1-\rho_R}$, where $\rho_R > 1$ is the coefficient of the relative risk aversion. This function does not satisfy $u(0) = 0$.

Constant returns to scale technology allows consumers/financial intermediaries to transform one unit of the good into one unit of the short-term or long-term asset in period 0. Short sales are not allowed.

2.3 Financial Intermediaries

In period 0, financial intermediaries design a banking-financial mechanism, which I call a banking-financial contract. The deposits can be split between a bank deposit contract and an asset market portfolio. The former is used for insurance against private-information liquidity shocks and may be subject to losses from a panic-based run. The latter is not used for liquidity insurance and hence is not subject to runs.

I assume that financial intermediation industry is perfectly competitive.[9] Thus, in equilibrium, intermediaries make zero profits and their objective is to maximize the expected utility of depositors. Intermediaries offer consumers a banking-financial contract in exchange for their endowments. The contract specifies: 1) a split of the endowment between bank deposits and an asset market portfolio; 2) a composition of the market portfolio; 3) a composition of the banking portfolio; and 4) period-one and period-two returns on bank deposits.

There are no asset markets in period 0. In period 1, an asset market opens in which the short-term and the long-term assets can be traded. Hence, once the private-information shock is realized, a consumer can request a portfolio rebalancing transaction.

Banks offer demand deposits. Following Freeman (1988), I assume that depositors cannot observe the pattern of withdrawals until period 2. It follows that they would not accept any contract which makes payments contingent on the depositor's place in line since they cannot verify that the payment is correct. Thus, banks promise a fixed return to any depositor withdrawing in period 1. Banks stand up to this obligation unless they have run out of funds.[10] In period 2, whatever is left in the bank is divided equally among the remaining depositors. Banks can access the asset market in period 1 and readjust their portfolio holdings depending on their liquidity

[9] One can apply the analysis either to a competitive financial intermediation sector or a single competitive intermediary (since intermediaries are homogeneous). For consistency, I will use the former label throughout the paper.

[10] Diamond and Rajan (2000, 2001) show that demand deposit structure can arise endogenously as a commitment device for bankers to repay depositors who have otherwise no skills to collect investments made by banks. In an earlier contribution, Calomiris and Kahn (1991) argue that sequential service provides incentives for depositors to monitor bankers.

needs.

After consumers have made their deposits, the so-called "post-deposit game" begins. Each individual learns his type at the beginning of period 1 and decides whether to withdraw his bank deposit in period 1 or in period 2. Early types withdraw in period 1. Late consumers have a choice to withdraw in period 1 or in period 2. Banks always offers a contract which is incentive-compatible, i.e. a late consumer's payoff from withdrawing in period 2 is higher than his payoff from withdrawing early given that all other late consumers wait until period 2 to withdraw. Following most of the literature, I look at symmetric, pure-strategy equilibria to the game played by late consumers.

Two types of equilibria are possible. In the "no run" Nash equilibrium, early consumers withdraw in period 1 and late consumers wait until period 2 to make their withdrawals. However, whenever the bank chooses an incentive-compatible contract such that if *all* depositors showed up at the bank in period 1, it would not have enough resources to pay off the promised return to everyone, there exists another, bank run, Nash equilibrium. The reason is that if a late consumer believes that all other late consumers will try to withdraw in period 1, it is in his best interest to withdraw, too (and store the good to consume it in period 2). If he waited until period 2, his return on the bank deposit would be zero (the bank runs out of funds by the end of period 1). When a late consumer chooses to withdraw from the bank in period 1, I say that he "runs". I present the definition of a bank run equilibrium below.

Definition 1 *Panic-based bank run equilibrium is a Nash equilibrium to the post-deposit game such that each late consumer finds it optimal to withdraw in period* 1 *because he believes that all other late consumers will withdraw in period* 1 *and hence the bank will run out of funds in period* 1*. Thus,* ***both*** *types withdraw from the bank in period* 1*.*

Intermediaries can always offer a contract which is immune to runs (the so called run-proof contract). However, I will show that this contract provides no liquidity insurance. Under the run-proof contract, the expected payoff of a late consumer from withdrawing in period 2 is at least as high as his expected payoff from withdrawing in period 1 regardless of when other late consumers make their withdrawals. This induces a unique (no run) equilibrium in the post-deposit game.

Whenever there are two equilibria to the post-deposit game, we need a device that will coordinate consumers' actions. A common approach in the literature is to assume an existence of a publicly observed sunspot signal.[11] After consumers have made their deposits, a number σ is drawn from a uniform distribution on $[0, 1]$. The draw itself is unrelated to any other variable in the economy. Given σ, late consumers behave according to the following decision rule:

$$\text{run if } \sigma \leq \pi,$$
$$\text{don't run otherwise}$$

for some number $\pi \in [0, 1]$. Variable π is the probability of a run outcome. Naturally, a run-proof contract induces $\pi = 0$ and the sunspot realization is ignored by individuals. Note that depending on the realization of σ, there are two possible states of the world in period 1: a no run state $(\sigma > \pi)$ and a run state of the world.

For now, I assume that π as a fixed, exogenous parameter of the economy following Peck and Shell (2003). Banks take π as given when choosing the optimal contract and consumers have rational expectations of the following form: They expect a bank run with exogenous probability π whenever the banking contract admits a run equilibrium. In Section 5, I allow π to vary depending on the properties of the contract offered and fundamentals of the economy.

The timing of events is summarized in Table 1, p. 31.

3 The Banking-Financial Economy

In this section, I first characterize an asset market equilibrium in period 1. The price of the asset will depend on the amount of liquidity in the market, following Allen and Gale (1994).[12] I then formulate the problem of financial intermediaries in period 0.

I am using the following notation. Let W denote a fraction of the consumer's endowment that is invested in a market portfolio. Let α be a fraction of this amount invested in the short-term asset in period 0 (the remaining fraction is invested in the long-term asset). Let β be a fraction of bank deposits invested in the short-term asset in period 0. Let c_1^B and c_2^B denote multiples of the period-one return and period-two returns on demand deposits, respectively.[13] Let θ denote a state of the world in period

[11] For the analysis of the sunspot equilibrium concept in general and how sunspots can be used to select among multiple equilibria in particular, see the seminal paper by Cass and Shell (1983).

[12] In the paper, they study an economy populated by heterogeneous consumers with Diamond-Dybvig preferences and examine volatility of asset prices in financial markets.

[13] I.e. a consumer is entitled to withdrawing an amount equal to $(1 - W)\, c_1^B$ from the bank in period 1. The period-two return on the withdrawal from the bank is equal to $(1 - W)\, c_2^B$.

1, where $\theta \in \{\text{no run, run}\}$.

3.1 Asset Market Equilibrium

The asset market opens in period 1, after private-information shocks are realized. I normalize the price of period-one consumption to 1. It is easy to show that the price of the short-term asset coincides with the price of period-one consumption and is equal to 1.[14] Let $P_1(\theta)$ denote the price of one unit of the long-term asset in terms of the period-one consumption in state θ. I assume that consumers and banks know and take as given price function P_1.

Consumers adjust their individual portfolio holdings given their type. Banks adjust their portfolio holdings given liquidity demand they face in period 1. In particular, if upon covering all the withdrawals in period 1, banks still have some short-term asset holdings (i.e. $\beta - \lambda c_1^B > 0$), they can use them to acquire more long-term asset in the asset market. On the contrary, if banks run out of the short-term asset before serving all withdrawals, they can acquire more liquidity in the asset market by selling off some or all of their long-term asset holdings.[15] Price $P_1(\theta)$ must then clear the asset market.

Before trading begins, each consumer holds a portfolio $(\alpha, 1-\alpha)$ and banks hold an (aggregate) portfolio $(\beta, 1-\beta)$. For now, I assume that aggregate holdings of the short-term and long-term asset are positive and that $P_1(\theta) \leq R$. I prove that this is indeed the case below (Proposition 1). It is easy to see that for any $P_1 > 0$, early consumers offer their entire long-term asset holdings for sale. Let $\gamma(P_1)$, $0 \leq \gamma(P_1) \leq 1$, denote a fraction of the long-term asset holdings banks need to liquidate in period 1. Note that $\gamma = 1$ in case there is a bank run. Let S be the total amount of the long-term asset early consumers and banks supply to the market in period 1. Then,

$$S \equiv \lambda(1-\alpha)W + \gamma(P_1)(1-\beta)(1-W).$$

For $P_1 \leq R$, late consumers keep their initial long-term asset holdings until period 2. They may choose to acquire more long-term asset using their short-term asset holdings. Let $a(P_1)$, $0 \leq a(P_1) \leq 1$, be the optimal fraction of the short-term asset holdings to be sold. The remaining part of the short-term asset hold-

[14] I thus talk about the short-term asset and period-one consumption interchangeably.

[15] Once the bank runs out of the short-term asset, it gives out "slips", i.e. promises of c_1^B, to consumers. This way, it finds out how much more liquiditty it needs to acquire.

ings, $(1-a(P_1))\alpha W \geq 0$ is reinvested for one more period. Similarly, let $b(P_1)$, $0 \leq b(P_1) \leq 1$, be the optimal fraction of the banks' excess short-term asset holdings to be sold, with $b(P_1) = 0$ whenever $\beta - \lambda c_1^B < 0$. Note that $a(P_1) = b(P_1) = 1$ if $P_1 < R$; and $a(P_1) \in [0,1]$ and $b(P_1) \in [0,1]$ if $P_1 = R$. Let D denote the total amount of the short-term asset supplied to the asset market:

$$D \equiv (1-\lambda)\, a(P_1)\, \alpha W + b(P_1)\left(\beta - \lambda c_1^B\right)(1-W).$$

The price that clears the market is given by

$$P_1(\theta)\, S = D,$$

i.e. the value of period 2 claims in period 1 has to equal to the value of period 1 claims in period 1.

I denote the equilibrium price of the long-term asset in the no run state of the world by P_1. I use P_1^L to denote the equilibrium price of the long-term asset in the run state of the world. In the run state of the world, banks liquidate their entire long-term asset holdings (superscript L stands for "liquidation") and $\gamma = 1$.

The next proposition provides characterization of an asset market equilibrium.

Proposition 1 *An asset market equilibrium satisfies the following properties: 1) the aggregate amount of the long-term assets held in period* 0 *is positive; 2) the aggregate amount of the short-term asset held in period* 0 *is positive; 3)* $P_1(\theta) \leq R$ *for all* θ*; 4) it must be the case that* $P_1 > 1$ *and* $P_1^L < 1$*.*

Given the expectations about the equilibrium price, it is optimal for consumers and banks to diversify their portfolios in period 0 by investing in both the short-term and the long-term asset.[16] The equilibrium price then reveals whether the state of the world is "run" or "no run". Moreover, the long-term asset is more valued in the "no run" state of the world, $P_1^L < 1 < P_1$.

3.2 Banking-Financial Contract

Intermediaries take prices as given and choose α, W, β, and demand deposit multiples $\left(c_1^B, c_2^B\right)$ to maximize the expected utility of consumers. Let $\underline{\lambda}$ denote a proportion

[16] Aggregate asset holdings are uniquely determined whereas individual asset holdings are not. I assume that the asset holdings are the same among consumers and banks, respectively.

of consumers who get c_1^B before the bank runs out of funds: $\underline{\underline{\lambda}} = \min\left\{\frac{\beta+P_1^L(1-\beta)}{c_1^B}, 1\right\}$, $c_1^B > 0$. Intermediaries' optimization problem (BFP) is formulated below:[17]

$$\max_{\alpha, W, \beta, c_1^B, c_2^B} (1-\pi)\left[\lambda u(c_1) + (1-\lambda) u(c_2)\right] + \pi\underline{\underline{\lambda}}\left[\lambda u\left(c_1^S\right) + (1-\lambda) u\left(c_2^S\right)\right] \quad (BFP)$$

$$+\pi\left(1-\underline{\underline{\lambda}}\right)\left[\lambda u\left(c_1^N\right) + (1-\lambda) u\left(c_2^N\right)\right] \text{ subject to}$$

$$0 \le \alpha \le 1,\ 0 \le \beta \le 1,\ 0 \le \gamma \le 1,\ 0 < W \le 1,$$

$$c_2^B = \begin{cases} \frac{(1-\gamma)(1-\beta)R}{1-\lambda} \text{ if } \lambda c_1^B - \beta > 0 \\ \frac{\left[\frac{\beta - \lambda c_1^B}{P_1} + 1 - \beta\right]R}{1-\lambda} \text{ otherwise} \end{cases}, \quad (RC)$$

$$c_2^W \le c_2. \quad (ICC)$$

With probability $(1-\pi)$ there is no bank run in period 1. Variables c_1 and c_2 represent the total consumption of an early and late consumer, respectively, in the no run state of the world. With probability π a bank run occurs in period 1. If a bank run occurs and a consumer is served, his consumption is equal to c_1^S if he is an early type and c_2^S if he is a late type. In case there is a run and a consumer is not served, his consumption is determined solely by the return on his market portfolio. I let c_1^N and c_2^N denote consumption allocations of an early and late type, respectively, for this case.

I summarize the definitions of the total consumption allocations below:

$$\begin{aligned}
c_1 &= \left[\alpha + (1-\alpha) P_1\right] W + (1-W) c_1^B, \\
c_2 &= \left[\left(\frac{a\alpha}{P_1} + 1 - \alpha\right) R + (1-a)\alpha\right] W + (1-W) c_2^B, \\
c_2^W &= \left[\left(\frac{a\alpha}{P_1} + 1 - \alpha\right) R + (1-a)\alpha\right] W + (1-W) c_1^B, \\
c_1^S &= \left[\alpha + (1-\alpha) P_1^L\right] W + (1-W) c_1^B, \\
c_2^S &= \left(\frac{\alpha}{P_1^L} + 1 - \alpha\right) RW + (1-W) c_1^B, \\
c_1^N &= \left[\alpha + (1-\alpha) P_1^L\right] W, \\
c_2^N &= \left(\frac{\alpha}{P_1^L} + 1 - \alpha\right) RW.
\end{aligned}$$

The resource constraint (RC) simply states that all funds that are in the bank in period 2 are divided evenly among the late depositors. The incentive-compatibility constraint (ICC) is defined in terms of total consumptions of a late consumer (it includes the corresponding return on the market portfolio).

[17] The first-order conditions for this problem are in the Appendix, p. 26.

In the economy with banks alone, the asset market is non-existent and the optimal contract is a triple $\left\{\beta, c_1^B, c_2^B\right\}$. In this case, banks must make sure that they have enough liquidity to at least cover period-one withdrawals, $\beta - \lambda c_1^B \geq 0$, as they cannot acquire more liquidity in the asset market. If banks choose to hold excess liquidity, they can costlessly store the unused balances until period 2. In the event of a run, they can liquidate their long-term asset holdings at a fixed value equal to $P_1^L < 1$.

Run-proof banking-financial contracts are contracts such that it is a dominant strategy for a late consumer to wait until period 2 to withdraw from the bank. I denote the run-proof banking-financial problem by (RPF). I let c_1^{RP} and c_2^{RP} denote the returns on run-proof deposits. Financial intermediaries maximize the following objective function:

$$\max_{\alpha, W, \beta^{RP}, c_1^{RP}, c_2^{RP}} \lambda u\left(c_1\right) + \left(1-\lambda\right) u\left(c_2\right) \tag{RPF}$$

subject to the set of constraints in problem (BFP), $c_1^{RP} \geq 0$, and the run-proof constraint:

$$c_1^{RP} \leq \beta + \left(1-\beta\right) P_1^L.$$

The constraint states that the expected payoff of a late consumer from withdrawing in period 2 is at least as high as his expected payoff from withdrawing in period 1 regardless of when other late consumers make their withdrawals. Thus, there is only one, no run, equilibrium to the post-deposit game.

4 Results and Discussion

Given consumers' preferences and asset structure, I now ask: What are the properties of the first-best allocation in an economy in which the realization of the idiosyncratic liquidity shock (early/late) is publicly observable? I consider a problem of a social planner who chooses how to split consumers' endowments between a short-term and a long-term asset to maximize consumers' expected utility. Let c_1^{FB} and c_2^{FB} be optimal period-one and period-two consumption allocations, respectively. In the next Lemma, I state the result that is well-known in the literature since Diamond and Dybvig (1983).

Lemma 1 *In the first-best allocation, there is a demand for ex ante liquidity insurance (a cross-subsidy between early and late consumers), i.e.* $c_1^{FB} > 1$ *and* $c_2^{FB} < R$.

The optimal allocation exhibits preference for a higher period-one consumption at the expense of a somewhat lower period-two consumption. Risk-averse consumers want *ex ante* insurance against the “bad luck” of becoming early consumers in period 1. Note that the allocation $c_1 = 1$ and $c_2 = R$ is feasible but is not chosen.

Let us return to the case when liquidity shocks are observed privately. If the probability of a run π *is set* to be zero, bank deposit contract alone can implement the first-best allocation. Hence, for $\pi = 0$, consumers choose to exchange their entire endowment for the bank deposit contract. What happens if the probability of a run is greater than zero? Next Proposition shows that in an economy where runs are a possibility, a mechanism which allows for an asset market investment is welfare-improving.

Proposition 2 *Suppose a bank run is possible, i.e. $\pi > 0$. If consumers are sufficiently risk-averse, they are better off in the banking-financial economy as compared to the economy with banks alone.*

The intuition behind Proposition 2 is as follows. Consider the economy with banks alone. If consumers are not served by the bank in the event of a run, their consumption is equal to zero regardless of their *ex post* type. When consumers are sufficiently risk-averse, they can achieve a large increase in their utility by investing even a small amount in an asset market portfolio. Of course, decreasing the amount deposited with banks leads to a relatively less smooth intertemporal consumption profile. However, this utility loss is small compared to the gains from ensuring that consumption is positive in all states of the world.

Proposition 3 *Consider a run-admitting banking-financial contract. If the probability of a run π is sufficiently high, then the entire endowment is invested in the market portfolio, i.e. the contract specifies $W^* = 1$.*

What consumption profile are consumers able to achieve if they invest their entire endowment in the asset market? It is easy to show that the asset market allocation has $c_1 = 1$ and $c_2 = R$. Liquidity insurance is impossible to achieve in the competitive market since type-contingent insurance contracts cannot be written. The intuition behind Proposition 3 is as follows: As the probability of a run increases, bank deposits become more risky. For small run probabilities, it is optimal to take advantage of liquidity insurance offered by bank deposits by investing part of the endowment in bank deposits and the rest in the market portfolio. However, for high run probabilities, it is optimal to pass up on liquidity insurance and invest everything in the market portfolio.

I now consider the case of a run-proof banking-financial contract.

Proposition 4 *The optimal run-proof banking-financial contract is weakly dominated by the contract which specifies that the entire endowment be invested in the market portfolio, i.e.* $W^* = 1$.

The formal proof is in appendix. Intuitively, the reason to deposit with banks is to acquire *ex ante* liquidity insurance. Since a run-proof contract eliminates liquidity insurance to prevent runs, it offers no advantage over the asset market allocation. More precisely, consumers' investment is immune to runs regardless of whether they invest in the market portfolio or deposit with the run-proof bank. If consumers invest the entire endowment in the market portfolio, early types are guaranteed $c_1 = 1$ and late types are guaranteed $c_2 = R$. To be run-proof, any contract must have $c_1 \leq 1$ and $c_2 \geq R$. Thus, it cannot deliver higher utility than the asset market allocation.

Propositions 3 and 4 together imply that there exists a threshold level of the probability of a run such that for all run probabilities above this threshold, the optimal allocation has $c_1 = 1$ and $c_2 = R$ and liquidity insurance is no longer provided.

I now examine properties of the run-admitting banking-financial contract along two dimensions: banks' portfolio holdings and the provision of liquidity insurance.

Proposition 5 *If the probability of a run is sufficiently small, then banks hold no excess liquidity, i.e.* $\lambda c_1^B - \beta \geq 0$.

The long-term asset is more productive than the short-term asset over the long run. However, holding substantial amounts of the long asset is costly in the run state of the world since banks need to liquidate their entire holdings. When the probability of a run is small, banks can invest freely in the long asset. If they need to acquire more liquidity in the no run state of the world, they can sell some of their long-term asset holdings at a favorable price $P_1 > 1$.

Next Proposition gives the conditions under which banks in the banking-financial economy choose to *fully* specialize into the provision of liquidity insurance.

Proposition 6 *Suppose that a consumer's endowment is split between the bank deposit and the market portfolio, i.e.* $W^* < 1$*. If* π *is sufficiently small, incentive-compatibility constraint* (ICC) *must bind in the optimum.*

In other words, banks provide as much liquidity as enabled by the incentive-compatibility constraint. Note that the fact that (ICC) binds implies that $c_1^B = c_2^B$ in the optimum. It is straightforward to show that incentive-compatibility constraint never binds in the optimum of the economy with banks alone, i.e. $c_1^B < c_2^B$. Thus, we have the following corollary.

Corollary 1 *Suppose that the optimal contract has $W^* < 1$. If π is sufficiently small, bank deposits provide a higher degree of liquidity insurance in the banking-financial economy when compared to the economy with banks alone.*

Note that banks are able to provide more liquidity insurance without holding excess liquidity for small run probabilities (Proposition 5). This is only possible due to the presence of an asset market where additional liquidity can be acquired when needed.

5 Probability of a Run

In this Section, I investigate how presence of the asset market affects banking sector instability. The goal is to compare the probability of a run that banks face in the economy with banks alone to that in the banking-financial economy. To do so, I abandon the assumption of the fixed probability of a run maintained so far. The probability of a run is now influenced by the contract intermediaries offer.

Two alternative ways to tie the probability of a run to the contract and fundamentals emerged in the literature so far. Goldstein and Pauzner (2005) use global games methodology, following Carlsson and van Damme (1993) and Morris and Shin (1998), to pin down a unique equilibrium in the post-deposit game. This approach requires introducing a continuum of noisy signals about fundamentals. A consumer's signal not only conveys information about the fundamentals but also about what other consumers will do. Depending on the signal realization, a late consumer decides whether or not to run on the bank. For a range of fundamentals, a different proportion of late consumers runs on the bank. Applying this approach to the environment considered in this paper would lead to a dramatic increase in the number of states of the world in period 1. The demand for liquidity and thus asset prices would differ across each of these states. This would complicate the analysis and make it difficult to derive analytical results.

An alternative approach was developed by Ennis and Keister (2003).[18] They modify a concept of risk dominance, used as an equilibrium selection mechanism in Harsanyi and Selten (1988), to link consumers' beliefs about the likelihood of a run to the parameters of the contract. There are only two states of the world in period 1: either all late consumers run or none does. The probability of a run depends on relative payoffs late consumers get under different scenarios (run/no run, served/not served). As there are only two states to consider, this approach offers a simpler way to model probability of a run in an environment with both banks and markets while still providing a link to the contract and fundamentals. Since my goal is a comparison between an economy with and without an asset market and not the determination of the probability of a run per se, I follow this approach here (see also Remark 3, p. 24).

The intuition behind the determination of the probability of a run is as follows: A late consumer has to decide whether or not to run on the bank. Running is only optimal if other late consumers run, too. This is the essence of panic-driven runs. Given that a late consumer does not know whether others will run, his incentive to run is related to his beliefs about how likely it is that other late consumers will run. I am looking for a cutoff level of a late consumer's beliefs about actions of others such that he is exactly indifferent between running and not running. The higher is the incentive to run, the higher is the *ex ante* probability of a run π that intermediaries face.

A technical device used to formalize a relation between π and the parameters of the contract is the so-called risk-factor of the run equilibrium, denoted by ϕ. The risk-factor represents how "risky" running is from the point of view of a late consumer. The riskiness stems from the fact that he does not know what other late consumers will do.

The risk-factor ϕ is determined as follows. Consider a contract $\left(\alpha, W, \beta, c_1^B, c_2^B\right)$ offered by financial intermediaries in period 0. Let ξ denote a late consumer's prior belief about the probability that all other late consumers will run in period 1. For $\xi = \phi$, a late consumer is indifferent between withdrawing in period 1 and period 2, i.e. the risk-factor is a cutoff level of ξ such that for all $\xi > \phi$ running is the unique optimal action.

I use ϕ^{BF} to denote the risk-factor of the run equilibrium in the banking-financial

[18] In the paper, they study how a possibility of a bank run affects capital formation and economic growth.

economy. It is determined by the following equation:

$$\phi^{BF}\left[\underline{\underline{\lambda}}u\left(c_2^S\right)+\left(1-\underline{\underline{\lambda}}\right)u\left(c_2^N\right)\right]+\left(1-\phi^{BF}\right)u\left(c_2^W\right)=\phi^{BF}u\left(c_2^N\right)+\left(1-\phi^{BF}\right)u\left(c_2\right).$$

The left hand side is a late consumer's expected payoff of running when he believes that with probability ϕ^{BF} all other late consumers will run. The right hand side gives the expected payoff of waiting until period 2 given the same belief. Rearranging yields

$$\phi^{BF}=\frac{u\left(c_2\right)-u\left(c_2^W\right)}{u\left(c_2\right)-u\left(c_2^W\right)+\underline{\underline{\lambda}}\left(u\left(c_2^S\right)-u\left(c_2^N\right)\right)}.$$

Note that $\phi^{BF}\in[0,1]$. If the risk-factor is low, probability that $\xi>\phi^{BF}$ increases and a late consumer chooses to run for a wider range of beliefs about the actions of other late consumers.

It is straightforward to derive the risk-factor for the economy with banks alone:

$$\phi^{B}=\frac{u\left(\widetilde{c}_2\right)-u\left(\widetilde{c}_1\right)}{u\left(\widetilde{c}_2\right)-u\left(\widetilde{c}_1\right)+\underline{\underline{\lambda}}u\left(\widetilde{c}_1\right)},$$

where $\widetilde{c}_1$ and $\widetilde{c}_2$ are the returns on bank withdrawals in the economy with banks alone.

Note that the risk-factor in the banking-financial economy is determined by: 1) the difference between $u\left(c_2\right)$ and $u\left(c_2^W\right)$ and 2) the difference between $u\left(c_2^S\right)$ and $u\left(c_2^N\right)$. The former reflects the degree of liquidity insurance provided (the smaller is the difference, the higher is the degree of liquidity insurance provided). The latter reflects the degree of insurance against runs provided by the market portfolio (the smaller the difference, the higher is the degree of insurance against runs). In the economy with banks alone, if there is a run and a consumer is not served, his consumption is equal to zero. In particular, $c_2^N=0$. Thus, there is a significant difference between the consumption level that a consumer gets if he is served by the bank $(\widetilde{c}_1)$ and if he is not (0). In the banking-financial economy, this difference is smaller.

The lower ϕ is, the higher is a late consumer's *ex ante* incentive to run. Hence, I assume that probability of a run π is a decreasing function of the risk-factor ϕ:

$$\pi\left(\phi\right)=m-hf\left(\phi\right),$$

where $m>0$, $h\geq 0$, $f'(\phi)>0$, and $\frac{m-1}{h}<f\left(\phi\right)<\frac{m}{h}$ must hold for all ϕ to ensure

$0 < \pi < 1$. Essentially, this amounts to re-scaling ϕ which is determined endogenously. I am not interested in the value of π. My aim is to compare the probability of a run that banks face in the economy with banks alone to the probability of a run in the banking-financial economy. Note that a special case of the constant probability of a run considered in the previous Section is obtained for $h = 0$ and $m \in (0, 1)$.

Whether or not the run actually happens depends on the realization of the sunspot variable σ (if $\sigma \leq \pi$, a bank run takes place). This approach retains Diamond and Dybvig (1983) spirit that bank runs are to some extent chance events (this is represented by the randomness of the sunspot variable σ). At the same time, portfolio decisions of banks and fundamentals of the economy affect the determination of the *ex ante* probability of a run. This allows to model bank runs as chance events after taking fundamentals into account.

I consider systemic runs, i.e. the situation when the probability of a run on a particular bank is determined by the economy-wide (average) contract. When choosing the optimal contract, intermediaries know and take as given the relation between their contract and the risk-factor ϕ and thus the probability of a run π. The probability of a run is determined by the rational expectations condition that requires the probability of a run, which is taken as given by intermediaries, to be the same as the probability implied by the contract all intermediaries choose.

Three remarks are in order. First, note that it is only sensible to consider how risky is running on the bank if the banking allocation $\left(c_1^B, c_2^B\right)$ admits two equilibria to the post-deposit game. If it is always optimal for a late consumer to run, then $\phi = 0$ (this is the case of a contract which is not incentive-compatible).

Remark 2: What is the relation between the risk-factor of the run equilibrium and a fixed probability of a run considered in the previous Section? The difference lies in the type of payoffs which a late consumer takes into account when deciding whether or not to run on the bank. In principle, there are four payoffs to consider: 1) a payoff from withdrawing in period 1 given that other late consumers withdraw in period 2; 2) a payoff from withdrawing in period 2 given that others withdraw in period 2; 3) a payoff from withdrawing in period 1 given that others run; and 4) a payoff from withdrawing in period 2 given that others run. Note that the incentive-compatibility requires that the first payoff does not exceed the second. When the probability of a run is fixed, only first two payoffs enter the decision-making through the incentive-compatibility constraint. In the risk-factor approach, all four payoffs play a role in the analysis. They are combined in the way described above.

Remark 3: In Goldstein and Pauzner (2005), a late consumer also compares his payoff from running versus his payoff from not running given his expectations about fundamentals and what other late consumers will do. Extending their model to allow for an investment opportunity outside the run-prone banking sector would change those payoffs and thus affect the threshold below which a consumer runs. This threshold would be now determined not only by the degree of liquidity insurance provided by the contract but also by the degree of insurance against runs. In other words, the mechanisms behind the decision about whether or not to run appear to be the same in the two frameworks. However, a full analysis is beyond the scope of this paper.

Next Proposition provides a comparison between the probability of a run banks face in the banking-financial economy and in the economy with banks alone.

Proposition 7 *Suppose that for given parameters, run-admitting contracts are offered in both the banking-financial economy and the economy with banks alone. If the probability of a run implied by the banking-financial contract, π^{BF}, is sufficiently small, then $\pi^{BF} > \pi^{B}$ must hold.*

Proposition 7 gives conditions under which the probability of a run that intermediaries face in the banking-financial economy, π^{BF}, is greater than the probability of a run that banks face in the economy with banks alone, π^{B}. For small probabilities of a run, bank deposits provide maximum degree of liquidity insurance (Proposition 5). This implies that the risk-factor in the banking-financial economy is equal to zero, i.e. $\phi^{BF} = 0$. The liquidity insurance effect dominates the insurance against runs effect. On the other hand, in the economy with banks alone, incentive-compatibility constraint does not bind and so we have $\phi^{B} > 0$. It follows that the probability of a run banks face in the economy with banks alone must be lower than the probability of a run in the banking-financial economy, since $\pi(\phi) = m - hf(\phi)$ and f is strictly increasing.

The intuition behind the result is as follows. Consumers have a preference for liquidity insurance. It is optimal for banks to *fully* specialize in its provision since it cannot be provided by the market investment. Reducing liquidity insurance would lower incentives to run and thus the probability of a run but this effect is not strong enough to outweigh the reduction in welfare due to the loss of intertemporal consumption smoothing.

6 Implementation

In this Section, I address the question of how to implement the banking-financial contract as an equilibrium in an economy with banks and an asset market. In particular, one has to worry about a potential arbitrage opportunity for the late consumers: If the deposit contract provides *ex ante* liquidity insurance, a late consumer has an incentive to withdraw from the bank in period 1 and sell his period-one consumption in the market in exchange for the period-two consumption. By doing so he is able to achieve a return equal to $\frac{c_1^B R}{P_1}$ on his bank deposits which is strictly greater than the return he would get if he postponed his withdrawal until period 2, c_2^B (this arbitrage possibility was first pointed out by Jacklin (1987)).

One way to prevent this "cashing-out" under the banking-financial contract is to charge a fee whenever a consumer's expenditures on buying the long-term asset in period 1 exceed the value of his short-term holdings. Any fee $T > 0$ such that $0 < \frac{c_1^B R}{P_1} - T < c_2^B$ would render arbitrage unprofitable. Under this arrangement, late consumers rebalance only their original portfolio holdings. In case of a run, those served do not use the withdrawal to acquire more long-term asset. They store it to consume in period 2. Another way to prevent arbitrage is to directly impose a restriction on trading in period 1 so that consumers are only allowed to re-trade their original portfolio holdings.

Competition in the banking industry forces banks to maximize the expected utility of consumers when designing the optimal contract. Banks offer to consumers a contract specifying period-one and period-two returns on withdrawals and banks' portfolio decisions. Given a fee T, consumers choose optimally 1) how much to invest in bank deposits and the asset market and 2) composition of their private portfolio. The resulting equilibrium allocation is the one characterized in the previous Sections.

7 Concluding remarks

The starting point of the analysis in this paper is an economy in which consumers invest their entire savings in a bank. Banks provide liquidity insurance but that makes them susceptible to sunspot-triggered runs. I add the possibility for consumers to invest part of their savings in an asset market portfolio. Such diversification carries novel implications for markets, banks, and consumers' welfare.

Markets are incomplete since there is no market for sunspot-contingent securities.

They are also incomplete from the point of view of a consumer who faces liquidity shocks since those are observed privately and thus it is impossible to write type-contingent insurance contracts. In the economy with markets alone, this implies that markets cannot provide liquidity insurance. However, in the presence of run-prone banks, markets can provide some liquidity insurance in the no run state of the world. This is because the long-term asset is relatively more valuable in this state which benefits early consumers who are selling the long-term asset in the market.

Banks have more flexibility in their decisions about investments and deposit returns in an economy with an asset market. This is for two reasons. One is simply that banks can adjust their portfolio in the interim period by trading in the asset market. This is similar in spirit to Allen and Gale (2004) except that in their model trading takes place among banks with heterogeneous portfolios whereas in my model banks trade with consumers. The other reason is that banks can choose to offer higher period-one return even though that implies that less people will be served in a run since consumers who are not served are insured through their market investment. For small probabilities of a run, banks indeed choose to fully specialize into the provision of liquidity insurance, which is their comparative advantage.

The stability of banks in the economy with an asset market is determined by two forces. Insurance against runs through the market investment on one hand and a bigger scope for intertemporal consumption smoothing on the other hand. The former effect can dominate at the optimum and the probability of a run is higher when compared to the economy with banks alone. Thus, there is a tradeoff between diversification and the risk of runs.

I argue in this paper that market investments can act as insurance against banking sector instability. Other investment opportunities outside the run-prone banking sector, e.g. keeping a fraction of savings “under the mattress”, would serve the same purpose. But since the return on such investment may be unattractive, consumers would only invest a small fraction of their savings in it. Some of the effects highlighted in this paper would remain present albeit to a quantitatively smaller extent.

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Appendix

Proof of Proposition 1. I prove the claims by contradiction. First, suppose that consumers and banks find it optimal to invest solely in the short-term asset, i.e. $\alpha = \beta = 1$. In period 1, then, price P_1 must be high enough so as to induce zero demand for the long-term asset, i.e. $a(P_1) = b(P_1) = 0$. A sufficient condition would be $P_1(\theta) > R > 1$. But then, the long-term asset dominates the short-term asset. Contradiction.

Now suppose that consumers and banks invest solely in the long-term asset, i.e. $\alpha = \beta = 0$. Since $\beta = 0$, also $b(P_1) = 0$. Hence, the demand for the long-term asset in period 1 is equal to zero. Since λ is always greater than 0, the supply of the long-term asset must be strictly positive: Early consumers offer their long-term holdings for sale and/or banks sell part of their long-term holdings to pay out period-one withdrawals (for $W < 1$). Since nothing can be offered in exchange (no one holds the short-term asset), the price in the market must be equal to zero. But then, short-term asset in fact dominates the long-term asset. Contradiction.

Furthermore, suppose that $P_1(\theta) > R$ for some θ. In this state of the world, short-term asset dominates the long-term asset between period 1 and 2. Thus, demand for the long-term asset in period 1 must be zero. By market clearing, the supply must be zero, too. However, we know that a positive amount of the long-term asset is held in period 0. These entire holdings will be offered for sale. Contradiction.

Finally, I show that $P_1 > 1$ and $P_1^L < 1$ must hold. I do so in two steps. I prove that the equilibrium price must reveal the state. I then show that $P_1 > 1 > P_1^L$ must hold.

Suppose first that the equilibrium price does not reveal the state, i.e. $P_1 = P_1^L$. Then, late consumers know only their own type and they choose the same a for both states. Banks must liquidate their entire long-term holdings in the run state of the world: $\gamma = 1$. For P_1 to be constant across states, it must be the case that $a = b = 0$ and the resulting $P_1 = 0$. But then, $a = 0$ is not optimal. This cannot be an equilibrium.

I now show that price in one of the states must be above 1 and in the other below 1. Suppose for a moment that $P_1(\theta) \geq 1$ for all θ with the strict inequality for at least one θ. Then the short-term asset is dominated by the long-term asset between period 0 and 1 and no short-term asset will be held. Contradiction. Similarly, it cannot be the case that $P_1(\theta) \leq 1$ for all θ with the strict inequality for at least one

θ. Suppose otherwise. Then the long-term asset is dominated by the short-term asset between period 0 and 1 and no long-term asset will be held. Contradiction.

I now show that $P_1 > P_1^L$ which implies $P_1 > 1$ and $P_1^L < 1$. Suppose otherwise, in particular $P_1^L = R > P_1$. Then, $a\left(P_1^L\right) \in (0,1]$, $b\left(P_1^L\right) = 0$ and $\gamma\left(P_1^L\right) = 1$. Since $P_1 < R$, we have that $a\left(P_1\right) = 1$, $b\left(P_1\right) \geq 0$ and $\gamma\left(P_1\right) < 1$, since γ is strictly decreasing in P_1. But then, the demand for the long-term asset is at least as high and the supply is strictly lower at P_1 as compared to P_1^L and thus $P_1 > P_1^L$. Contradiction. The proof is identical for the case $R > P_1^L > P_1$. ■

Proof of Lemma 1. (Sketch) The social planner chooses β, the fraction of resources to be invested in the short-term asset, so as to maximize expected utility of consumers. Since a strictly positive fraction of population becomes early in period 1 and the return on the short-term asset dominates the return on the long-term asset in the short-run, it must be that $\beta > 0$. Also, since a strictly positive fraction of population prefers to consume in period 2 (i.e. $1 - \lambda > 0$) and the return on the long-term asset dominates the return on the short-term asset in the long-run, it must be that $\beta < 1$. Thus, the social planner solves:

$$\max_{c_1, c_2, \beta} \lambda u\left(c_1\right) + (1-\lambda) u\left(c_2\right)$$

subject to $\lambda c_1 = \beta$, $(1-\lambda) c_2 = (1-\beta) R$, and $0 < \beta < 1$. Let φ be the Lagrangian multiplier on the first constraint and let μ_3 be the Lagrangian multiplier on the second constraint. Then, first-order conditions yield:

$$u'\left(c_1^{FB}\right) = \varphi > 0,\ u'\left(c_2^{FB}\right) = \mu_3 > 0, \text{ and } u'\left(c_1^{FB}\right) = Ru'\left(c_2^{FB}\right).$$

Since $cu'(c)$ is decreasing in c (coefficient of the relative risk-aversion $\rho_R > 1$), we must have that $c_1^{FB} > 1$ and $c_2^{FB} < R$. ■

FOCs for the problem (BFP) Let μ_2 denote a Lagrangian multiplier on the constraint $\beta - \lambda c_1^B \geq 0$. Let μ_3 denote a multiplier on the constraint (RC). Let $\mu_4 \geq 0$ denote a multiplier associated with the constraint $W \leq 1$. Note that it must be the case in equilibrium that $\alpha^* > 0$ since otherwise $P_1^L = 0$ and $\alpha = 0$ cannot be optimal. Let $\mu_5 \geq 0$ denote a multiplier associated with $\alpha \leq 1$.

FOCs of the problem (BFP) with respect to c_1^B, c_2^B, β, W, and α are then given by:

$$(1-\pi)\lambda u'(c_1)(1-W)+\frac{\pi}{c_1^B}\underline{\underline{\lambda}}\left[\lambda\left(u'\left(c_1^S\right)(1-W)c_1^B-u\left(c_1^S\right)\right)+(1-\lambda)\right. \\ \left(u'\left(c_2^S\right)(1-W)c_1^B-u\left(c_2^S\right)\right)+\lambda u\left(c_1^N\right)+(1-\lambda)u\left(c_2^N\right)\left.\right]-\mu_6=0,\ \mu_6\geq 0, \\ \mu_6\left(c_2-c_2^W\right)=0 \text{ and } \lambda c_1^B-\beta>0, \tag{1}$$

$$(1-\pi)\lambda u'(c_1)(1-W)+\frac{\pi}{c_1^B}\underline{\underline{\lambda}}\left[\lambda\left(u'\left(c_1^S\right)(1-W)c_1^B-u\left(c_1^S\right)\right)+(1-\lambda)\right. \\ \left(u'\left(c_2^S\right)(1-W)c_1^B-u\left(c_2^S\right)\right)+\lambda u\left(c_1^N\right)+(1-\lambda)u\left(c_2^N\right)\left.\right]-\mu_2\lambda-\mu_3\frac{\lambda}{P_1}-\mu_6 \\ =0 \text{ for } \beta-\lambda c_1^B\geq 0, \tag{2}$$

$$(1-\pi)(1-\lambda)u'(c_2)(1-W)-\mu_3(1-\lambda)+\mu_6=0,\ \mu_3\geq 0 \tag{3}$$

$$\frac{\pi}{c_1^B}\left(1-P_1^L\right)\left[\lambda u\left(c_1^S\right)+(1-\lambda)u\left(c_2^S\right)-\lambda u\left(c_1^N\right)-(1-\lambda)u\left(c_2^N\right)\right]+\mu_1 \\ -\mu_3(1-\gamma)R=0,\ \mu_1,\acute{\mu}_1\geq 0,\ \mu_1\beta=0,\ \text{and } \lambda c_1^B-\beta>0, \tag{4}$$

$$\frac{\pi}{c_1^B}\left(1-P_1^L\right)\left[\lambda u\left(c_1^S\right)+(1-\lambda)u\left(c_2^S\right)-\lambda u\left(c_1^N\right)-(1-\lambda)u\left(c_2^N\right)\right]-\acute{\mu}_1 \\ +\mu_2-\mu_3\left(1-\frac{1}{P_1}\right)R=0,\ \acute{\mu}_1(1-\beta)=0,\ \text{and } \beta-\lambda c_1^B\geq 0, \tag{5}$$

$$(1-\pi)\left[\lambda u'(c_1)\left(\alpha+(1-\alpha)P_1-c_1^B\right)+(1-\lambda)u'(c_2)\left(\left(\frac{a\alpha}{P_1}+1-\alpha\right)R\right.\right. \\ \left.\left.+(1-a)\alpha-c_2^B\right)\right]+\pi\underline{\underline{\lambda}}\left[\lambda u'\left(c_1^S\right)\left(\alpha+(1-\alpha)P_1^L-c_1^B\right)+(1-\lambda)u'\left(c_2^S\right)\right. \\ \left.\times\left(\left(\frac{\alpha}{P_1^L}+1-\alpha\right)R-c_1^B\right)\right]+\pi\left(1-\underline{\underline{\lambda}}\right)\left[\lambda u'\left(c_1^N\right)\left(\alpha+(1-\alpha)P_1^L\right)+\right. \\ \left.(1-\lambda)u'\left(c_2^N\right)\left(\frac{\alpha}{P_1^L}+1-\alpha\right)R\right]-\mu_4=0, \tag{6}$$

$$(1-\pi)\left[\lambda u'(c_1)(1-P_1)+(1-\lambda)u'(c_2)\left(\left(\frac{a}{P_1}-1\right)R+1-a\right)\right]+\pi\underline{\underline{\lambda}} \\ \times\left[\lambda u'\left(c_1^S\right)\left(1-P_1^L\right)+(1-\lambda)u'\left(c_2^S\right)\left(\frac{1}{P_1^L}-1\right)R\right]+\pi\left(1-\underline{\underline{\lambda}}\right)\left[\lambda u'\left(c_1^N\right)\right. \\ \left.\times\left(1-P_1^L\right)+(1-\lambda)u'\left(c_2^N\right)\left(\frac{1}{P_1^L}-1\right)R\right]-\mu_5=0. \tag{7}$$

Note that (3) implies that $\mu_3=(1-\pi)(1-W)u'(c_2)+\frac{\mu_6}{1-\lambda}>0$ and thus the resource constraint (RC) must bind.

Proof of Proposition 2. Suppose a triple $\left(\beta,c_1^B,c_2^B\right)$ constitutes the optimal contract in the economy with banks alone. I show that in the banking-financial economy, a higher level of welfare can be achieved.

First, suppose that for a given π, a run proof contract is offered in the economy with banks alone. This contract must have $c_1^B\leq\beta+(1-\beta)P_1^L\leq 1$ with strict inequality for $\beta=1$. If $\beta=1$, then $c_2^B=1$. Allocation $\left(c_1^B=1,c_2^B=1\right)$ is clearly dominated by the allocation $(c_1=1,c_2=R)$ which is always feasible in the banking-financial economy (just let $W=1$). If $\beta<1$, then $c_1^B<1$ and, by the resource constraint, $c_2^B > R$. Given the properties of the utility function, it is easy to see that this allocation is also strictly dominated by the allocation which has $c_1 = 1$ and $c_2 = R$. In summary, if a run proof contract is optimal in the economy with banks alone, there exists a banking-financial contract that yields a strictly higher welfare.

Now, suppose that for a given π, a run-admitting contract is offered in the economy with banks alone. Suppose that financial intermediaries in the banking-financial economy choose the same β, c_1^B, and c_2^B. Consider the case when $c_1^B \geq P_1$. Then, we have:

$$\begin{aligned}
c_1 &= [\alpha + (1-\alpha) P_1] W + (1-W) c_1^B \leq c_1^B, \\
c_2 &= \left[\left(\tfrac{a\alpha}{P_1} + 1 - \alpha\right) R + (1-a)\alpha\right] W + (1-W) c_2^B \geq c_2^B, \\
c_1^S &= \left[\alpha + (1-\alpha) P_1^L\right] W + (1-W) c_1^B < c_1 \leq c_1^B, \\
c_2^S &= \left(\tfrac{\alpha}{P_1^L} + 1 - \alpha\right) RW + (1-W) c_1^B > c_1^B, \\
c_1^N &= \left[\alpha + (1-\alpha) P_1^L\right] W > 0, \\
c_2^N &= \left(\tfrac{\alpha}{P_1^L} + 1 - \alpha\right) RW > 0.
\end{aligned}$$

I now claim, contrary to the claim in the Proposition, that for all $W > 0$ and all utility functions u, it must be the case that:

$$\begin{aligned}
&(1-\pi)\left[\lambda u\left(c_1^B\right) + (1-\lambda) u\left(c_2^B\right)\right] + \pi \underline{\underline{\lambda}} u\left(c_1^B\right) - (1-\pi)\left[\lambda u(c_1) + (1-\lambda) u(c_2)\right] \\
&-\pi \underline{\underline{\lambda}}\left[\lambda u\left(c_1^S\right) + (1-\lambda) u\left(c_2^S\right)\right] - \pi\left(1 - \underline{\underline{\lambda}}\right)\left[\lambda u\left(c_1^N\right) + (1-\lambda) u\left(c_2^N\right)\right] > 0.
\end{aligned} \tag{8}$$

Dividing through by $\pi\left(1 - \underline{\underline{\lambda}}\right)$ and rearranging yields:

$$\begin{aligned}
&\tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)} \lambda\left[u\left(c_1^B\right) - u(c_1)\right] + \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}} \lambda\left[u\left(c_1^B\right) - u\left(c_1^S\right)\right] > \lambda u\left(c_1^N\right) + (1-\lambda) u\left(c_2^N\right) \\
&+ \tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)} (1-\lambda)\left[u(c_2) - u\left(c_2^B\right)\right] + \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}} (1-\lambda)\left[u\left(c_2^S\right) - u\left(c_1^B\right)\right].
\end{aligned}$$

The following set of inequalities hold for the left hand side of the above expression:

$$\begin{aligned}
&\tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)} \lambda\left[u\left(c_1^B\right) - u(c_1)\right] + \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}} \lambda\left[u\left(c_1^B\right) - u\left(c_1^S\right)\right] < \tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)} \lambda u'(c_1)\left(c_1^B - c_1\right) \\
&+ \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}} \lambda u'\left(c_1^S\right)\left(c_1^B - c_1^S\right) < 2\lambda \max\left\{\tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)}, \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}}\right\} u'\left(c_1^S\right)\left(c_1^B - c_1^S\right) = 2\lambda \\
&\times u'\left(c_1^S\right) \max\left\{\tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)}, \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}}\right\}\left[c_1^B - \left(\alpha + (1-\alpha) P_1^L\right)\right] W.
\end{aligned}$$

Let K be a constant such that $K \equiv 2\lambda \max\left\{\tfrac{1-\pi}{\pi\left(1-\underline{\underline{\lambda}}\right)}, \tfrac{\underline{\underline{\lambda}}}{1-\underline{\underline{\lambda}}}\right\}$. I claim that there exists a utility function $u(\cdot)$ and a $W > 0$ such that:

$$K u'\left(c_1^S\right)\left[c_1^B - \left(\alpha + (1-\alpha) P_1^L\right)\right] W < u\left(\left(\left(\frac{\alpha}{P_1^L} + 1 - \alpha\right) RW\right)\right) = u\left(c_1^N\right)$$
$$< \lambda u\left(c_1^N\right) + (1-\lambda) u\left(c_2^N\right).$$

Any $u(\cdot)$ and $W > 0$ satisfying the following two conditions would work: $u'(0) > K u'\left(c_1^S\right)\left[c_1^B - \left(\alpha + (1-\alpha) P_1^L\right)\right]$ and $c_1^N > u^{-1}\left[K u'\left(c_1^S\right)\left[c_1^B - \left(\alpha + (1-\alpha) P_1^L\right)\right] W\right]$. For example, take $W \to 0$ and $u(\cdot)$ such that $u'(0) \to \infty$. Then,

$$\lim_{W\to 0} u^{-1}\left(K u'\left(c_1^B\right)\left(c_1^B - 1\right) W\right) < \lim_{W\to 0} RW.$$

In this case, welfare is higher under the banking-financial contract than under the banking contract and Equation 8 cannot hold. The proof proceeds analogously for the case when $c_1^B < P_1$. ■

Proof of Proposition 3. Suppose otherwise, i.e. $W^* < 1$ in the optimum of the run-admitting contract. Since $c_1^S = c_1^N + (1-W) c_1^B$ and $c_2^S = c_2^N + (1-W) c_1^B$, I have that

$$u\left(c_1^S\right) - u\left(c_1^N\right) > u'\left(c_1^S\right)(1-W) c_1^B,$$
$$u\left(c_2^S\right) - u\left(c_2^N\right) > u'\left(c_2^S\right)(1-W) c_1^B.$$

Then, all terms in the Equations (1) and (2) are negative except the first term, which is multiplied by $(1-\pi)$. Clearly, there exists a sufficiently high π such that for all higher π's, those Equations cannot hold. This cannot be an optimum. Contradiction. ■

Proof of Proposition 4. Let β^{RP}, c_1^{RP} and c_2^{RP} denote β, c_1^B and c_2^B solving problem (RPF), respectively. Recall that P_1^L must be smaller than 1 (Proposition 1). Since $c_1^B \leq \beta + (1-\beta) P_1^L$ must hold for a contract to be run-proof, this implies that $c_1^{RP} \leq 1$ with equality for $\beta = 1$. By the resource constraint, $c_2^{RP} = \frac{\left[1-\lambda c_1^{RP}\right] R}{1-\lambda} \geq R$.

It is straightforward to show using the properties of the utility function u that allocation $c_1 = 1$, $c_2 = R$ is strictly preferred to any allocation which has $c_1 < 1$ and $c_2 > R$. Thus, the best banking-financial run-proof contract must have $\beta^{RP} = 1$. Since $\beta^{RP} = 1$, it follows that $\alpha^* < 1$ since otherwise there is no equilibrium in the asset market. Proposition 1 implies that $P_1 = 1$ (there is only one state of the world in period 1). Then, any allocation satisfying $\beta^{RP} = 1$, $c_1^{RP} = 1$, $c_2^{RP} = R$, and $\alpha < 1$ and $W > 0$ such that $W(1-\alpha) = 1 - \lambda$ constitutes an optimum.

Note, however, that allocation $c_1 = 1$ and $c_2 = R$ is also attainable by investing solely through the asset market. This completes the proof. ■

Proof of Proposition 5. Suppose not, i.e. $\beta - \lambda c_1^B > 0$ in the optimum. Then, $\mu_2 = 0$. By Equation 5, there exists a sufficiently small π, such that the weight of the first term becomes negligible. Then, the equation cannot hold. Contradiction. ■

Proof of Proposition 6. Suppose otherwise, i.e. (ICC) does not bind and $c_1^B < c_2^B$ and $\mu_6 = 0$. Note that in case $\lambda c_1^B - \beta > 0$, equation 1 becomes

$$\begin{aligned}&(1-\pi)\,\lambda u'(c_1)\,(1-W) + \tfrac{\pi}{c_1^B}\underline{\lambda}\left[\lambda\left(u'\left(c_1^S\right)(1-W)\,c_1^B - u\left(c_1^S\right)\right) + (1-\lambda)\right. \\ &\left.\left(u'\left(c_2^S\right)(1-W)\,c_1^B - u\left(c_2^S\right)\right) + \lambda u\left(c_1^N\right) + (1-\lambda)\,u\left(c_2^N\right)\right] = 0.\end{aligned} \tag{9}$$

There exists a sufficiently small π such that for all smaller π's, the sign of the first term dominates the left hand side of the above equation and thus it cannot hold. Contradiction.

Now consider the case when $\beta - \lambda c_1^B \geq 0$. Equation 2 becomes

$$\begin{aligned}&(1-\pi)\,(1-W)\,\lambda\left[u'(c_1) - u'(c_2)\left(R + (1-R)\,\tfrac{1}{P_1}\right)\right] + \pi\tfrac{\underline{\lambda}}{c_1^B}\left[\lambda\left(u'\left(c_1^S\right)(1-W)\,c_1^B\right.\right. \\ &\left.-u\left(c_1^S\right)\right) + (1-\lambda)\left(u'\left(c_2^S\right)(1-W)\,c_1^B - u\left(c_2^S\right)\right) + \lambda u\left(c_1^N\right) + (1-\lambda)\,u\left(c_2^N\right)\Big] \\ &+\pi\tfrac{\lambda}{c_1^B}\left(1-P_1^L\right)\left[\lambda u\left(c_1^S\right) + (1-\lambda)\,u\left(c_2^S\right) - \lambda u\left(c_1^N\right) - (1-\lambda)\,u\left(c_2^N\right)\right] + \lambda\acute{\mu}_1 = 0\end{aligned} \tag{10}$$

Multiplying Equation 7 through by α and subtracting from the Equation 6 yields:

$$\begin{aligned}&(1-\pi)\left[\lambda u'(c_1)\left(P_1 - c_1^B\right) + (1-\lambda)\,u'(c_2)\left(R - c_2^B\right)\right] + \pi\left\{\underline{\lambda}\left[\lambda u'\left(c_1^S\right)\right.\right. \\ &\times\left(P_1^L - c_1^B\right) + (1-\lambda)\,u'\left(c_2^S\right)\left(R - c_1^B\right)\Big] + \left(1-\underline{\lambda}\right)\left[\lambda u'\left(c_1^N\right)P_1^L + (1\right. \\ &\left.\left.-\lambda)\,u'\left(c_2^N\right)R\right]\Big\} + \alpha\mu_5 = 0\end{aligned} \tag{11}$$

as $\mu_4 = 0$ for $W^* < 1$.

Note that Equation 11 implies that $c_1^B > P_1$ if π is sufficiently small. This is because $R - c_2^B > 0$ holds (if not, i.e. $c_2^B \geq R$, then $c_1^B \leq 1$ and we know from the proof of Proposition 4 that this cannot be the case in the optimum of the run-admitting banking-financial contract).

Multiplying Equation 10 by c_1^B and Equation 11 by $(1-W)$ and adding the two yields:

$$\begin{aligned}&(1-\pi)\,(1-W)\left\{\lambda u'(c_1)\,P_1 + u'(c_2)\left[(1-\lambda)\left(R - c_2^B\right) - \lambda c_1^B R\right.\right. \\ &\left.\left.+\lambda c_1^B\,(R-1)\,\tfrac{1}{P_1}\right]\right\} + \pi\left\{\underline{\lambda}\,(1-W)\left[\lambda u'\left(c_1^S\right)P_1^L + (1-\lambda)\,u'\left(c_2^S\right)R\right]\right. \\ &+\left(1-\underline{\lambda}\right)(1-W)\left[\lambda u'\left(c_1^N\right)P_1^L + (1-\lambda)\,u'\left(c_2^N\right)R\right] - \underline{\lambda}\left[u\left(c_1^S\right) + u\left(c_2^S\right)\right.\end{aligned} \tag{12}$$

$$-\lambda u\left(c_1^N\right)-(1-\lambda)\,u\left(c_2^N\right)\big]+\lambda\left(1-P_1^L\right)\left[\lambda u\left(c_1^S\right)+(1-\lambda)\,u\left(c_2^S\right)\right.$$
$$\left.-\lambda u\left(c_1^N\right)-(1-\lambda)\,u\left(c_2^N\right)\big]\right\}+\lambda \acute{\mu}_1 c_1^B+\alpha\mu_5\,(1-W)=0.$$

The last two terms in the above equation are non-negative. I now show that when consumers are sufficiently risk-averse, the sign of the first term, multiplied by $(1-\pi)(1-W)$, is strictly positive. Using the fact that $\beta-\lambda c_1^B \geq 0$ and $(1-\lambda)\,c_2^B = \left[\frac{\beta-\lambda c_1^B}{P_1}+1-\beta\right]R$ yields:

$$\begin{aligned}
&\lambda \acute{u}(c_1)\,P_1+\acute{u}(c_2)\left[(1-\lambda)\left(R-c_2^B\right)-\lambda c_1^B R+\lambda c_1^B\,(R-1)\,\tfrac{1}{P_1}\right]=\lambda \acute{u}(c_1)\,P_1\\
&+\acute{u}(c_2)\left[(1-\lambda)\,R-\left(\tfrac{\beta-\lambda c_1^B}{P_1}+1-\beta\right)R-\lambda c_1^B R+\lambda c_1^B\,(R-1)\,\tfrac{1}{P_1}\right]=\\
&=\lambda \acute{u}(c_1)\,P_1+\acute{u}(c_2)\left[\left(\beta-\lambda c_1^B\right)R\left(1-\tfrac{1}{P_1}\right)-R\lambda+\lambda c_1^B\,(R-1)\,\tfrac{1}{P_1}\right]=\\
&=\lambda \acute{u}(c_1)\,P_1+\acute{u}(c_2)\left[\left(\beta-\lambda c_1^B\right)R\left(1-\tfrac{1}{P_1}\right)+\lambda\left((R-1)\,\tfrac{c_1^B}{P_1}-R\right)\right]\\
&\geq \lambda \acute{u}(c_1)\,P_1+\lambda \acute{u}(c_2)\left((R-1)\,\tfrac{c_1^B}{P_1}-R\right)
\end{aligned}$$

since $1-\frac{1}{P_1}>0$ (recall that $P_1>1$ holds in equilibrium).

We know that for a sufficiently small π, $c_1^B > P_1$ implying $\frac{c_1^B}{P_1}>1$. But then, either $(R-1)\,\frac{c_1^B}{P_1}-R\geq 0$, in which case

$$\lambda \acute{u}(c_1)\,P_1+\lambda \acute{u}(c_2)\left((R-1)\,\frac{c_1^B}{P_1}-R\right)>0,$$

or $(R-1)\,\frac{c_1^B}{P_1}-R<0$ and thus

$$\begin{aligned}
&\lambda \acute{u}(c_1)\,P_1+\lambda \acute{u}(c_2)\left((R-1)\,\tfrac{c_1^B}{P_1}-R\right)\\
&\geq \lambda \acute{u}(c_1)\,P_1+\lambda \acute{u}(c_2)\,[(R-1)-R]\\
&=\lambda \acute{u}(c_1)\,P_1-\lambda \acute{u}(c_2)>\lambda\,[\acute{u}(c_1)-\acute{u}(c_2)]
\end{aligned}$$

which is positive by the concavity of $u(\cdot)$.

Now, there exists a sufficiently small π such that for all smaller π's the sign of the second term, multiplied by π, is dominated by the other terms. But then, Equation 12 cannot hold. Contradiction. ■

Table 1: Timing of events	
Deposit with a financial intermediary	t=0
Sunspot signal σ, private types observed Consumers decide the amount of the long-term asset to buy/sell Early withdraw from the bank, late choose withdraw/not Banks decide the amount of the long-term asset to buy/sell Batch clearing in the centralized market for the long-term asset Early types consume	t=1
Late types consume	t=2

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ISSN 1561-0810
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