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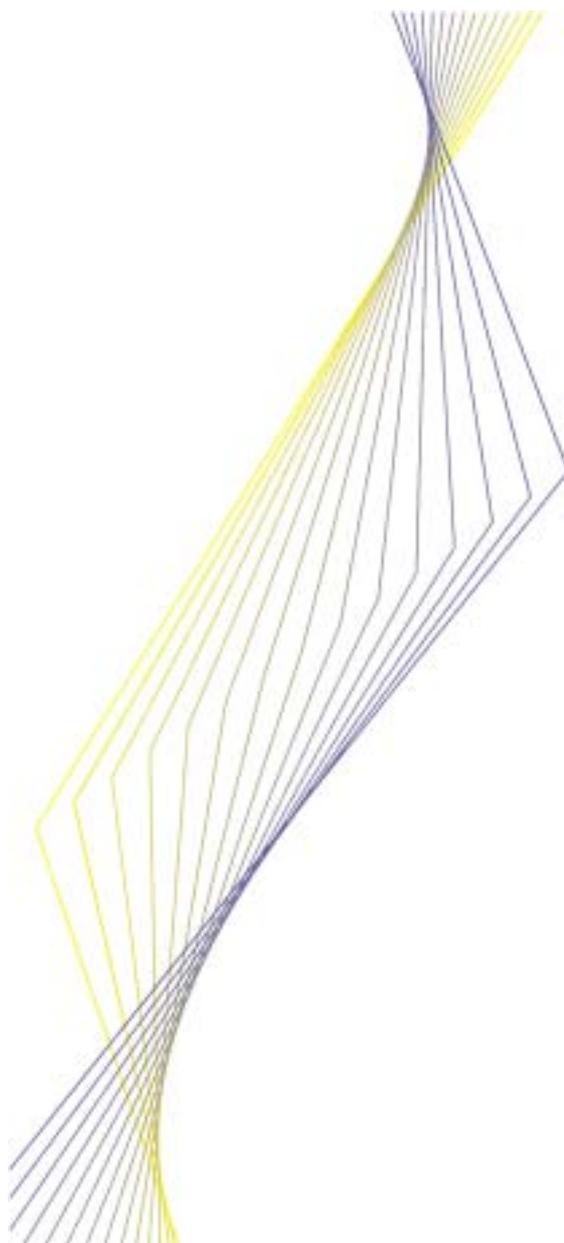
**WORKING PAPER NO. 173**

**OPENNESS AND EQUILIBRIUM  
DETERMINACY UNDER  
INTEREST RATE RULES**

**BY FIORELLA DE FIORE  
AND ZHENG LIU**

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ISSN 1561-0810

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## Abstract

This paper shows that the conditions under which inflation-targeting interest rate rules lead to equilibrium uniqueness in a small open economy in general differ from those in a closed economy. As the monetary authority adjusts nominal interest rates in response to inflation, the real interest rate changes. The overall effect of this change on aggregate demand has important implications for equilibrium determinacy. In an open economy, an increase in the real interest rate is transmitted to aggregate demand through an intertemporal substitution effect, as in a closed economy, but also through a terms of trade effect that is absent in the closed economy. These effects move aggregate demand in opposite directions. We find that, in a broad class of models, the conditions for local equilibrium uniqueness depend crucially on the degree of openness to international trade. Openness matters not only quantitatively, but also qualitatively.

*JEL Classification Numbers:* E52, E58, F41.

*Keywords:* Indeterminacy, Interest Rate Rules, Small Open Economy, Terms of Trade.

## Non-Technical Summary

The most recent decade has witnessed an increasing popularity of inflation-targeting policy. Some central banks have explicitly switched to inflation targeting (e.g., New Zealand, Canada, the United Kingdom, Sweden, Australia, the Czech Republic, and Poland), and many other countries are moving toward making price stability a primary goal of their central banks. Countries have also become increasingly interconnected through trade. Among the countries that have adopted an explicit inflation targeting policy, the degree of openness (as measured by the share of imports in GNP) ranges from 20% for Australia to around 40% in Sweden and Canada, and to more than 60% in the Czech Republic.

In practice, inflation targeting can be implemented through an interest rate rule, where the central bank adjusts the short-term nominal interest rate in response to changes in the inflation rate. It is well known that, depending on the activeness of the policy, these feedback rules can potentially lead to multiple equilibria. In the literature, conditions for local equilibrium uniqueness under this class of policy rules have been widely explored in a closed economy context. In light of the evidence that most of the countries that adopt inflation targeting rules have also a large trade share, a natural question arises: does openness make a qualitative difference in the determinacy properties of interest rate rules?

In this paper, we argue that openness indeed plays an important role in equilibrium determinacy under inflation-targeting interest rate rules. The main reason is that the transmission mechanism of monetary policy in an open economy differs from that in a closed economy. As the central bank adjusts nominal interest rates in response to inflation, the real interest rate changes. In the closed economy, an increase in the real interest rate tends to *lower* current aggregate demand through intertemporal substitution. In the open economy, a higher real interest rate also leads to an appreciation of the country's currency and an improvement of its terms of trade. The improved terms of trade tend to *increase* domestic aggregate demand as the country will be able to export less (leaving more domestic resources to home residents) and to import more. The net effect of a higher real interest rate on aggregate demand therefore depends on the relative importance of the terms of trade effect, which in turn depends on the degree of openness.

# 1 Introduction

The most recent decade has witnessed an increasing popularity of inflation-targeting policies, particularly in countries that are highly open to international trade (e.g., New Zealand, Canada, the United Kingdom, Sweden, Australia, the Czech Republic, and Poland). These countries share two common features. First, they are mostly small open economies with a large trade share. The degree of openness measured by the share of imports in GNP ranges from 20% in Australia, to around 40% in Sweden and Canada, and to more than 60% in the Czech Republic. Second, with few exceptions, the central banks in these countries have adopted CPI inflation as a primary targeting variable,<sup>1</sup> while allowing exchange rates to be freely floating.

In practice, the goal of price stability is commonly achieved through interest rate rules, where the central bank adjusts the short-term nominal interest rate in response to changes in inflation. The role of these feedback rules in stabilizing macroeconomic fluctuations has been extensively studied in the literature both for closed-economies (e.g., Clarida, Gali, and Gertler (2000), Orphanides (2001), and Rotemberg and Woodford (1997)) and for open economies (e.g., Ball (1999), Erceg (2002), McCallum and Nelson (2001), and Taylor (2001)). It is well known that, depending on the activeness of the policy, these rules can potentially lead to multiple equilibria.<sup>2</sup> A conventional wisdom holds that an active interest rate rule under which the monetary authority raises the nominal interest rate by more than the increase in inflation above its targeted value leads to a unique local equilibrium (e.g., Clarida, et al. (2000), Kerr and King (1996), and Woodford (2000)). The validity of this view has been challenged in the more recent literature. It has been shown that whether an active rule can ensure determinacy of local equilibrium may depend on, for example, whether the policy is forward-looking or backward-looking (e.g., Bernanke and Woodford (1997) and Carlstrom and Fuerst (2002)), whether prices are sticky or flexible (e.g., Carlstrom and Fuerst (2001)), or how money enters the utility function (e.g., Carlstrom and Fuerst (2001)). It is remarkable, however, that this strand of literature has focused almost exclusively on closed economy models

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<sup>1</sup>A notable exception is the United Kingdom, where the official target is the retail price index, excluding mortgage interest.

<sup>2</sup>There are two notions of multiple equilibria, one concerns about the global stability (e.g., Benhabib, et al. (2001) and Christiano and Rostagno (2001)), and the other about local equilibrium uniqueness. We focus here on the latter. We also choose to focus on the determinacy of real allocations instead of nominal determinacy.

economy models despite the empirical evidence that most inflation-targeting countries have also a large trade share.

In this paper, we argue that whether a feedback interest rate rule ensures equilibrium determinacy depends in general on whether and to what extent the country is open to international trade. In a broad class of small open economy models, we find that the degree of openness interacts in important ways with the activeness of interest rate rules to generate local equilibrium determinacy. The main reason behind this finding is that the transmission mechanism of monetary policy in the open economy differs from that in the closed economy. In particular, as the central bank adjusts nominal interest rates in response to inflation, the real interest rate changes. In the closed economy, an increase in the real interest rate tends to *lower* current aggregate demand through intertemporal substitution. In the open economy, a higher real interest rate also leads to an appreciation of the country's currency and an improvement of its terms of trade. The improved terms of trade tend to *increase* domestic aggregate demand as the country will be able to export less (leaving more domestic resources to home residents) and to import more. The net effect of a higher real interest rate on aggregate demand therefore depends on the relative importance of the terms of trade effect, which in turn depends on the substitutability between domestic goods and imported goods and, more importantly, on the degree of openness.

The link between openness and equilibrium determinacy under inflation-targeting interest rate rules survives variations of model environment and alternative choices of targeting variables. In particular, we begin by establishing this link in a small open economy with pure exchange. In this model environment, openness plays an important role in equilibrium determinacy, regardless of whether the interest rate rule targets expected inflation, current period inflation, or past inflation. The same result also arises when the target variable is domestic price inflation instead of CPI inflation. We then extend the analysis to a model with endogenous labor supply and flexible prices, and finally, to a model with sticky prices. In all cases, openness turns out to be crucial to ensure local equilibrium uniqueness.

The models that we consider share two features that are essential in building the link between openness and equilibrium determinacy. First, monetary policy affects real activity through a terms of trade effect, which arises because of imperfect substitutability between domestic and foreign goods in the representative agent's consumption basket (e.g., Clarida, et al. (2001) and Gali and Monacelli (2002)). Second, money plays a transaction role since

we assume that the money balances entering the representative household's utility function are those left after asset market transactions and before the opening of the goods market (as in Carlstrom and Fuerst (2001)). We show that both features help generate the interactions between the degree of openness and the activeness of interest rate rules to induce equilibrium determinacy.

Our work is closely related to Carlstrom and Fuerst (1999), who also examine the issue of equilibrium determinacy under inflation-targeting interest rate rules in a small open economy. They find that the conditions for equilibrium determinacy in a small open economy are similar to those in a closed economy. The primary reason for this similarity is that their model features a single traded good and thus the terms of trade effect is absent. Our paper is also related to Clarida, et al. (2001), who examine issues of optimal monetary policy design and gains from commitment in a small open economy model with sticky prices. Their model is similar to ours in that domestic goods and imported goods are imperfect substitutes in the representative agent's consumption basket, although it differs because of their implicit assumption on the timing of transactions. In their model, an active interest rate rule ensures equilibrium uniqueness, regardless of the degree of openness. In the class of models that we consider, however, we find that whether an active rule can lead to determinacy depends nonlinearly on the degree of openness.

The paper proceeds as follows. We first present in Section 2 a simple small open economy model with pure exchange, and establish the link between openness and equilibrium determinacy under an interest rate rule that targets future CPI inflation. Our model nests the closed economy model in Carlstrom and Fuerst (2001) as a special case. In such a closed economy, a passive rule leads to local equilibrium determinacy but an active rule does not. In an open economy, we find that a passive rule is required for equilibrium determinacy only if the degree of openness is below a certain threshold, not if it's above; and the threshold is determined by the fundamental parameters, including the representative agent's relative risk aversion parameter and the elasticity of substitution between home produced goods and imported goods. When the degree of openness is large enough, an active rule can also lead to determinacy.

In Section 3, we show that, in establishing the link between openness and equilibrium determinacy under inflation-targeting interest rate rules, the terms of trade effect and the timing of transactions are both crucial. In Section 4, we examine the robustness of the results by considering interest rate rules that target current or past CPI inflation rates as well as rules

that target domestic price inflation. We then extend the analysis to a model with endogenous labor supply, with either flexible or sticky prices. In all cases, we find that openness interacts in important ways with the activeness of the policy rules to induce local equilibrium determinacy. Finally, we conclude the paper in Section 5.

## 2 A Small Open Economy with Pure Exchange

To illustrate the role of openness in the equilibrium determinacy properties of inflation-targeting interest rate rules, we first consider a discrete-time, small open economy model with pure exchange. The home country is populated by a continuum of identical and infinitely lived households, each endowed with a homogenous good  $y_t$  in each period  $t$ . The country trades with the rest of the world in a competitive goods market. The exchange rate system is perfectly flexible. All agents in the world economy have access to a common asset market that provide complete insurance against country-specific income risks.

### 2.1 The Domestic Economy

The representative household in the small open economy derives utility from consumption and real money balances, with the utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \frac{A_t}{P_t}), \quad 0 < \beta < 1, \quad (1)$$

where  $\beta$  is a discount factor,  $c_t$  is consumption,  $A_t$  is the holding of nominal money balances,  $P_t$  is the price level, and  $E_0$  is an expectation operator. We assume that the period utility function satisfies

**(A1)**  $U : \mathcal{R}_{++}^2 \rightarrow \mathcal{R}$  is strictly increasing, strictly concave, twice continuously differentiable with respect to both arguments, and satisfies the usual Inada conditions;

**(A2)**  $U(c, A/P)$  is additively separable in its two arguments. Further, it displays constant relative risk aversion in  $c$  with the risk aversion parameter given by  $\sigma = -U_{cc}c/U_c$ .

The consumption good is produced by a perfectly competitive aggregation sector, using domestic goods and imported goods as inputs, with an aggregation technology

$$c_t = \left[ \omega_1 c_{Ht}^{\frac{\eta-1}{\eta}} + \omega_2 c_{Ft}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (2)$$

where  $c_{Ht}$  and  $c_{Ft}$  denote the consumption of domestic goods and imported goods, respectively, and  $\eta > 0$  is the elasticity of substitution between the two types of goods.

The aggregation sector chooses  $c_{Ht}$  and  $c_{Ft}$  to maximize its profit  $P_t c_t - P_{Ht} c_{Ht} - P_{Ft} c_{Ft}$ , subject to (2), taking as given the price of domestic goods  $P_{Ht}$ , the price of imported goods  $P_{Ft}$ , and the price level  $P_t$ . The resulting demand functions for the two types of goods are given by

$$c_{Ht} = (1 - \gamma) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} c_t, \quad c_{Ft} = \gamma \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} c_t, \quad (3)$$

where  $\gamma \equiv \omega_2^\eta$  and we have chosen the normalization so that  $\omega_1^\eta + \omega_2^\eta = 1$ . Thus, as the price of domestic goods relative to the price level rises, the demand for home goods falls, and the speed at which the demand falls is measured by the parameter  $\eta$ , the elasticity of substitution between the two types of goods.

The zero-profit condition in the aggregation sector implies that the price level is linked to the prices of domestic and foreign goods through

$$P_t = \left[ (1 - \gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \quad (4)$$

The household begins period  $t$  with nominal cash balances  $M_t$  accumulated during the previous period and nominal bond holdings  $B_t$ . It first enters a financial market to purchase new state-contingent bonds  $B_{t+1}$  and to receive a lump-sum nominal transfer  $X_t$  from the monetary authority. At the end of the financial market transactions and before the opening of the goods market, the household's cash balances are given by

$$A_t = M_t + X_t + B_t - E_t D_{t,t+1} B_{t+1}, \quad (5)$$

where  $D_{t,t+1}$  is a stochastic discount factor,  $B_{t+1}$  denotes state-contingent bonds, each unit of which is a promise of a delivery of one unit of domestic currency upon the realization of a certain state in period  $t + 1$ , and  $E_t D_{t,t+1} B_{t+1}$  is the total cost of all state-contingent bonds. It follows that the return to a risk-free bond (i.e., the nominal interest rate) is given by  $R_t = 1/E_t D_{t,t+1}$ .

When the financial market is closed, the household enters the goods market where it buys consumption goods and sells its endowment.<sup>3</sup> The remaining cash balances  $M_{t+1}$  after the

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<sup>3</sup>We adopt here the timing assumption proposed by Carlstrom and Fuerst (2001) to capture the transaction role of money. It differs from the conventional timing used in open-economy models in that the money balances entering the utility function are  $A_t/P_t$  instead of the end-of-period balances  $M_{t+1}/P_t$ . For similar timing assumptions that are intended to reflect the transaction role of money in models with money entering the utility function, see Lucas (2000) and Correia and Teles (1999).

following sequence of budget constraints:

$$M_{t+1} \leq M_t + X_t + B_t - E_t D_{t,t+1} B_{t+1} - P_t c_t + P_{Ht} y_t, \quad (6)$$

for all  $t \geq 0$ . In the budget constraint, since the consumption good is a composite of domestic goods and imported goods, the relevant price of the consumption good is the consumer price index (i.e., the CPI)  $P_t$ , while the relevant price of the endowment goods is the price of domestic goods  $P_{Ht}$ .

In the goods market, the household also decides the amount of the endowment good to be sold in the home market ( $c_{Ht}$ ) and the amount of export (denoted by  $c_{Ht}^*$ ). The price of exported goods is set in the buyers' local currency. There is a spot foreign exchange market so that the revenue from exporting can be instantaneously converted into home currency. In particular, the household chooses  $c_{Ht}$  and  $c_{Ht}^*$  to maximize  $P_{Ht} c_{Ht} + S_t P_{Ht}^* c_{Ht}^*$  subject to  $c_{Ht} + c_{Ht}^* = y_t$ , where  $S_t$  denotes the nominal exchange rate (units of home currency per unit of foreign currency), and  $P_{Ht}^*$  is the price of exported goods in foreign currency units. The solution, along with its foreign analogue, yields the law of one price

$$P_{Ht} = S_t P_{Ht}^*, \quad P_{Ft} = S_t P_{Ft}^*, \quad (7)$$

where  $P_{Ft}$  and  $P_{Ft}^*$  are the price of foreign goods in home currency and in foreign currency, respectively.

The domestic monetary authority follows a forward-looking interest rate rule, where it sets the nominal interest rate  $R_t$  to respond to changes in expected inflation. In practice, most inflation-targeting countries adopt the CPI inflation rate as a target variable. Thus, we assume that the policy rule is described by

$$R_t = \kappa E_t \pi_{t+1}^\tau, \quad (8)$$

where  $\pi_{t+1} \equiv P_{t+1}/P_t$  denotes the CPI inflation rate,  $\kappa$  is a constant determined by the steady state values of the nominal interest rate and inflation, and  $\tau > 0$  measures the activeness of the monetary policy. An active rule corresponds to  $\tau > 1$ , under which the monetary authority raises the nominal interest rate by more than the increase in the inflation rate. A passive rule corresponds to  $\tau < 1$ .

To commit itself to an interest rate rule, the monetary authority adjusts money supply to accommodate money demand. The newly created money is injected into the economy through lump-sum transfers so that  $X_t = M_{t+1} - M_t$ .

## 2.2 The Rest of the World

The structure of the foreign economy (i.e., the rest of the world) is formally identical to that of the home economy except that the weight on the imported goods in the foreign households' consumption basket is negligible. We denote the foreign variables with an asterisk. The consumption basket of the representative household in the foreign country is given by  $c_t^* = \left[ \omega_1^* c_{Ft}^* \frac{\eta-1}{\eta} + \omega_2^* c_{Ht}^* \frac{\eta-1}{\eta} \right]^{\frac{\eta}{\eta-1}}$ , and the corresponding consumer price index is given by  $P_t^* = \left[ (1 - \gamma^*) P_{Ft}^{* 1-\eta} + \gamma^* P_{Ht}^{* 1-\eta} \right]^{\frac{1}{1-\eta}}$ , where  $c_{Ft}^*$  and  $c_{Ht}^*$  are the foreign households' consumption of foreign and home goods,  $P_{Ft}^*$  and  $P_{Ht}^*$  are the corresponding prices, and the weights  $\omega_2^*$  and  $\gamma^* = \omega_2^{*\eta}$  are negligible.<sup>4</sup>

## 2.3 The Optimizing Conditions

The representative household in the home country maximizes the utility (1) subject to (5) and (6), taking all prices and foreign variables as given. The first order condition with respect to bond-holdings is given by

$$\frac{U_c(t) + U_m(t)}{P_t} D_{t,t+1} = \beta \frac{U_c(t+1) + U_m(t+1)}{P_{t+1}}, \quad (9)$$

where  $U_c(t)$  and  $U_m(t)$  denote the marginal utility of consumption and of real money balances, respectively. Using this condition, along with the relation  $1/R_t = E_t D_{t,t+1}$ , we obtain an intertemporal Euler equation

$$U_c(t) + U_m(t) = \beta E_t \left\{ [U_c(t+1) + U_m(t+1)] \frac{R_t}{\pi_{t+1}} \right\}. \quad (10)$$

Using the first order condition with respect to money holdings, we get a money demand equation

$$R_t = 1 + \frac{U_m(t)}{U_c(t)}. \quad (11)$$

The world asset market enables the households in all countries to pool country-specific risks. With perfect capital mobility, there is a no-arbitrage condition given by

$$D_{t,t+1}^* = D_{t,t+1} \frac{S_{t+1}}{S_t}, \quad (12)$$

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<sup>4</sup>Although the share of imported goods in the foreign country's consumption basket is negligible, the foreign household still buys home goods and derives a demand function for home goods through expenditure minimization.

where  $D_{t,t+1}^*$  denotes the price of the foreign country's state-contingent bonds. Denote  $Q_t = S_t P_t^*/P_t$  the consumption-based real exchange rate. Then, under the no-arbitrage condition (12), we can combine the first order conditions with respect to bond holdings in the two countries (i.e., (9) and its foreign analogue) to obtain

$$Q_t = \phi_0 \frac{U_c^*(t) + U_m^*(t)}{U_c(t) + U_m(t)}, \quad t \geq 0, \quad (13)$$

where  $\phi_0 = Q_0[U_c(0) + U_m(0)]/[U_c^*(0) + U_m^*(0)]$ .

The endowment good in each country is a homogeneous good that is freely traded, so that the law of one price holds as in (7). Yet, since domestic goods and imported goods are imperfect substitutes in the consumption basket of the representative household, the relative price of imported goods in terms of home goods (i.e., the terms of trade) plays an important role in determining aggregate consumption and thus the consumption-based real exchange rate. Let  $\mathcal{T}_t = \frac{P_{Ft}}{P_{Ht}}$  denote the home country's terms of trade. It turns out that there is a one-to-one mapping between the terms of trade and the real exchange rate. To see this, combine the CPI equation (4) with the relation  $\mathcal{T}_t = Q_t \frac{P_t}{P_{Ht}}$  to obtain

$$\mathcal{T}_t = Q_t \left[ \frac{1 - \gamma}{1 - \gamma Q_t^{1-\eta}} \right]^{\frac{1}{1-\eta}}. \quad (14)$$

It is straightforward to show that  $\mathcal{T}_t$  is increasing in  $Q_t$ . Thus, as the real exchange rate appreciates (i.e.,  $Q_t$  falls), the terms of trade will be improved (i.e.,  $\mathcal{T}_t$  falls). Given the one-to-one relation between real appreciation and the terms of trade improvement, we use the two terms interchangeably in the rest of the paper.

Finally, the endowment good in each country is divided between domestic consumption and exports, so that the goods market clearing conditions are given by

$$c_{Ht} + c_{Ht}^* = y_t, \quad c_{Ft}^* + c_{Ft} = y_t^*. \quad (15)$$

The home country's demand functions for domestic consumption goods  $c_{Ht}$  and imported goods  $c_{Ft}$  are given by (3). The foreign country's demand functions for  $c_{Ft}^*$  and  $c_{Ht}^*$  take similar forms:

$$c_{Ft}^* = (1 - \gamma^*) \left( \frac{P_{Ft}^*}{P_t^*} \right)^{-\eta} c_t^*, \quad c_{Ht}^* = \gamma^* \left( \frac{P_{Ht}^*}{P_t^*} \right)^{-\eta} c_t^*. \quad (16)$$

The first order conditions (10)-(11), the risk sharing condition (13), the market clearing conditions (15), the consumption demand functions (3) and (16), the definition of the consumer price index (4), along with the policy rule (8) completely summarize the equilibrium conditions of the underlying small open economy.

## 2.4 Equilibrium Determinacy

Under an interest rate rule, the monetary authority adjusts the nominal interest rate in response to changes in inflation. Since inflation is endogenous, an interest rate rule can potentially lead to multiple equilibria. This possibility has been widely discussed in the literature, mostly in a closed economy context. Here, we generalize the discussion to a small open economy and try to identify potential interactions between openness and the activeness of monetary policy in generating local equilibrium uniqueness.

To see how openness may potentially affect the determinacy properties of interest rate rules, we begin by reducing the optimizing conditions into a single dynamic equation, and then examine the relation between a measure of the degree of openness and the characteristic root(s) of the dynamic equation. For this purpose, we focus on the case with no intrinsic uncertainty and assume that all agents have perfect foresight throughout the rest of the analysis.

We first use the small open economy assumption (so that  $P_t^* \approx P_{Ft}^*$ ) together with the law of one price to rewrite the consumer price index equation (4) as

$$\frac{P_{Ht}}{P_t} = \left[ \frac{1 - \gamma Q_t^{1-\eta}}{1 - \gamma} \right]^{\frac{1}{1-\eta}}, \quad (17)$$

so that the home relative price  $P_{Ht}/P_t$  is a function of the real exchange rate  $Q_t$  only. Similarly, the foreign relative price  $P_{Ht}^*/P_t^*$  can also be expressed as a function of  $Q_t$ . We can then replace the relative price terms in the demand functions for  $c_{Ht}$  in (3) and for  $c_{Ht}^*$  in (16) and substitute the resulting expressions into the home market clearing condition (15) to get

$$\left[ \frac{1 - \gamma Q_t^{1-\eta}}{1 - \gamma} \right]^{\frac{\eta}{\eta-1}} [(1 - \gamma)c_t + \gamma^* Q_t^\eta c_t^*] = y_t. \quad (18)$$

Since  $y_t$  and  $c_t^*$  are taken as given, (18) implies that  $c_t = c(Q_t)$ , that is, home consumption is a function of the real exchange rate only. It is easy to verify that  $c_t$  is a decreasing function of  $Q_t$  so that a real appreciation (i.e., a lower  $Q_t$ ) leads to a higher level of consumption. To understand the negative relation between  $c_t$  and  $Q_t$ , observe that, following a real appreciation, (17) implies that the price of home goods rises relative to the price level. Under the law of one price, a higher  $P_{Ht}/P_t$  implies a higher  $P_{Ht}^*/P_t^*$  and thus a lower export demand  $c_{Ht}^*$  (since  $c^*$  is unchanged). With a fixed endowment, a fall in  $c_{Ht}^*$  implies a rise in  $c_{Ht}$  according to (15). Meanwhile, the rise in  $P_{Ht}/P_t$  implies a fall in  $c_{Ht}/c_t$  according to (3), so that  $c_t$  has to rise by more than the increase in  $c_{Ht}$  (through importing).

Next, denote  $z_t = U_c(t) + U_m(t)$  (similar for  $z_t^*$ ) so that the risk-sharing condition (13) can be rewritten as  $Q_t = \phi_0 z_t^*/z_t$ . Thus, (18) implies that  $c_t = c(Q_t) = c(\phi_0 z_t^*/z_t)$ , that is,  $c_t$  is a function of  $z_t$  (since the foreign variables are taken as given). The money demand equation (11) then implies that

$$z_t = U_c(c(\phi_0 z_t^*/z_t))R_t, \quad \text{or} \quad z_t = z(R_t). \quad (19)$$

Finally, use the intertemporal Euler equation (10) to obtain

$$z(R_t) = \beta z(R_{t+1}) \frac{R_t}{\pi_{t+1}}, \quad (20)$$

which, under the forward-looking interest rate rule (8), can be rewritten as

$$z(\kappa\pi_{t+1}^\tau) = \beta z(\kappa\pi_{t+2}^\tau) \kappa\pi_{t+1}^{\tau-1}, \quad (21)$$

where we have used the assumption of perfect foresight so that the expectations operator drops out.

Thus, the optimizing conditions reduce to a first order difference equation in the inflation rate, and the determinacy properties of the interest rate rule can be examined by calculating the slope of the dynamic inflation path evaluated at a balanced-trade steady state (in this case, the slope is also the characteristic root of the dynamic system). In such a steady state,  $P_H = P_F = P$ ,  $Q = \mathcal{T} = 1$ , and export equals import so that  $c_H^* = c_F$ .<sup>5</sup> The demand equations (3) and (16) imply that  $c_H = (1 - \gamma)c$  and  $c_H^* = c_F = \gamma c$ . We measure the degree of openness by the steady state share of imported goods in the gross domestic product (GDP), which is given by  $c_F/y = \gamma$  (where we have used the market clearing condition (15) to obtain  $c = y$  in the steady state). The following proposition establishes a link between openness and equilibrium determinacy.

**Proposition 1:** *In the small open economy with pure exchange, where the monetary authority follows a forward-looking interest rate rule (8), the following conditions are necessary and sufficient to achieve local equilibrium determinacy:*

(i)  $\tau < 1$  if  $0 < \gamma \leq \bar{\gamma}$ ;

(ii)  $\tau < 1$  or  $\tau > \bar{\tau} > 1$  if  $\bar{\gamma} < \gamma < 1$ , where

$$\bar{\gamma} = 1 - \sqrt{\frac{\sigma\eta}{1 + \sigma\eta}}, \quad \bar{\tau} \equiv \frac{\xi + 1}{\xi - 1}, \quad \xi = \frac{\sigma\eta\gamma(2 - \gamma)}{(1 - \gamma)^2}. \quad (22)$$

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<sup>5</sup>For a derivation of the balanced-trade steady state in a similar model, see Galí and Monacelli (2002).

**Proof:** A unique local equilibrium attains if and only if the characteristic root  $e$  of equation (21) lies outside the unit circle. To find the root, we take total differentiation in (21) and compute the derivative  $\frac{d\pi_{t+2}}{d\pi_{t+1}}$  evaluated at the balanced-trade steady state. This process involves the derivative terms  $z'(R)$  and  $c'(Q)$  (both evaluated at the steady state), which can be found by differentiating (19) and (18), respectively. The resulting characteristic root is given by

$$e = \frac{1}{\tau} [1 + \xi(1 - \tau)], \quad (23)$$

where  $\xi \equiv \frac{\sigma\eta\gamma(2-\gamma)}{(1-\gamma)^2}$ , as in (22).

Equilibrium determinacy requires that  $|e| > 1$ . Clearly, (23) implies that  $e > 1$  if and only if  $\tau < 1$ , and thus a passive rule always leads to determinacy. Determinacy can also be achieved if  $e < -1$ , which occurs if and only if  $\xi > 1$  and  $\tau > \bar{\tau} \equiv \frac{\xi+1}{\xi-1}$ . Thus, if  $\xi \leq 1$ , or equivalently, if  $0 < \gamma \leq \bar{\gamma}$  with  $\bar{\gamma} = 1 - \sqrt{\frac{\sigma\eta}{1+\sigma\eta}}$ , determinacy achieves if and only if  $\tau < 1$ ; if  $\xi > 1$ , or equivalently, if  $\bar{\gamma} < \gamma < 1$ , determinacy achieves if either  $\tau < 1$  or  $\tau > \bar{\tau}$ . *Q.E.D.*

Observe that, in the special case with  $\gamma = 0$ , equation (23) implies that the characteristic root reduces to  $e = 1/\tau$ . Thus, local equilibrium determinacy attains if and only if  $\tau < 1$ . This observation, along with Proposition 1, implies that a passive rule always leads to equilibrium determinacy, regardless of the degree of openness.

Proposition 1 also shows that an active rule can lead to determinacy only if the degree of openness exceeds a certain threshold, with the threshold determined by fundamental parameters in preferences (the relative risk aversion parameter  $\sigma$ ) and in technologies (the elasticity of substitution between home goods and imported goods  $\eta$ ). Under an active rule, equilibrium determinacy requires not only that the degree of openness exceeds the threshold level  $\bar{\gamma}$ , but the activeness of the policy rule needs also to exceed a critical level  $\bar{\tau}$ , and both  $\bar{\gamma}$  and  $\bar{\tau}$  are functions of the fundamental parameters including  $\sigma$ ,  $\eta$ , and  $\gamma$ .

To get a quantitative feel of the model's predictions, we present in Figure 1 the combinations of  $\tau$  and  $\gamma$  under which a unique equilibrium exists, where we have set  $\sigma = 1$  and  $\eta = 1.5$  following standard international business cycle literature (e.g., Backus, et al. (1995)). With the calibrated parameter values, an active rule can achieve determinacy only if  $\gamma$  exceeds  $\bar{\gamma} = 0.23$  (and  $\tau$  is large enough). For  $\gamma$  above 0.6, determinacy can be achieved if  $\tau$  is 1.3 or larger.

To illustrate the intuition for the results in Proposition 1, we begin by considering the special case of autarky where consumption is fixed by the endowment. In this case, the model

reduces to the closed economy model in Carlstrom and Fuerst (2001). As these authors find, a passive rule with  $\tau < 1$  achieves determinacy, while an active rule with  $\tau > 1$  does not. To see this, consider first a passive rule under which a rise in expected inflation leads to a fall in the real interest rate  $R_t/\pi_{t+1}$ . To be consistent with a lower real interest rate, the sum of the marginal utilities  $U_c(t) + U_m(t)$  in the intertemporal Euler equation (10) must fall. Since consumption is fixed by the endowment, with a separable utility function,  $U_c(t)$  stays constant and  $U_m(t)$  has to fall. Yet, a falling  $U_m(t)$  and a constant  $U_c(t)$  cannot be consistent with the money demand equation (11), since there, the rise in the nominal interest rate  $R_t$  requires a higher  $U_m(t)/U_c(t)$ . This contradiction implies that the initial fall in the real interest rate and the initial expectation of a higher inflation rate cannot be validated and there is a unique equilibrium path that converges to the steady state under a passive rule. Under an active rule, the reverse happens. The real interest rate rises in response to the expectation of higher inflation. For fixed consumption, the intertemporal Euler equation implies that  $U_m(t)$  rises, which is consistent with a higher  $R_t$  in the money demand equation, and thus validating the initial inflation expectation. In consequence, there are multiple equilibrium paths that converge to the steady state under an active rule in the closed economy.

When the economy opens to international trade, however, consumption is no longer fixed by the home endowment. Home consumption is now a basket of domestic goods and imported goods. The consumption of domestic goods is determined by the fraction of the endowment that is not exported. The export and import demand, according to (3) and (16), both depend on the relative prices or the terms of trade, which, as revealed by (14), is a function of the real exchange rate. Thus, the monetary policy can potentially influence consumption directly through changing the real interest rate (by varying the nominal interest rate in response to expected inflation) and indirectly through the effects of changes in the real interest rate on the real exchange rate and on the terms of trade.

The result that a passive rule can also lead to local equilibrium uniqueness in an open economy is similar to that in a closed economy, but for different reasons. Under a passive rule, the real interest rate falls in response to an increase in expected inflation. Thus, the term  $z_t = U_c(t) + U_m(t)$  in (10) must fall. A lower  $z_t$  leads to a real depreciation according to (13) and thus a fall in  $c_t$  according to (18). The consequent rise in  $U_c(t)$ , along with the fall in  $z_t$ , implies that  $U_m(t)$  must fall by more than the rise in  $U_c(t)$ . Yet, a lower  $U_m(t)$  and a higher

$U_c(t)$  cannot be consistent with a higher  $R_t$  in the money demand equation, invalidating the initial inflation expectation.

What makes the results in an open economy differ qualitatively from those in a closed economy is that an active rule can also lead to local equilibrium uniqueness if the degree of openness is sufficiently large. Under an active rule, the real interest rate rises in response to an increase in expected inflation. To be consistent with a higher real interest rate,  $z_t$  must rise, and therefore, according to (13),  $Q_t$  must fall. With this real appreciation, consumption rises and the marginal utility of consumption  $U_c(t)$  falls. If the fall in  $U_c(t)$  is so large that it more than offsets the increase in  $R_t$ , then, according to (11),  $z_t = R_t U_c(t)$  must fall, contradicting the initial rise in the real interest rate and thus invalidating the initial inflation expectation. To have such a large fall in  $U_c(t)$  requires a large increase in home consumption, and this is possible only if the degree of openness is large enough so that the improved terms of trade can actually induce a large amount of imports from the rest of the world while reducing the amount of exports.

### 3 The terms of trade effect and the timing of transactions

In the baseline model just described, openness interacts nonlinearly with the activeness of monetary policy in generating local equilibrium uniqueness. Two features of the model are essential for this result. One is the imperfect substitution between home goods and imported goods that allows policy induced changes in the real interest rate to generate a terms of trade effect, which is absent in a standard single-good open economy model. The second is the timing of household's decisions that reflects the transactions role for money. We now show that both features are crucial for openness to play any role in equilibrium determinacy under interest rate rules.

#### 3.1 The terms of trade effect

To understand the importance of the terms of trade effect, we consider a version of our model with a single traded good between the countries. In this case, home goods and imported goods are perfect substitutes so that  $P_{Ht} = P_{Ft} = P_t$  and  $\mathcal{T}_t = 1$ . In addition, the purchasing power parity holds so that  $Q_t = S_t P_t^* / P_t = 1$ . To examine the conditions for equilibrium determinacy, we begin by substituting the money demand equation (11) into the intertemporal

Euler equation (10) to obtain

$$\frac{R_{t+1}}{\pi_{t+1}} = \frac{U_c(t)}{\beta U_c(t+1)}. \quad (24)$$

Next, since  $Q_t = 1$ , the market clearing condition (15) implies that  $c_t$  is fixed by  $y_t$  and  $c_t^*$ . Thus,  $U_c(t)$  is also fixed. Under the forward-looking interest rate rule (8),  $R_{t+1} = \kappa \pi_{t+2}^r$  and equation (24) reduces to a first-order dynamic equation in the inflation rate, with a characteristic root given by  $1/\tau$ . Clearly, real equilibrium allocations are uniquely determined if and only if  $\tau < 1$ , regardless of the degree of openness.<sup>6</sup>

The lack of interactions between openness and the activeness of monetary policy rules resembles the results obtained by Carlstrom and Fuerst (1999) in a small open economy with a single good and limited participation. In our baseline model with imperfect substitution between home goods and imported goods, the purchasing power parity fails to hold. Thus, changes in the real interest rate will lead to changes in the real exchange rate and in the terms of trade, so that  $c_t$  can no longer be fixed by the endowment. The direction and the magnitude of the change in  $c_t$  depend on the degree of openness and the activeness of the monetary policy, with important implications on the determinacy properties of the policy rule.

### 3.2 The timing of transactions

The other feature that distinguishes our model from the standard open-economy literature lies in the timing of transactions. To illustrate its implications on equilibrium determinacy, we now consider a variation of the baseline model with an alternative timing assumption commonly used in the open economy macro literature. In particular, we replace  $A_t$  in the utility function with  $M_{t+1}$ . Under this assumption, the intertemporal Euler equation becomes

$$U_c(t) = \beta U_c(t+1) \frac{R_t}{\pi_{t+1}}, \quad (25)$$

and the international risk sharing condition becomes

$$Q_t = \phi_0 \frac{U_c^*(t)}{U_c(t)}, \quad t \geq 0, \quad (26)$$

where  $\phi_0 = Q_0 U_c(0)/U_c^*(0)$ . The money demand equation remains the same (as in (11)). Since money balances do not enter any other equilibrium conditions, under the separability assumption (A2), the money demand equation is a “residual” equation that determines the real money balances once  $R_t$  and  $c_t$  are determined.

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<sup>6</sup>In this case, we still have nominal indeterminacy as  $\pi_t$  is not determined.

Substituting the risk sharing condition (26) into the market clearing condition (15), we obtain a solution for  $c_t$  as a function of  $c_t^*$  and  $y_t$ . Thus,  $c_t$  is fixed, so is  $U_c(t)$ . It follows that, under the forward-looking interest rate rule (8), the intertemporal Euler equation (25) reduces to

$$\kappa\pi_{t+1}^{\tau-1} = \frac{U_c(t)}{\beta U_c(t+1)} = \frac{1}{\beta}.$$

Clearly, equilibrium determinacy obtains if and only if  $\tau \neq 1$ , regardless of the degree of openness.

To summarize, both the terms of trade effect and the timing of transactions are crucial in establishing the link between openness and equilibrium determinacy.

## 4 Robustness

We have thus far shown that openness interacts in important ways with the activeness of interest rate rules to achieve equilibrium determinacy. For simplicity, we have worked with a small open economy model with pure exchange, and we have considered an interest rate rule that targets future CPI inflation. We now examine the robustness of the results by considering rules that target different measures of the inflation rate and model environments that allow for endogenous labor supply, with either flexible or sticky prices. In each of the extensions considered, we find that openness in general plays an important role in equilibrium determinacy.

### 4.1 The timing of the policy rules

In addition to the forward-looking rule such as (8), the literature has also proposed rules that target current period CPI inflation (e.g., Taylor (1993)) or past CPI inflation (e.g., Carlstrom and Fuerst (2000, 2002)). We now examine the conditions for equilibrium determinacy under these alternative rules in the small open economy with pure exchange. In particular, we consider interest rate rules represented by

$$R_t = \kappa\pi_{t-j}^{\tau}, \quad j = 0, 1. \quad (27)$$

The policy is a current-inflation targeting rule if  $j = 0$  and backward-looking if  $j = 1$ . The next proposition summarizes our findings.

**Proposition 2:** *In the small open economy with pure exchange, the following conditions are necessary and sufficient for equilibrium determinacy:*

- (i) *Under a current inflation targeting rule with  $R_t = \kappa\pi_t^\tau$ , the policy parameter  $\tau$  satisfies  $1 < \tau < -\bar{\tau}$  if  $0 < \gamma < \bar{\gamma}$ , and  $\tau > 1$  if  $\bar{\gamma} \leq \gamma < 1$ .*
- (ii) *Under a backward-looking rule with  $R_t = \kappa\pi_{t-1}^\tau$ , the policy parameter  $\tau$  satisfies  $\tau > 1$  if  $0 < \gamma \leq \bar{\gamma}$ , and  $1 < \tau < \bar{\tau}$  if  $\bar{\gamma} < \gamma < 1$ ,*

where  $\bar{\gamma}$  and  $\bar{\tau}$  are given by (22).

**Proof:** Under the current inflation targeting rule, we first substitute the policy rule  $R_t = \kappa\pi_t^\tau$  into the reduced form intertemporal Euler equation (20). Then we take total differentiation in the resulting dynamic equation and evaluate it at the balanced-trade steady state to obtain

$$(1 - \tau + \xi)d\pi_{t+1} = \xi\tau d\pi_t, \quad (28)$$

where  $\xi$  is the same as in (22). In an open economy with  $\gamma > 0$ , we have  $\xi > 0$ . If  $\tau = 1 + \xi$ , then we have  $d\pi_t = 0$  for all  $t \geq 0$ , and thus determinacy is guaranteed. We focus on the case with  $\tau \neq 1 + \xi$ , where the characteristic root of the dynamic inflation equation (28) is given by

$$e = \frac{d\pi_{t+1}}{d\pi_t} \Big|_\pi = \frac{\xi\tau}{1 + \xi - \tau}. \quad (29)$$

Determinacy requires  $e > 1$  or  $e < -1$ . It is straightforward to verify that, if  $0 < \xi < 1$ , or equivalently, if  $0 < \gamma < \bar{\gamma}$ , then determinacy requires  $1 < \tau < \frac{1+\xi}{1-\xi} = -\bar{\tau}$ . On the other hand, if  $\xi \geq 1$ , or equivalently, if  $\bar{\gamma} \leq \gamma < 1$ , then determinacy requires  $\tau > 1$ . This proves the result under the current inflation targeting rule.

Under the backward-looking rule, we substitute the policy rule  $R_t = \kappa\pi_{t-1}^\tau$  into (20) and take total differentiations to get

$$(1 + \xi)d\pi_{t+1} = \tau d\pi_t + \tau\xi d\pi_{t-1}, \quad (30)$$

which is a second order difference equation in  $\pi_t$ . The characteristic equation is given by

$$H(e) = (1 + \xi)e^2 - \tau e - \xi\tau. \quad (31)$$

Since there is one predetermined variable, determinacy requires one root to lie outside the unit circle and the other inside. Denote the two roots by  $e_1$  and  $e_2$ . It is easy to verify that both roots are real and distinct. Since  $e_1 e_2 = -\frac{\xi\tau}{1+\xi} < 0$ , they have opposite signs. Note that

$H(-1) = 1 + \xi + \tau(1 - \xi)$ ,  $H(0) = -\xi\tau < 0$ ,  $H(1) = (1 - \tau)(1 + \xi)$ , and  $H''(e) = 2(1 + \xi) > 0$ . If  $\xi \in (0, 1]$ , then  $H(-1) > 0$  and thus  $e_1 \in (-1, 0)$ , and determinacy requires that  $e_2 > 1$  or that  $H(1) < 0$ . This implies that  $\tau > 1$ . If  $\xi > 1$ , we need to consider two cases. When  $\tau < 1$ , we have  $H(1) > 0$  and therefore  $e_1 \in (0, 1)$  and we need to have  $e_2 < -1$  or  $H(-1) < 0$ . This implies that  $\tau > \frac{\xi+1}{\xi-1} > 1$ , which contradicts the assumption that  $\tau < 1$ . Thus,  $\tau < 1$  implies indeterminacy. When  $\tau > 1$ , we have  $H(1) < 0$  and thus  $e_1 > 1$  and we need to have  $e_2 \in (-1, 0)$  or  $H(-1) > 0$ . This implies that  $\tau < \frac{\xi+1}{\xi-1}$ . Thus, when  $\xi > 1$ , determinacy requires  $1 < \tau < \frac{\xi+1}{\xi-1}$ . *Q.E.D.*

In the special case when the economy is closed (i.e.,  $\gamma = 0$ ), the dynamic inflation equation (28) under a current inflation targeting rule reduces to  $(1 - \tau)d\pi_{t+1} = 0$  so that real determinacy achieves if and only if  $\tau \neq 1$ . Thus, the uniqueness of equilibrium in a closed economy can be attained under either a passive rule with  $\tau < 1$  or an active rule with  $\tau > 1$ . Yet, according to Proposition 2, when the economy opens to trade, a passive rule always leads to multiplicity of equilibria if the monetary authority targets at the current period inflation. An active rule, on the other hand, can lead to local equilibrium uniqueness under the current inflation targeting rule, and the range of  $\tau$  values that achieves determinacy increases as  $\gamma$  increases. If  $\gamma$  exceeds a critical level  $\bar{\gamma}$ , equilibrium uniqueness is guaranteed for any  $\tau > 1$ .

When the inflation targeting policy is backward-looking, a passive rule never leads to determinacy, regardless of the degree of openness. In a closed economy with  $\gamma = 0$ , the dynamic inflation equation (30) reduces to  $d\pi_{t+1} = \tau d\pi_t$  so that determinacy achieves if and only if  $\tau > 1$ . In an open economy with  $\gamma > 0$ , Proposition 2 shows that equilibrium uniqueness is also ensured by an active rule with  $\tau > 1$ . Nonetheless, the range of  $\tau$  values that achieves determinacy varies with the degree of openness. The backward-looking rule differs from the current inflation targeting rule in that the determinacy region becomes smaller as  $\gamma$  increases. We plot in Figure 2 the combinations of  $\gamma$  and  $\tau$  that ensures equilibrium determinacy under these alternative policy rules. The figure shows that the activeness of the policy rules interacts nonlinearly with the degree of openness to ensure equilibrium determinacy.

## 4.2 Targeting domestic price inflation

The observation that most inflation-targeting countries have been using CPI inflation as a primary target has motivated our focus on CPI inflation-targeting rules in our baseline model.

The literature has suggested domestic price inflation as an alternative target (e.g., Clarida, et al. (2001)). We now show that, in the small open economy that we have constructed, the determinacy properties of interest rate rules that target domestic price inflation are qualitatively similar to those under a CPI-inflation targeting rule.

Consider the interest rate rule given by

$$R_t = \kappa \pi_{H,t+j}^\tau, \quad j = -1, 0, 1, \quad (32)$$

where  $\pi_{H,t+j} \equiv P_{H,t+j}/P_{H,t+j-1}$  denotes the domestic price inflation from period  $t+j-1$  to  $t+j$ . In light of (20), to obtain a difference equation analogous to (21), we need to relate the domestic price inflation  $\pi_{Ht}$  to the CPI inflation  $\pi_t$ . This relation can be obtained from (17) and is given by

$$\pi_t = \pi_{Ht} \left[ \frac{1 - \gamma Q_{t-1}^{1-\eta}}{1 - \gamma Q_t^{1-\eta}} \right]^{\frac{1}{1-\eta}}, \quad (33)$$

where, from (13) and (19), the real exchange rate  $Q_t$  can be expressed as a function of the nominal interest rate  $R_t$  given by

$$Q_t = Q(R_t) \equiv \phi_0 \frac{z_t^*}{z(R_t)}. \quad (34)$$

Substituting (33) and (34) into (20) and using the policy rules (32), we obtain a (nonlinear) difference equation in  $\pi_{Ht}$  given by

$$z(\kappa \pi_{H,t+j}^\tau) = \beta z(\kappa \pi_{H,t+j+1}^\tau) \frac{\kappa \pi_{H,t+j}^\tau}{\pi_{H,t+1}} \left[ \frac{1 - \gamma Q(\kappa \pi_{H,t+j+1}^\tau)^{1-\eta}}{1 - \gamma Q(\kappa \pi_{H,t+j}^\tau)^{1-\eta}} \right]^{\frac{1}{1-\eta}}, \quad (35)$$

from which we can prove the following results:

**Proposition 3:** *Under the interest rate rules (32), the following conditions are necessary and sufficient for local equilibrium determinacy:*

- (i) *Under a forward-looking rule with  $R_t = \kappa \pi_{H,t+1}^\tau$ , the policy parameter  $\tau$  satisfies  $\tau < 1$  if  $0 < \gamma \leq \bar{\gamma}$ , and either  $\tau < 1$  or  $\tau > \bar{\tau} > 1$  if  $\bar{\gamma} < \gamma < 1$ ;*
- (ii) *Under a current inflation targeting rule with  $R_t = \kappa \pi_{Ht}^\tau$ , the policy parameter  $\tau$  satisfies  $1 < \tau < -\bar{\tau}$  if  $0 < \gamma < \bar{\gamma}$ , and  $\tau > 1$  if  $\bar{\gamma} \leq \gamma < 1$ ;*
- (iii) *Under a backward-looking rule with  $R_t = \kappa \pi_{H,t-1}^\tau$ , the policy parameter  $\tau$  satisfies  $\tau > 1$  if  $0 < \gamma \leq \bar{\gamma}$ , and  $1 < \tau < \bar{\tau}$  if  $\bar{\gamma} < \gamma < 1$ , where*

$$\bar{\gamma} = \frac{\sigma\eta - \sqrt{\sigma\eta(\sigma\eta - 1) + 1}}{\sigma\eta - 1}, \quad \bar{\tau} \equiv \frac{\tilde{\xi} + 1}{\tilde{\xi} - 1}, \quad \tilde{\xi} = (1 - \gamma)(1 + \xi) - 1, \quad (36)$$

with  $\xi$  given by (22).

**Proof:** Under each of the three alternative policy rules, we can obtain a characteristic equation for (35) through total differentiation and evaluating the derivatives at the balanced-trade steady state. It is straightforward to show that the characteristic equations are formally identical to (23), (29), and (31), the counterparts under CPI inflation targeting rules, with the term  $\xi$  there replaced by  $\tilde{\xi}$ . The rest of the proof is identical to those of Propositions 1 and 2. *Q.E.D.*

Proposition 3 reveals that targeting domestic price inflation has qualitatively similar implications on equilibrium determinacy as targeting CPI inflation. In particular, under both types of inflation targeting rules, the degree of openness (measured by  $\gamma$ ) and the activeness of monetary policy (measured by  $\tau$ ) interact in important ways in generating local equilibrium uniqueness.<sup>7</sup>

### 4.3 Endogenous labor supply

The baseline model with pure exchange captures the intertemporal substitution effects by allowing for asset trading and a terms of trade effect by assuming imperfect substitution between home produced goods and imported goods. Yet, it abstracts away from intratemporal substitutions between leisure and consumption, and in the business cycle literature, such intratemporal margins are also important. We now extend the analysis by introducing endogenous labor supply with flexible prices, and show that the qualitative results we have obtained in the pure exchange economy do not change.

To introduce endogenous labor supply, we modify the utility function to include leisure. In particular, the utility function takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U(c_t, \frac{A_t}{P_t}) - V(n_t) \right], \quad (37)$$

where  $c_t$  denotes the household's consumption basket as in (2),  $A_t$  denotes the household's cash holding as in (5), and  $n_t$  denotes labor hours. To help exposition, we maintain the technical assumptions (A1) and (A2), and make a further assumption that the function  $V(n)$  is strictly increasing, twice continuously differentiable, and that  $\psi = V''(n)n/V'(n) > 0$  is a constant.

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<sup>7</sup>Since CPI inflation is a function of domestic price inflation and the real exchange rate (see (33)), policy rules that target CPI inflation are equivalent to rules that target a combination of domestic price inflation and a measure of real exchange rate.

Since labor income has now become a source of the household's total income, the budget constraint (6) becomes

$$M_{t+1} \leq M_t + X_t + B_t - E_t D_{t,t+1} B_{t+1} - P_t c_t + W_t n_t, \quad (38)$$

where  $W_t$  is the nominal wage.

The production of home goods requires home labor as the only input, with the production function given by the constant returns to scale technology  $y_t = n_t$ . With perfect competition in both the goods market and the labor market, the real wage in units of home goods equals the marginal product of labor. Thus, the nominal wage is given by

$$W_t = P_H t. \quad (39)$$

The foreign economy has similar preferences and technologies, with the import share arbitrarily close to zero. In addition to the equilibrium conditions in the exchange economy, here we have an extra equation representing the optimal intratemporal substitution between leisure and consumption:

$$\frac{-V_n(t)}{U_c(t)} = \frac{W_t}{P_t}, \quad (40)$$

where  $V_n(t)$  denotes the marginal disutility of working.

The following proposition establishes that the qualitative results we have obtained in the pure exchange economy remain unchanged.

**Proposition 4:** *In the small open economy with production, the following conditions are necessary and sufficient for local equilibrium determinacy:*

- (i) *Under a forward-looking rule with  $R_t = \kappa \pi_{t+1}^T$ , the policy parameter  $\tau$  satisfies  $\tau < 1$  if  $0 < \gamma \leq \bar{\gamma}$ , and either  $\tau < 1$  or  $\tau > \bar{\tau} > 1$  if  $\bar{\gamma} < \gamma < 1$ , where*

$$\bar{\gamma} = 1 - \frac{\sigma}{\psi(\sigma\eta + 1)} \left[ \sqrt{1 + (1 + \psi\eta)\psi(\sigma\eta + 1)/\sigma} - 1 \right], \quad (41)$$

and

$$\bar{\tau} = \frac{\xi + 1}{\xi - 1}, \quad \xi = \frac{\sigma\gamma}{1 - \gamma} \left[ \frac{\psi\eta(2 - \gamma) + 1}{\psi(1 - \gamma) + \sigma} \right]. \quad (42)$$

- (ii) *Under a current inflation targeting rule with  $R_t = \kappa \pi_t^T$ , the policy parameter  $\tau$  satisfies  $1 < \tau < -\bar{\tau}$  if  $0 < \gamma < \bar{\gamma}$ , and  $\tau > 1$  if  $\bar{\gamma} \leq \gamma < 1$ .*

- (iii) *Under a backward-looking rule with  $R_t = \kappa \pi_{t-1}^T$ , the policy parameter  $\tau$  satisfies  $\tau > 1$  if  $0 < \gamma \leq \bar{\gamma}$ , and  $1 < \tau < \bar{\tau}$  if  $\bar{\gamma} < \gamma < 1$ .*

**Proof:** (see the Appendix)

It is straightforward to show that, when  $\gamma = 0$ , local equilibrium uniqueness attains if and only if  $\tau < 1$  under forward-looking rules,  $\tau \neq 1$  under current inflation targeting rules, and  $\tau > 1$  under backward-looking rules, as in the pure exchange model. To get a quantitative feel of the results, we plot in Figures 3 the combinations of  $\gamma$  and  $\tau$  that ensure equilibrium determinacy under CPI inflation targeting, with various timing assumptions in the policy rules.<sup>8</sup> Evidently, all the qualitative results that we have obtained in the pure exchange economy carry over to the production economy.

#### 4.4 Sticky prices

To establish the connections between openness and equilibrium determinacy under interest rate rules, we have assumed in our baseline model that prices are perfectly flexible. In the literature, the determinacy issue is often examined in closed economy models with sticky prices. We now consider an extension of the standard Calvo's (1983) sticky price model to a small open economy.

The key ingredients of the model and optimizing conditions are described in the Appendix. To gain insights, we log-linearize the optimizing conditions around the balanced-trade steady state, and examine the determinacy properties of alternative interest rate rules in the system of log-linear dynamic equations.

First, denote a hatted variable as the log-deviation of the level variable from its steady state value. As shown in the Appendix, in a perfect foresight equilibrium, the optimal linearized price-setting rule is given by

$$\hat{\pi}_{Ht} = \beta \hat{\pi}_{H,t+1} + \lambda \hat{v}_t, \quad \lambda = \frac{(1 - \beta\theta)(1 - \theta)}{\theta}, \quad (43)$$

where  $\hat{\pi}_{Ht}$  is the deviation of domestic price inflation,  $\hat{v}_t$  is the deviation of real unit production cost, and  $\theta$  is the fraction of firms that cannot adjust prices in each period. This equation can be interpreted as the open-economy Phillips curve. It differs from the closed economy counterpart in that the CPI inflation  $\hat{\pi}_t$  is here replaced by the domestic price inflation  $\hat{\pi}_{Ht}$  and the unit production cost is here defined as the real wage in units of home produced goods.

Under the benchmark policy rules that we consider, the monetary authority targets the CPI inflation rate, which is related to the domestic price inflation rate through (33). The

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<sup>8</sup>In plotting the figures, we use the calibrated parameter values  $\sigma = 1$ ,  $\eta = 1.5$ , and  $\psi = 2$ .

log-linearized version of (33) is

$$\hat{\pi}_{Ht} = \hat{\pi}_t - \frac{\gamma}{1-\gamma} \Delta \hat{q}_t, \quad (44)$$

where  $\hat{q}_t$  is the log-deviation of the real exchange rate from steady state and  $\Delta \hat{q}_t = \hat{q}_t - \hat{q}_{t-1}$ .

To express the real unit cost  $\hat{v}_t$  as a function of aggregate demand  $\hat{c}_t$ , we log-linearize the labor supply equation (40) around the steady state and get

$$\hat{v}_t = \sigma \hat{c}_t + \frac{\gamma}{1-\gamma} \hat{q}_t, \quad (45)$$

where, to simplify presentation, we have assumed that labor is indivisible so that  $\psi = 0$  (e.g., Hansen (1985)).

The remaining optimizing conditions include the risk-sharing condition (13), the money demand equation (11), and the intertemporal Euler equation (10), the log-linearized version of which are respectively given by

$$\hat{q}_t = -\hat{z}_t, \quad (46)$$

$$\hat{z}_t = \hat{r}_t - \sigma \hat{c}_t, \quad (47)$$

$$\hat{z}_t - \hat{z}_{t+1} = \hat{r}_t - \hat{\pi}_{t+1}, \quad (48)$$

where  $\hat{z}_t$  is the log-deviation of the term  $z_t = U_m(t) + U_c(t)$  from the steady state,  $\hat{r}_t$  is the deviation of the nominal interest rate.

Finally, we specify the interest rate rules in linearized forms:

$$\hat{r}_t = \tau \hat{\pi}_{t+j}, \quad j = -1, 0, 1. \quad (49)$$

To illustrate the role of openness in generating equilibrium determinacy, we focus on a forward-looking rule that targets CPI inflation (with  $j = 1$ ). The following proposition characterizes the determinacy conditions under such a policy rule.

**Proposition 5:** *In the small open economy with sticky prices, if the monetary authority follows the rule with  $\hat{r}_t = \tau \hat{\pi}_{t+1}$ , then the following conditions are necessary and sufficient for local equilibrium determinacy:*

$$1 < \tau < \frac{1-\beta}{(1-\beta)\gamma + \lambda(1-\gamma)} \quad \text{and} \quad \lambda < 1-\beta. \quad (50)$$

**Proof:** (See the Appendix)

When the economy is closed, that is, when  $\gamma = 0$ , the determinacy conditions become

$$1 < \tau < \frac{1-\beta}{\lambda} \quad \text{and} \quad \lambda < 1-\beta. \quad (51)$$

Comparing this condition with that in the open economy reveals that, as  $\gamma$  changes, the range of  $\tau$  values that achieves determinacy also varies. Yet, since determinacy requires  $\lambda < 1 - \beta$ , regardless of the degree of openness, it is unlikely for a forward-looking rule to achieve determinacy with reasonable parameter values. For instance, if  $\beta = 0.99$ , then the restriction on  $\lambda$  would require  $\theta > 0.9$ . Such values of  $\theta$  correspond to an extremely high degree of price stickiness: prices would be fixed on average for ten quarters, in contrast to the empirical evidence that price contracts typically last no more than four or five quarters (e.g., Taylor (1999b)).

Although a forward-looking rule in general fails to achieve determinacy under reasonable parameter values, we find that a rule that targets current or past period inflation can easily achieve determinacy. Under these policy rules, as under the forward-looking rule, the system of optimizing conditions (43)-(48) can be reduced to a single dynamic difference equation in inflation, albeit of a higher order. Although we cannot analytically characterize the determinacy conditions under these alternative rules, it is easy to compute the roots of the characteristic equations for given parameter values. To do this, we follow the standard business cycle literature and set  $\beta = 0.99$ ,  $\theta = 0.75$ ,  $\sigma = 1$ , and  $\eta = 1.5$ . We plot in Figure 4 the determinacy regions under the interest rate rules that target current or past inflation. The figure reveals that an active rule that targets current period inflation can always ensure equilibrium determinacy, regardless of the degree of openness. Under a backward-looking rule, however, the region of equilibrium determinacy shrinks as  $\gamma$  increases. Thus, openness in general matters for equilibrium determinacy in the presence of sticky prices.

## 5 Conclusion

We have established that the determinacy properties of inflation-targeting interest rate rules depend in general on the degree of openness. Such dependence arises from a policy-induced terms of trade effect when the central bank adjusts interest rates in response to inflation. We have also shown that, in a broad class of model environments, openness plays an important role in equilibrium determinacy under interest rate rules that target various measures of the inflation rate.

The model-dependent relation between openness and the conditions for equilibrium determinacy prevents us from making a general policy prescription for a small open economy. Yet,

our experiments suggest cautions in pursuing active inflation-targeting interest rate rules, particularly for a small open economy. A rule that aggressively reacts to current or past inflation rates may be a sound monetary policy for a large and relatively closed economy such as the United States, but the same set of policy rules may not always ensure equilibrium determinacy in a small and relatively open economy such as Canada.

## Appendix

In this appendix, we derive the optimal price-setting rule in the sticky price model and prove Propositions 4 and 5.

### A1. The small open economy with sticky prices

To derive the optimal price-setting rule, we assume that there is a continuum of firms, each producing a differentiated intermediate good indexed by  $j \in [0, 1]$ , using labor as the only input. Firm  $j$  can sell its output either to the home market or to the foreign market. Denote  $c_{Ht}(j)$  and  $c_{Ht}^*(j)$  the quantities of sales to the two markets. The production function is given by  $c_{Ht}(j) + c_{Ht}^*(j) = n(j)$ , where  $n(j)$  is the firm's labor input supplied by the representative household. The household has utility function given by (37), with the consumption good  $c_t$  being a composite of home produced good  $c_{Ht}$  and imported good  $c_{Ft}$ , as in (2). The home good and the imported good are each produced using an aggregation technology given by

$$c_{Ht} = \left( \int_0^1 c_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad c_{Ft} = \left( \int_0^1 c_{Ft}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (52)$$

where  $\epsilon > 1$  is the elasticity of substitution between different types of goods. Cost-minimization of the aggregation sector implies that the demand for each type of intermediate goods is given by

$$c_{Ht}^d(j) = \left( \frac{P_{Ht}(j)}{\bar{P}_{Ht}} \right)^{-\epsilon} c_{Ht}, \quad c_{Ft}^d(j) = \left( \frac{P_{Ft}(j)}{\bar{P}_{Ft}} \right)^{-\epsilon} c_{Ft}, \quad (53)$$

where  $\bar{P}_{Ht} = \left( \int_0^1 P_{Ht}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ ,  $\bar{P}_{Ft} = \left( \int_0^1 P_{Ft}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$  are the price indices of home produced goods and imported goods, respectively. As in the flexible-price models, the demand for the finished goods  $c_{Ht}$  and  $c_{Ft}$  are given by (3), and the price level  $P_t$  is related to the price indices as in equation (4).

In the intermediate goods sector, firms are price takers in the factor market and monopolistic competitors in the product markets. In each period, each firm receives an iid random

signal that enables it to set a new price, taking the demand for its product as given. The signal arrives with probability  $1 - \theta$ . Thus, by the law of large numbers, there is always a fraction  $\theta$  of firms that cannot adjust prices. When a firm  $j$  in the home country can set a price, it chooses its price  $P_{Ht}(j)$  for the home market and  $P_{Ht}^*(j)$  for the foreign market to maximize the discounted future profits

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \theta^{\tau-t} D_{t,\tau} \{ [P_{Ht}(j) - W_{\tau}(j)] c_{H\tau}^d(j) + [S_{\tau} P_{Ht}^*(j) - W_{\tau}(j)] c_{H\tau}^{*d}(j) \}, \quad (54)$$

subject to the demand equation (53) (with the foreign counterpart), where the term  $D_{t,\tau} = \beta^{\tau-t} \frac{U_c(\tau)/P_{\tau}}{U_c(t)/P_t}$  is a discount factor. The resulting optimal pricing rule (for the goods sold in the home market) is given by

$$P_{Ht}(j) = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} \theta^{\tau-t} D_{t,\tau} c_{H\tau}^d(j) W_{\tau}}{\mathbb{E}_t \sum_{\tau=t}^{\infty} \theta^{\tau-t} D_{t,\tau} c_{H\tau}^d(j)}. \quad (55)$$

Thus, the optimal price is a weighted average of future marginal costs (here, the marginal cost is the nominal wage rate since labor is the only input).

The household's optimizing conditions remain the same as in the model with flexible prices. These conditions are summarized by the intertemporal Euler equation (10), the money demand equation (11), and the labor supply equation (40). By log-linearizing (55) around the balanced-trade steady state with zero inflation, we obtain (43) in the text.

## A2. The proofs

**Proof of Proposition 4:** By substitution, the market clearing condition in the production economy reduces to an analogue of (18) and is given by

$$\left[ \frac{1 - \gamma Q_t^{1-\eta}}{1 - \gamma} \right]^{\frac{\eta}{\eta-1}} [(1 - \gamma)c_t + \gamma^* Q_t^{\eta} c_t^*] = n_t. \quad (56)$$

Using this equation, along with (39), (40), and (17), we can express  $c_t$  as a function of  $Q_t$  only. The rest of the proof is identical to those in the pure exchange economy. The only difference is that, in the characteristic equations here, the parameter  $\xi$  is given by (42) instead of (22). *Q.E.D.*

**Proof of Proposition 5:** Under the policy rule (49), the system of optimizing conditions (43)-(48) can be reduced to a second order difference equation

$$[\beta + \tau(\lambda(1 - \gamma) - \beta\gamma)] \hat{\pi}_{t+2} + (\tau\gamma - 1)(1 + \beta + \lambda) \hat{\pi}_{t+1} + (1 - \tau\gamma) \hat{\pi}_t = 0, \quad (57)$$

the characteristic equation of which is given by

$$H(e) = H_2e^2 + H_1e + H_0, \quad (58)$$

where  $H_2 = \beta + \tau(\lambda(1 - \gamma) - \beta\gamma)$ ,  $H_1 = (\tau\gamma - 1)(1 + \beta + \lambda)$ , and  $H_0 = 1 - \tau\gamma$ . Since there is no predetermined variable, determinacy requires the two roots  $e_1$  and  $e_2$  to both lie outside the unit circle.

A passive rule cannot achieve determinacy. With  $\tau < 1$ , the two roots are real and distinct. Since  $H(0) = 1 - \tau\gamma > 0$ ,  $H'(0) = H_1 < 0$ , and  $H(1) = H_2 + H_1 + H_0 = \lambda(\tau - 1) < 0$ , one root must lie inside the unit circle.

Consider an active rule with  $\tau > 1$ . There are two subcases. If  $\tau > 1/\gamma$ , then it is easy to verify that the two roots are both real and distinct. In this case,  $H(0) = 1 - \tau\gamma < 0$  and  $H(1) = \lambda(\tau - 1) > 0$ . Thus, one root lies in the interval  $(0, 1)$  and there is indeterminacy.

In the other case with  $1 < \tau < 1/\gamma$ , the roots may be either real or complex. If the roots are complex, determinacy requires that  $e_1e_2 = H_0/H_2 > 1$ , or  $1 < \tau < \frac{1-\beta}{(1-\beta)\gamma + \lambda(1-\gamma)}$ . For the set of such  $\tau$ 's to be non-empty, we need to have  $\lambda < 1 - \beta$ . If the roots are real, they must have the same sign since  $e_1e_2 = H_0/H_2 > 0$ . With  $1 < \tau < 1/\gamma$ ,  $H(0) = 1 - \tau\gamma > 0$ ,  $H'(0) = (1 + \beta + \lambda)(\tau\gamma - 1) < 0$ , and  $H(1) = \lambda(\tau - 1) > 0$ . Thus, the two roots are both positive and they can be both greater than one if and only if  $e_1e_2 > 1$ , which is guaranteed if (50) holds. *Q.E.D.*

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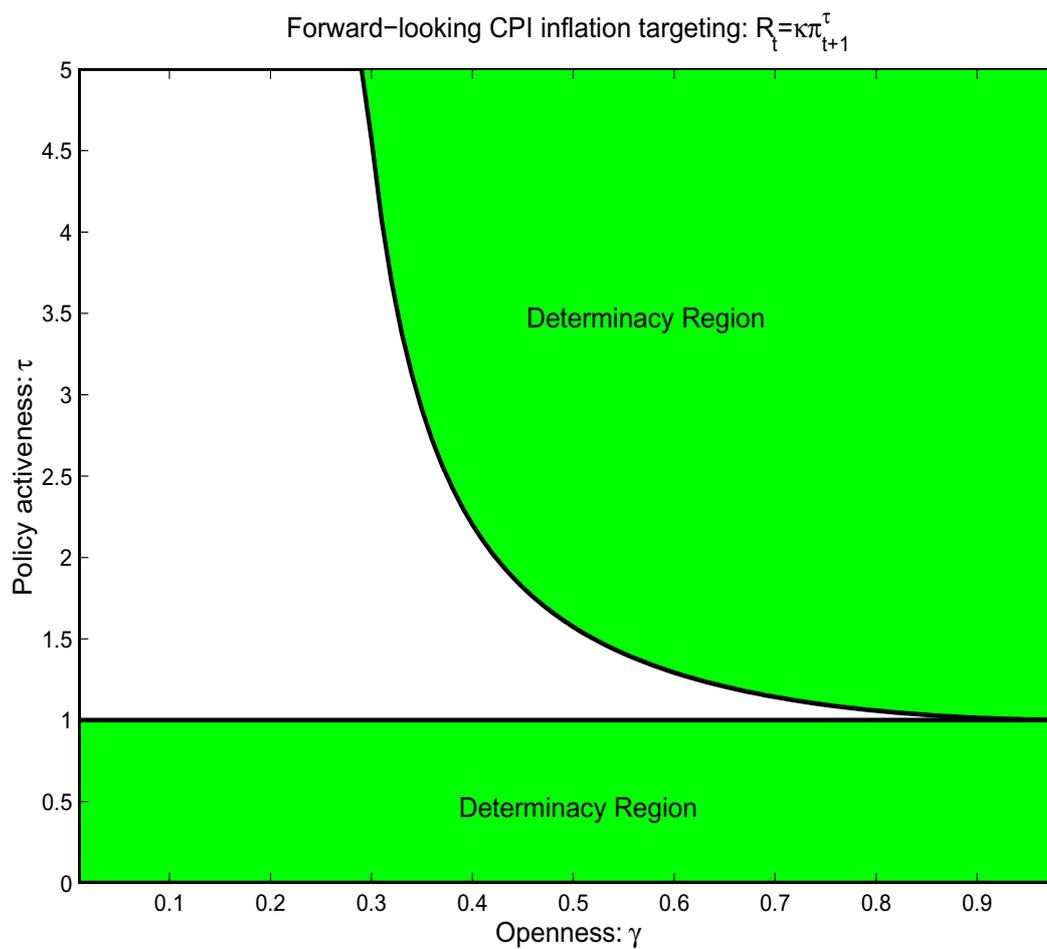


Figure 1:—Equilibrium determinacy under the forward-looking interest rate rule in the exchange economy.

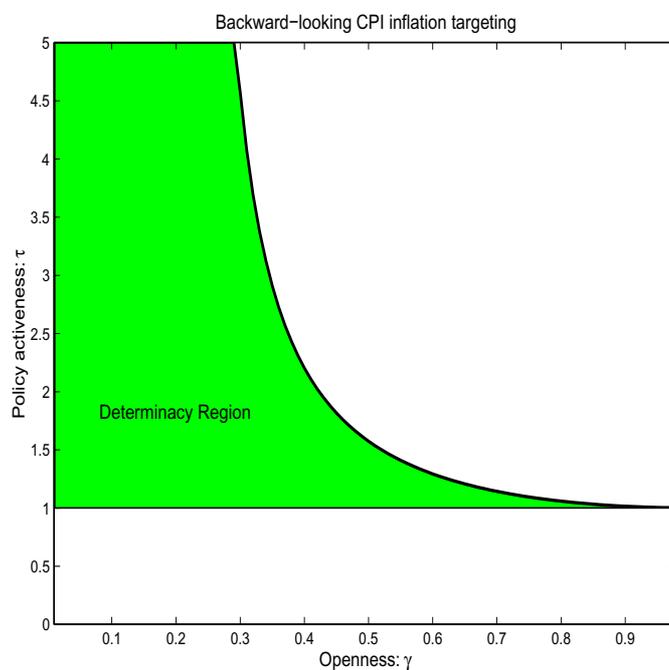
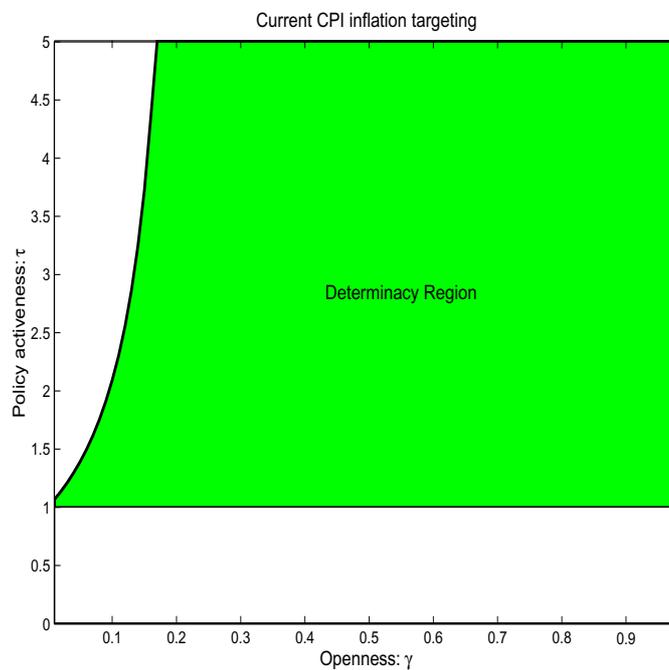


Figure 2:—Equilibrium determinacy under current and backward-looking rules in the exchange economy.

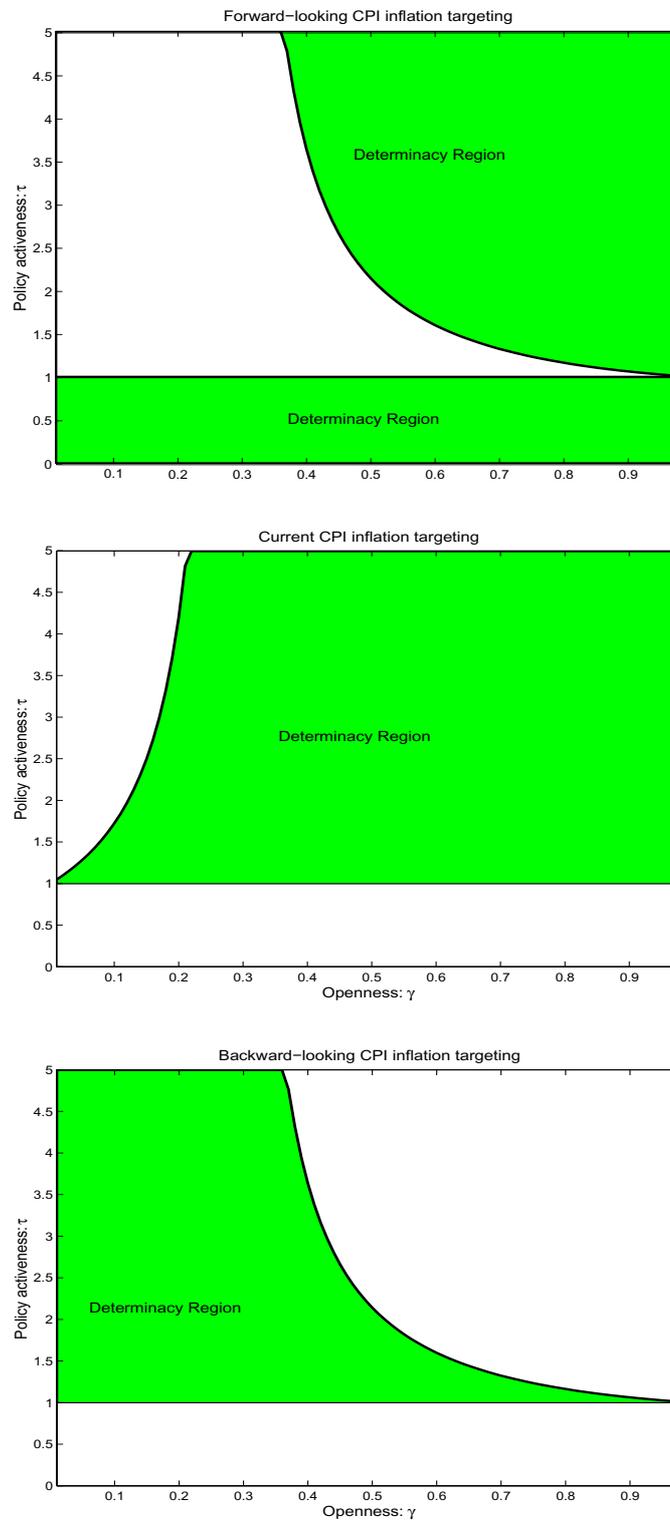


Figure 3:—Equilibrium determinacy under alternative interest rate rules in the production economy with flexible prices.

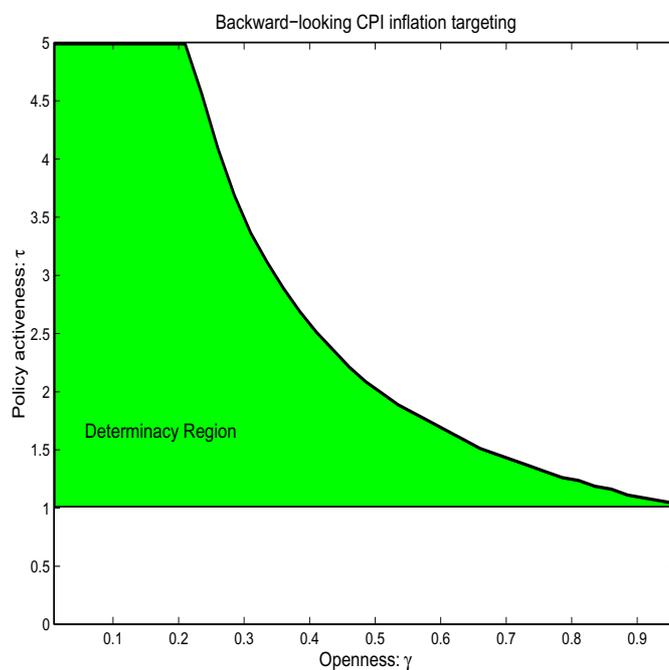
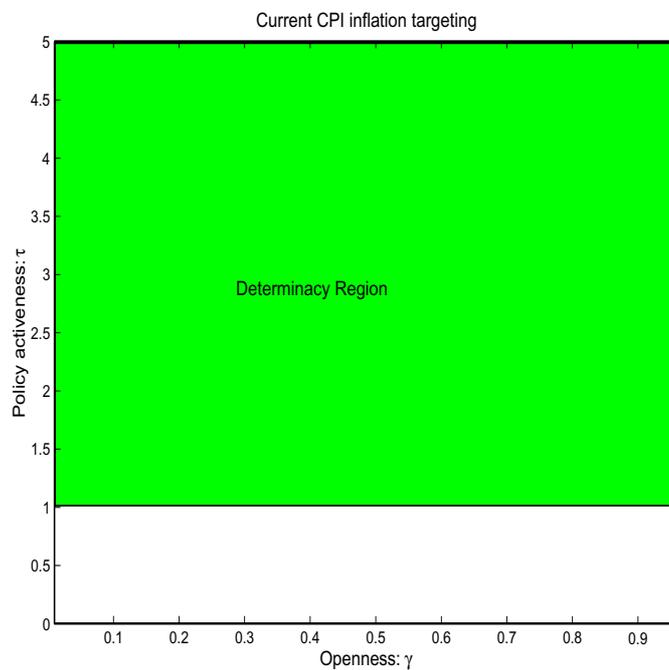


Figure 4:—Equilibrium determinacy under current and backward-looking rules in the sticky price economy.

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