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MATCHING MODEL?**

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by Olivier l'Haridon², Franck Malherbet³
and Sébastien Pérez-Duarte⁴

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Abstract

In this article, we use a stylized model of the labor market to investigate the effects of three alternative and well-known bargaining solutions. We apply the Nash, the Egalitarian and the Kalai-Smorodinsky bargaining solutions in the small firm's matching model of unemployment. To the best of our knowledge, this is the first attempt to implement and systematically compare these solutions in search-matching economies. Our results are twofold. First from the theoretical and methodological viewpoint, we extend a somewhat flexible search-matching economy to alternative bargaining solutions. In particular, we prove that the Egalitarian and the Kalai-Smorodinsky solutions are easily implementable within search-matching economies. Second, our results show that even though the traditional results of bargaining theory apply in this context, they are generally qualitatively different from the standard results, and the differences are quantitatively weaker than expected. This is of particular relevance in comparison with the results established in the earlier literature.

JEL Classification: C71; C78; J20; J60;

Keywords: Search and matching models, Bargaining theory, Nash, Egalitarian, Kalai-Smorodinsky

Non-technical summary

Search-matching models of unemployment have become the standard tool for studying labor markets. A major feature of these models is to consider inflows and outflows, rather than stocks, as natural starting points for any question about the labor market. They have been widely used to study a wide range of both theoretical and policy questions. Among them, a large literature has been devoted to the determinants of structural unemployment in Europe, to labor market reforms, and to the characteristics of cyclical unemployment dynamics. This paper discusses an hypothesis about the bargaining structure of the search-matching model.

At the heart of the model is the concept of *local monopoly rent*, which insures the stability of the worker–firm relationship. This rent emerges from the existence of transaction costs to find the appropriate partner for productive activities. The rent is crucial for the length of the employment relationship: as long as it is positive, the match is sufficiently productive to be economically relevant. Once set, the rent has to be shared between the firm and the worker by a wage rule.

In the bulk of the search and matching literature, the conventional approach to determine wages is based on the use of a specific bilateral bargaining scheme, initially proposed by Nobel-prize winner John Nash. Although this Nash criterion is used almost exclusively in economic applications, there is *a priori* no particular rationale – either empirical or theoretical – to restrict the bargaining scheme to this criterion alone. To our knowledge, the Nash solution to the wage bargaining problem is popular for two reasons. First, it offers a flexible technical solution and allows to calculate the bargained wage on the basis of marginal information only. Second, it can be recovered from an extensive game where both the firm and the worker play a well-defined sequence of moves and have preferences over both the time and the terms of agreement.

However, these arguments do not justify restricting the bargaining scheme for worker-firms pairs; other solutions exist in the literature. Taking the Nash solution as a benchmark, we consider the two other key alternative solutions that have emerged in bargaining theory, namely the Egalitarian and the Kalai-Smorodinsky solutions. The first one, the egalitarian bargaining solution, imposes equal gains for both parties. Wages are chosen so as to give an equal return from reaching an agreement. The second one, the Kalai-Smorodinsky solution, is characterized by equal proportional concessions for both agents from their respective aspiration levels. These most favored bargaining points can be conceptualized as utopia points.

Our central results are twofold. First from a theoretical and methodological viewpoint, we extend a somewhat flexible search-matching economy to alternative bargaining solutions. Second, our results show that even though the traditional results derived from Nash bargaining

apply in this context, they differ remarkably in both qualitative and quantitative terms as compared to the results established in the earlier literature. In particular, we show that if for risk-neutral workers, the three bargaining schemes lead to the same labor outcomes, things turn out to be different when workers are risk-averse. However, based on simulated labor markets, we conclude that these differences appear to be small. A main consequence is that the policy implications are not very sensitive to the choice of the bargaining solution used within the canonical search-matching model.

1 Introduction

Search-matching models of unemployment have become the standard for studying aggregate labor markets. The existence of transaction costs on the labor market results in a local monopoly rent which is shared through bilateral bargaining between the firm and the worker. In the bulk of the search and matching literature, the conventional approach to determine wages is based on the use of a generalized Nash criterion (see for instance the surveys by either Mortensen and Pissarides, 1999 or Rogerson, Shimer and Wright, 2005). Although this criterion is used almost exclusively in economic applications, there is, *a priori*, no particular rationale – either empirical or theoretical – to restrict to the Nash criterion alone. Consequently one may wonder why the Nash solution is so popular. Usually, two lines of arguments are put to the fore. First, the Nash solution is flexible and allows to calculate the bargained wage on the basis of marginal information only. Second, it has a natural strategic counterpart (see *e.g.* Osborne and Rubinstein, 1990). These two lines of argument, however, are not entirely compelling. As a matter of fact, the Nash solution has been challenged on theoretical grounds. On the one hand, various objections have been raised against his axiomatic treatment. Among others, the independence to irrelevant alternatives axiom (IIA) has been the object of considerable controversy mainly because of the fact that it implies independence to contractions of the feasible set¹. On the other hand, the precise meaning of the Nash rule, a product of utility gains, is not easily interpretable (Thomson, 2010).

As emphasized by Thomson (2010), others solutions² such as the Kalai-Smorodinsky (Kalai and Smorodinsky, 1975) and the Egalitarian (Kalai, 1977) solutions³ are important challengers to the Nash's solution in bargaining theory, and have, for the most part, been put aside.

Finally, the prevalence of the Nash solution is all the more questionable in that a number of classical controlled lab-experiments cast some doubts on its relevance (see *e.g.* Nydegger and Owen, 1974, Nydegger, 1977), and tend to support the Kalai-Smorodinsky (hereafter, KS) solution over the Nash solution (Heckathorn, 1978). This latter result appears however to be fragile as it is difficult to create an experimental design to disentangle the KS and the Nash bargaining solutions (Binmore, Swierzbinski, Hsu, and Proulx, 1993). More recently, Bruce and Clark (2010) show that subjects in a two-party, two-attribute experiment prefer Egalitarian outcomes to the Nash bargain.

Despite this, discussions have remained mostly confined to the social choice/bargaining lit-

¹Axioms and bargaining solutions are presented in a detailed form in section 3.

²Those solutions are briefly discussed below and then detailed in section 3.

³It is worth remarking that these two solutions: (i) have also strategic counterparts, see *e.g.* Moulin, 1984 or Miyagama, 2002; (ii) confute the (IIA) axiom and are characterized by the possibility of direct interpersonal comparison of utility (for the Egalitarian solution) and weaker requirement on the feasible set (for the Kalai-Smorodinsky solution), see section 3 for a detailed exposition.

erature and, until recently, have been largely ignored in labor economics. Early and notable exceptions are McDonald and Solow (1981) and Alexander (1992) who offer characterizations of the Nash and the KS solutions in an efficient contract and a right-to-manage model, respectively. Recent contributions in labor economics suggest however that the Nash solution may be too simplistic. In this perspective, the papers from Laroque and Salanié (2004), Gerber and Upmann (2006), and Dittrich (2010) are of particular interest.

Laroque and Salanié (2004) use the KS solution to characterize the spread of the minimum wage to the wage distribution in an efficient contract model *à la* McDonald and Solow (1981). They show that under the Nash solution, a marginal increase in the minimum wage does not affect the wage distribution due to a specific assumption regarding Nash's IIA axiom which makes the bargained wages independent of the minimum wage. The authors convincingly argue that such a property is counterfactual and that the KS solution is the one that is best suited from this particular standpoint. In what follows, we depart from these authors and consider only bargained wages, thereby excluding all others institutional wage agreements.

Gerber and Upmann (2006), again in an efficient contract model, challenge the innocuousness of the Nash solution. They argue that the labor market equilibrium hinges essentially on the bargaining solutions considered. In particular, they show that with the KS solution an increase in the reservation wage has an ambiguous effect on unemployment and that under reasonable parameter values employment may increase. This key result contradicts the traditional effect found with the Nash solution. They further demonstrate that alternative solution concepts such as the Egalitarian solution and the equal-loss solution (Chun, 1988) exhibit the same type of ambiguity. They conclude that this puzzling employment effect is by no means a pathological or degenerate case and is more likely to be the rule than the exception.

Dittrich (2010) considers a dual labor market with a minimum-wage sector and a unionized sector. This structure allows endogenizing the outside option in the efficient contract model as a weighted average of the minimum wage and unemployment benefits. The key result is that with the KS solution, an increase in the minimum wage has an ambiguous effect on employment.

There is hence a need to answer a more general question regarding the innocuousness of the bargaining solutions and assumptions in labor economics. Although the usual specification in the prevailing search-matching framework is Nash ex-post bargaining, the universality of this assumption needs to be questioned. Indeed, as emphasized by Mortensen and Pissarides (1999), wage determination is a major issue in the context of search equilibrium modeling. Existing literature (see Mortensen and Pissarides, 1999) has focused on alternative wage setting mechanisms (*e.g.* insider-outsider divide, monopoly union or efficiency wage) but has neglected alternative bargaining solutions. To the best of our knowledge, this paper is the first paper that attempts

to systematically characterize the way in which bargaining solutions affect equilibrium unemployment in a search and matching model of the labor market.⁴ Taking the Nash solution as a benchmark and following Thomson (1994, 2010), we consider the two other alternative solutions that have emerged in bargaining theory, namely the Egalitarian and the Kalai-Smorodinsky solutions. In a nutshell, the Egalitarian solution assigns equal gain to both parties whereas the KS solution grants the same percentage gain of the maximally attainable outcome over the disagreement point to each player.

At this point, it is important to emphasize that our approach has been kept simple on purpose, in order to obtain, whenever possible, analytically tractable results and to focus on comprehensible mechanisms. This voluntary parsimony is however not restrictive and enables us nonetheless to characterize a number of positive and normative properties of the three solutions.

Our central results are twofold. First from a theoretical and methodological viewpoint, we extend a somewhat flexible search-matching economy to alternative bargaining solutions. In particular and contrary to common beliefs, the Egalitarian and the KS solutions are proved to be easily implementable and mathematically tractable within search-matching economies. Second, our results show that even though the traditional results derived from Nash bargaining apply in this context, they differ remarkably in both qualitative and quantitative terms as compared to the results established in the earlier literature (see for instance, Gerber and Upmann, 2006 and Dittrich, 2010). More precisely, through the paper we contrast two economies: an economy with perfectly transferable utility which serves as our benchmark and an economy with a partially transferable utility only.⁵ In the former case, we show that the three bargaining solutions boil down to a unique solution and yield a Pareto optimal resource allocation when the economy is constrained efficient. In the latter case, when utility is not perfectly transferable because of workers' risk aversion, the three solutions are proved to diverge and a first-best resource allocation is not feasible when wages are bargained over. Interestingly, the nonlinear discrepancy between the Nash and the KS solutions is proved to be small and this from both a qualitative and a quantitative viewpoint. This is not only the case in the neighborhood of linearity as might be expected, but also generalizes at high (and reasonable) levels of risk aversion for plausible parameter values. Of secondary interest, but nevertheless worth mentioning, is that under Constant Relative Risk Aversion (CRRA) the Nash and KS solutions yield similar outcomes when the Arrow-Pratt coefficient of risk aversion is equal to 0 and 2, *i.e.* for boundary values in the

⁴Similar questions have been separately raised in search models of monetary exchange (see Aruoba, Rocheteau and Waller, 2007).

⁵Following Burdett and Wright (1998), transferable utility refers to the case where the frontier in utility is linear while non-transferable utility refers to the case where outside institutions or constraints yield non-linearity in the frontier. In this article, the frontier is nonlinear because of workers' preferences and because we do not introduce any additional constraints for discontinuity. To avoid any confusion between the linear and nonlinear case, we say that utility is partially transferable when workers are risk-averse.

range accepted in the literature.⁶ In general, the Nash and KS solutions behave similarly under fairly mild conditions. Therefore, and except for considerations specific to the axiomatic differences between bargaining solutions, the KS solution offers no significant improvement over the Nash solution for labor market analysis. Results are more contrasted as regards the Egalitarian solution. The difference between the Nash and the Egalitarian solutions is much stronger. This contrast owes much to the axiomatic difference between the two solutions, in particular the scale invariance property. Finally, the three solutions are proved to behave identically following a change in the key structural parameters. This is analytically true for the solutions considered in the neighborhood of linearity and globally for the Nash and the Egalitarian solutions. However, because of the proximity between the Nash and the KS solutions, the results generalize for the KS solution to a wide range of plausible parameter values. Hence our results again run counter to those previously established in the literature.

The outline of this article is as follows. Section 2 presents a stylized search model. Section 3 details the bargaining solutions under consideration. A particular emphasis is put on some key axioms. Section 4 focuses on the key properties of the model when utility is perfectly transferable. Section 5 extends the model to the case of partially transferable utility. The properties established in section 4 are then discussed and illustrated in the case of partially transferable utility. Finally, section 6 concludes.

2 The Model

The benchmark model builds on a continuous-time search and matching framework. The core of the model is in many ways similar to the canonical textbook model of Pissarides (2000, Chapter 1), the objective being to remain as close as possible to a well-established framework, except that alternative bargaining solutions are considered. We will consider a slightly modified framework in section 5 of this paper. For the remainder of the paper, we will restrict the analysis to the steady state.

2.1 Main assumptions

We consider an economy populated with a continuum of infinitely-lived and homogenous workers and a continuum of identical firms holding at most one vacancy. The size of the population is normalized to one. All agents discount future payoffs at the rate $r > 0$.

The labor market has matching frictions. Searching for a job and matching with a worker are costly activities. Frictions are captured by a customary matching function, H , with constant

⁶See Chetty (2006) for a review.

returns to scale. The flow of hires satisfies $H(\nu, u)$ where ν and u indicate the number of vacancies and unemployed, respectively. Let $\theta \equiv \nu/u$ be the labor market tightness. The transition rate for vacancies is $q(\theta)$ with $q(\theta) \equiv H(\nu, u)/\nu = H(1, 1/\theta)$. Similarly, the flow out of unemployment is $\theta q(\theta)u$ with $\theta q(\theta) \equiv H(\nu, u)/u = H(\theta, 1)$. The properties of the matching function imply that $q(\theta)$ and $\theta q(\theta)$ are decreasing and increasing in labor market tightness, respectively. Filled jobs are destroyed at a Poisson rate s . The evolution of the unemployment rate satisfies:

$$\dot{u} = s(1 - u) - \theta q(\theta)u \quad (1)$$

In a steady-state, the unemployment rate is obtained by equating the flows out of unemployment, $\theta q(\theta)u$, to the flows in unemployment, $s(1 - u)$, and verifies:

$$u = \frac{s}{s + \theta q(\theta)} \quad (2)$$

It is worth remarking here that the steady-state unemployment rate, u , is decreasing in the labor market tightness, θ , *i.e.* $\frac{\partial u}{\partial \theta} \Big|_u < 0$.

2.2 Value functions

Let U and $E(\cdot)$ denote the lifetime expected utility of an unemployed worker and an employed worker, respectively. The associated value functions verify:

$$rU = v(z) + \theta q(\theta) (E(w) - U), \quad (3)$$

$$rE(w) = v(w) + s(U - E(w)). \quad (4)$$

Workers' utility stems from an instantaneous von Neumann and Morgenstern utility function, $v(\cdot)$ with the usual properties. According to eq. (3), an unemployed worker earns an instantaneous real return z and changes state on the labor market at rate $\theta q(\theta)$. In turn, eq. (4) indicates that a worker earns an income flow w and changes state at rate s .

Let V and $J(\cdot)$ denote the lifetime expected value of an open vacancy and an occupied job, respectively. The associated value functions verify:

$$rV = -\gamma + q(\theta) (J(w) - V), \quad (5)$$

$$rJ(w) = y - w + s(V - J(w)). \quad (6)$$

Recruiting is costly. According to equation (5), advertising a vacant job incurs a flow cost γ . A vacancy changes state at a rate $q(\theta)$. Similarly, equation (6) states that a filled job yields an instantaneous profit $y - w$ and changes state at a rate s .



3 Bargaining

In this section, we describe the bargaining problem related to the settlement of a contract between a worker and a firm. We first detail the set of feasible outcomes and then apply three standard bargaining solutions to the problem – the Nash, Kalai-Smorodinsky, and Egalitarian solutions. It is important here to mention that two complementary approaches to bargaining coexist in the literature. The first one, initiated by Nash (1950), is axiomatic. Based on a set of behavioral properties, or axioms, it defines the class of solutions consistent with these properties, and is thus mainly normative. The second approach is strategic. It describes the necessary strategic conditions to reach an agreement in a non-cooperative game. In the present paper, we aim to analyze the equilibrium properties of the three different solutions. For the sake of parsimony, we restrict ourselves to the axiomatic approach and focus on the normative properties of the different solutions rather than on the specific non-cooperative bargaining conditions required to reach an agreement. It is worth noting nevertheless that the different solutions also have strategic foundations (see *e.g.* Moulin, 1984 or Miyagawa, 2002).

Upon matching, a job-worker pair generates a local monopoly rent. This rent originates from the matching frictions on the labor market and implies that the expected total return from productive activities, $J + E$, is strictly greater than the total return from search, $U + V$. This rent is then shared through a constant wage rate resulting from a decentralized firm-worker bargaining. Assuming symmetric equilibria, the aggregation of match-specific wage rules leads to an aggregate wage equation.⁷

3.1 The feasible set

The bargaining problem can be described in a two-dimensional space of the expected lifetime utilities of both agents. In order to characterize it, two elements are necessary: the disagreement point and the set of feasible allocations obtained when an agreement is reached. The disagreement point is equal to the values of continued search for the firm and the worker, namely $d = (V, U)$. It is worth observing from (4) that a wage set at $w = rU$ makes the worker indifferent between employment and unemployment: $E(rU) = U$. Similarly, according to (6), a wage set at $w = y - rV$ makes the value of a job to the firm identical to the value of vacant job, hence $J(y - rV) = V$. Consequently, the disagreement point can also be written as $d = (J(y - rV), E(rU))$ and the wage w must satisfy the following constraint in an agreement point: $y - rV \geq w \geq rU$. Let Ω define the set of allocations obtained when an agreement is

⁷For an in-depth explanation, see Pissarides (2000, Chapter 1). For the remainder of the paper, and in order to minimize the notations, we will consider only symmetric equilibria.

reached; Ω is the set of feasible allocations:

$$\Omega = \{x \in \mathbb{R}^2 | \exists w \in \mathbb{R}, rU \leq w \leq y - rV, x \preceq X(w) \text{ and } X(w) \succeq d\} \quad (7)$$

where $X(w) = (J(w), E(w))$ for $rU \leq w \leq y - rV$ is the Pareto frontier, and where $(a, b) \preceq (c, d)$ if and only if $a \leq c$ and $b \leq d$. (Ω, d) is a standard two-agent bargaining problem where Ω is convex and closed, and the individually rational utility allocations are bounded. The following two lemmas prove that (Ω, d) satisfies these two properties:

Lemma 1. *Ω is convex and closed.*

Proof. The Pareto efficient allocations are given by (4) and (6). The Pareto-frontier is defined as the set of all points $(J, E(J))$ with $E(J) = [v(y - (r + s)J + sV) + sU]/(r + s)$. Since $v' > 0$ and $v'' \leq 0$, the first and second derivatives of $E(J)$ are negative, the Pareto frontier is strictly decreasing and strictly concave. Convexity of Ω follows. By continuity of E and J , Ω is closed. \square

Lemma 2. *The set of individually rational utility allocations $\{\delta \in \Omega | \delta \succeq d\}$ is bounded and there exists $\delta \in \Omega$ with $\delta_j > d_j$ for $j = 1, 2$.*

Proof. The maximum value from a job to the firm is reached when worker is paid her reservation wage $w = rU$. The maximum value from a job to the worker is reached when $w = y - rV$ so that she captures the entire surplus from the match. If $\delta \in \Omega$ is individually rational ($\delta \succeq d$) then $\delta \preceq (J(rU), E(y - rV))$ and $\{\delta \in \Omega | \delta \succeq d\}$ is bounded. By (4) and (6), $E(w)$ is increasing in w and $J(w)$ is decreasing in w . There exists $\tilde{w} = rU + \epsilon$, $\epsilon \in (0, y - rV - rU)$ such that $(J(\tilde{w}), E(\tilde{w})) \succeq d$. Therefore there exists $\delta \in \Omega$ with $\delta_j > d_j$ for $j = 1, 2$. \square

We now proceed with the exposure of the three key solutions. We restrict ourselves to symmetric bargaining and choose as a benchmark a case where utility is perfectly transferable.⁸ This case constitutes the standard search and matching framework (see *e.g.* Pissarides, 2000, and Rogerson et al., 2005). Technically, it amounts to assuming that the utility function is increasing and linear in its argument; we will relax this hypothesis in section 5. Each solution yields an aggregate wage equation. In equilibrium, combined with a job creation condition, the aggregate wage equation determines the equilibrium values of labor market tightness, θ^* , and of the wage, w^* . Finally, given these values, one obtains thanks to eq. (2) the steady state unemployment rate.

⁸The results readily extend to asymmetric bargaining; proofs are available upon request from the authors.

3.2 The Nash solution

The Nash solution (1950) is the most common solution to determine wages in the bulk of the literature on labor markets with frictions. It follows from the maximization of the expected present value of the job to the worker and to the employer, net of the value of searching for an alternative partner. Wages are chosen so as to maximize the product of each partner's net return from agreement and cooperation. The Nash solution δ^N to the problem defined by the set of feasible allocations Ω and the disagreement point d is defined as:

$$\delta^N(\Omega, d) = \{x \in \Omega | \exists w^*, x = X(w^*) \text{ and } w^* = \arg \max_w (J(w) - V)(E(w) - U)\} \quad (8)$$

The wage w_N^* solves:

$$w_N^* = \arg \max_w (J(w) - V)(E(w) - U) \quad (9)$$

This solution is the only one that satisfies the four following axioms:

- Pareto-Optimality (PAR)
- Symmetry (SYM)
- Invariance to equivalent utility representation (INV)
- Independence of irrelevant alternatives (IIA)

The first two axioms, (PAR) and (SYM), are common to all the bargaining solutions we will consider. They are detailed in great depth *inter alia* in Osborne and Rubinstein (1990) and Thomson (1994). Roughly speaking, they indicate, respectively, that no party can be made better off without harming the other (PAR) and that the solution is invariant to a permutation in the players (SYM). The third axiom (INV), scale invariance or invariance to equivalent utility representation, implies that the solution is invariant under positive affine transformations. This property is inherited from the von Neumann-Morgenstern utility specification. It prevents basing compromises on interpersonal comparisons of utility. The fourth and last axiom is by far the most controversial.⁹ Generally speaking, the contraction independence or independence of irrelevant alternatives (IIA) states that the outcome of a bargain (in our context, the bargained wage) should remain the same if the set of feasible agreements shrinks and still encompasses the initial solution.

The Nash solution is represented on Figure 1. The solution to the bargaining problem satisfies a tangency condition between the Nash curve and the Pareto frontier.

⁹The next two solutions do not use the Independence of irrelevant alternatives (IIA) axiom. See below for a detailed presentation of these solutions.

Figure 1 about here

Under Nash bargaining, the aggregate equilibrium wage is given by (see Appendix A):

$$w_N^* = rU + \frac{1}{2}(y - rV - rU) \quad (10)$$

3.3 The Egalitarian solution

The origin of this solution is reputedly difficult to trace back. Arguments in favor of this solution are detailed with attention in Kalai (1977). The solution is characterized by equal gains for both parties. Wages are chosen so as to give an equal return from reaching an agreement. This concept is central to social choice economics and aims at endowing each agent with exactly the same welfare. The solution does not preserve all the axioms from the traditional Nash solution. As briefly mentioned above, the contraction independence has come under a lot of criticism. In the Egalitarian case this axiom is replaced by a strong monotonicity axiom (MON). It should be noted that contrary to the Nash solution, this solution is not scale invariant (see *e.g.* Thomson (1994) for details). It follows that the Egalitarian solution is the only solution that satisfies the following axioms:

- Pareto-Optimality (PAR)
- Symmetry (SYM)
- Strong monotonicity (MON)

The Egalitarian solution performs best from the viewpoint of monotonicity. The strong monotonicity (MON) axiom states that all players should benefit from an expansion of the feasible set irrespective of whether the expansion is biased toward a particular player. This solution satisfies:

$$\delta^E(\Omega, d) = \{x \in \Omega | \exists w^*, x = X(w^*) \text{ and } (J(w^*) - V) = (E(w^*) - U)\} \quad (11)$$

The Egalitarian solution is represented on Figure 2 and is given by the intersection between the Pareto frontier and the 45° line.

Figure 2 about here

The aggregate equilibrium wage is then defined by (see Appendix A):

$$w_E^* = rU + \frac{1}{2}(y - rV - rU) \quad (12)$$

3.4 The Kalai-Smorodinsky solution

The Kalai-Smorodinsky solution (1975) is characterized by equal proportional concessions for both agents from their respective most favored bargaining points, called utopia points. For each agent, this utopia point is defined by the maximum utility attainable subject to the constraint that no agent should receive less than her status-quo utility.

$$\delta^{KS}(\Omega, d) = d + \Lambda(x^* - d) \quad (13)$$

where $\Lambda = \max\{\lambda \in \mathbb{R} \mid d + \lambda(x^* - d) \in \Omega\}$ and where x^* , the utopia point, denotes the pair of best individual outcomes in Ω for each agent. From an axiomatic perspective, this solution is in many aspects similar to the Nash solution except as regards the contraction independence axiom. This axiom is replaced here with an individual monotonicity (IM) axiom. This solution is the only one that satisfies the following four axioms:

- Pareto-Optimality (PAR)
- Symmetry (SYM)
- Invariance to equivalent utility representation (INV)
- Individual Monotonicity (IM)

The individual monotonicity (IM) axiom states that an expansion of the feasible set biased toward a particular player always benefits her. This monotonicity axiom is therefore weaker than that of the Egalitarian case.

In the particular case of the KS solution, the first difficulty we are confronted with stems from the definition of the utopian ideal. This definition appears to be crucial since it affects the very nature of the disagreement points for each player. As has already been emphasized, the disagreement points are determined by the fall-back or status-quo position each player faces. In our framework, these positions are given by the expected utility of unemployment for a worker and the expected value of a vacancy for a firm. Yet these two values are themselves affected by the expectations of the agents about the labor market equilibrium. In what follows, in the case of a failure both parties return to their searching activities and expect to earn or to pay the equilibrium aggregate wage. Such an expectation is encompassed in the utopia point, which is therefore defined as *pragmatic*. However, a more radical definition of utopia could have been retained. In such a case, a breakdown in the bargaining leads both parties to expect to earn or to pay the utopian wage in the next match, *i.e.* the highest possible wage for the worker and the lowest possible wage for the firm. However this case, which is defined as *utterly utopian*, is difficult to justify. As a matter of fact, this definition leads firms to expect the market

wage to equal the reservation wage and conversely, workers to expect the market wage to equal productivity. As both expectations are degenerate labor market equilibria, we will focus solely on the more appealing *pragmatic* definition.

We therefore rewrite (4) and (6) to take into account the agents' most opportunistic expectation. Henceforth, \widehat{X}^k will refer, by convention, to agent's k utopian expectations for $X \in \{E, J\}$ and $k \in \{e, f\}$ where e stands for *employee* and f for *firm*. Similarly, \widehat{w}^k refers to the utopian wage for agent k . The firm's utopia solves:

$$r\widehat{J}^f = y - \widehat{w}^f + s[V - \widehat{J}^f], \quad (14)$$

where \widehat{J}^f denotes the firm's most opportunistic expectation when it pays the utopian wage \widehat{w}^f . The firm's utopia implies that the net surplus of the worker is nil. The firm appropriates all the rent from the match so that the utopian wage (for the firm) is exactly equal to the reservation wage rU . Formally, the wage is therefore equal to $\widehat{w}^f = rU$. On the worker's side, utopia solves:

$$r\widehat{E}^e = \widehat{w}^e + s[U - \widehat{E}^e], \quad (15)$$

where \widehat{E}^e denotes the worker's most opportunistic expectations when she gets the utopian wage \widehat{w}^e . The worker's utopia implies that the net surplus of the firm is nil. Hence, the worker seizes all the rent from the match so that the utopian wage (for the worker) is exactly equal to $y - rV$. Formally, the wage is therefore equal to $\widehat{w}^e = y - rV$. The Kalai-Smorodinsky solution satisfies:

$$\frac{J(w) - V}{\widehat{J}^f - V} = \frac{E(w) - U}{\widehat{E}^e - U} \quad (16)$$

The KS solution is represented on Figure 3. This solution is determined by the intercept between the line that connects the disagreement point $d = (V, U)$ and the utopian ideal $(\widehat{J}^f, \widehat{E}^e)$ and the Pareto frontier.

Figure 3 about here

The aggregate equilibrium wage is then defined by (see Appendix A):

$$w_{KS}^* = rU + \frac{1}{2}(y - rV - rU) \quad (17)$$

4 Labor Market Equilibria and Properties

4.1 Job creation curve (Jc)

Firms enter the labor market until all profit opportunities from new jobs are exhausted. In equilibrium, the rent from a vacant job is zero and the free-entry condition, $V = 0$, is satisfied.

Using (5) and (6) together with the free-entry condition yields the traditional job creation equation:

$$\frac{\gamma}{q(\theta)} = \frac{y - w}{r + s} \quad (18)$$

Labor demand decreases in the expected or average wage w , in the interest rate r , in the job destruction rate s and in the cost of a vacancy γ . Conversely, it increases in the labor productivity y . The job creation curve is downward sloping in the (θ, w) -space, hence the negative partial derivative $\frac{\partial \theta}{\partial w} \Big|_{J_c} < 0$.

4.2 Wage curves (wc)

In the previous section, we derived three aggregate wage equations. The Nash w_N^* , the Egalitarian w_E^* and the Kalai-Smorodinsky w_{KS}^* wage curves are defined by the relations (10), (12) and (17), respectively. Under each solution, the wage increases in the labor productivity, y , in the real return, z and in the advertising cost, γ . The three wage curves are upward sloping in the (θ, w) -space, hence the positive partial derivative $\frac{\partial \theta}{\partial w} \Big|_{wcj} > 0$, for $j \in \{N, E, KS\}$. For the sake of parsimony, we restrict our analysis to symmetric bargaining. All three solutions (Nash, Egalitarian and Kalai-Smorodinsky) have their counterpart in the asymmetric case and our results remains under these generalizations. The Nash solution becomes the *Generalized or Weighted Nash solution*, the Egalitarian solution becomes the *Proportional solution* and the Kalai-Smorodinsky solution becomes the *Weighted Kalai-Smorodinsky solution*.

4.3 Equilibria

Definition 1. A labor market equilibrium with perfectly transferable utility is made up of the labor market tightness, θ^* , defined by (18), and a wage equation, w_j^* , for all $j \in \{N, E, KS\}$, defined by one of (10), (12) or (17). *In other words, the labor market equilibrium is determined in the (θ, w) -space by:*

- *the intercept of the curves defined by (10) and (18) if the wage is bargained according to the Nash criterion.*
- *the intercept of the curves defined by (12) and (18) if the wage is bargained according to the Egalitarian criterion.*
- *the intercept of the curves defined by (17) and (18) if the wage is bargained according to the Kalai-Smorodinsky criterion.*

The labor market equilibria are represented on Figure 4. It is worth noting that when utility is perfectly transferable, the three wage equations have exactly the same shape (upward sloping

and linear) in the (θ, w) -space.¹⁰

Figure 4 about here

4.4 Properties

Proposition 1. *Equivalence.* *In a steady-state equilibrium with perfectly transferable utility, the three bargaining solutions collapse to a single aggregate wage equation. This equation verifies:*

$$w = \frac{1}{2}z + \frac{1}{2}(y + \gamma\theta) \quad (19)$$

Proof. Straightforward, using the free entry condition, $V = 0$, and noting that the reservation wage is equal to $rU = z + \gamma\theta$. \square

It follows that there is a single labor market equilibrium (θ^*, w^*) compatible with the three solutions that yields an unique equilibrium unemployment rate. This result show that when the boundary of the feasible set is linear, any scale invariant solution gives the same answer as Nash. Proposition 1 thus establishes a strict equivalence result between the three wage rules in the canonical search and matching model.

When the labor market has matching frictions, the tightness is not necessarily efficient due to trade externalities between firms and workers. Otherwise stated, one extra hiring firm makes workers better off and leaves other firms worse off. Conversely, one additional searching worker makes firms better off and leaves other workers worse off. In our benchmark economy, the objective of the benevolent social planner consists in maximizing total production with respect to θ subject to the same matching constraint on labor market flows as the decentralized economy. For the sake of simplicity, we restrict ourselves to the case where $r \rightarrow 0$, thereby ignoring transitional dynamics. The objective of the planner simplifies to ¹¹:

$$\max_{\{\theta, u\}} \Gamma = (1 - u)y - u\theta\gamma \quad (20)$$

subject to eq. (2). The instantaneous social output is made up of the market return of productive job-worker pairs, $(1 - u)y$, minus the aggregate search costs $\gamma\nu = \theta u\gamma$. Two remarks must be made when utility is perfectly transferable. First, the planner is not concerned by distributional issues. Wages and unemployment benefits are consequently excluded from the social welfare function. Second, Γ is similar here to a Bergson-Samuelson social welfare function and since

¹⁰A formal characterization of the labor market equilibria is given in Appendix C.

¹¹Note that we previously normalized the utility of leisure/home production to zero.

the planner has no aversion to inequity (by assumption), Γ is reduced to an Utilitarian welfare function.

Therefore the critical point here is to assess whether the decentralized (bargaining) equilibria is socially efficient, *i.e.* whether the social welfare is maximized or not. In the canonical search and matching model, it is well established that a necessary condition for a decentralized equilibrium with Nash bargaining to be efficient, is given by the so-called Hosios-Diamond-Pissarides (HDP hereafter) condition. Technically, this condition implies that the elasticity of the matching function with respect to unemployment must be equal to the bargaining power of the worker. It translates into $\eta = \frac{1}{2}$ in our symmetric setup where η is the elasticity of the matching function. The economy is said to be constrained efficient if the HDP condition holds. By virtue of Proposition 1, when utility is perfectly transferable, this result generalizes to the Egalitarian and the KS wage rules. An additional condition for efficiency is that the equilibrium is not affected by unemployment benefits, z . It follows that social output is maximized under the three bargaining solutions when the HDP condition is fulfilled and $z = 0$. The following proposition summarizes this point.

Proposition 2. *Efficiency.* *When utility is perfectly transferable, $z = 0$, and the economy is constrained efficient, the Nash, the Egalitarian and the Kalai-Smorodinsky solutions all maximize social welfare. It follows that job creation and unemployment are socially efficient whatever the bargaining solution considered.*

Proof. See Appendix D. □

Proposition 2 demonstrates that when congestion/search externalities are internalized in a decentralized equilibrium, then the market creates an efficient number of jobs and yields a socially optimal unemployment rate. For instance, Hosios (1990) argues in his seminal paper that under perfectly transferable utility, none of the results should depend on any particular rationalization for the wage.¹² Our results generalize Hosios' results to the Kalai-Smorodinsky and Egalitarian bargaining solutions.

To complete the comparison between the three bargaining solutions, we now turn to some classical comparative statics exercises. The critical point here is to check whether or not the results provided by Gerber and Upmann (2006) hold in the standard search and matching framework. Recall that these authors argue that under fairly mild conditions economic policy implications (*e.g.* a change in the reservation wage) hinge entirely on the bargaining solution

¹²In his seminal article, Hosios (1990) considers the Nash solution only under perfectly transferable (linear) utility. See Hosios (1990) p. 283.

considered. The next proposition summarizes the comparative statics results for the key exogenous (policy) parameters.

Proposition 3. Comparative statics. *Whatever the bargaining solution considered:*

- *The aggregate wage increases in the match productivity y and in the real return z , and decreases in the vacancy cost γ , the interest rate r , and in the destruction rate s .*
- *The labor market tightness increases in the match productivity y , and decreases in the real return z , in the vacancy cost γ , in the interest rate r , and in the destruction rate s .*

Proof. See Appendix E. □

By virtue of Proposition 1, it is easy to conclude that modeling wage setting through the Nash, the Egalitarian or the Kalai-Smorodinsky bargaining solution does not affect the sign or the value of the derivative of the wage or the tightness with respect to the main exogenous variables. The usual results apply (see Pissarides, 2000). Comparative statics results are summarized in Table 4 of Appendix E.

Hence, the three preceding propositions stress the strict equivalence between the bargaining solutions in the standard search and matching model when utility is perfectly transferable. This last assumption is fairly standard and from this standpoint, using one solution rather than another one seems to be neutral. This obviously runs counter to the arguments developed by Gerber and Upmann (2006). Where does the divergence between the two models come from? We will examine this issue in the next section.

5 Matching Under Partially Transferable Utility

So far, we have restricted the analysis to the case of perfectly transferable utility between the firms and the workers. In the previous development, such an assumption consists in assuming a linear utility function for each player in the bargaining games. This hypothesis is fairly standard in the search and matching literature (see *e.g.* Pissarides, 2000 and Rogerson et al., 2005), and amounts to assuming complete financial markets where workers may fully insure against income variation and unemployment hazard. In this section, we relax this hypothesis and consider the case where risk averse workers have no access to financial markets. In this slightly modified framework, we characterize the three different solutions when utility is partially transferable¹³

¹³The case of partially transferable utility has recently attracted much attention in search equilibrium models of the marriage market –see for instance Coles and Burdett (1997) and Burdett and Wright (1998)– but none of these papers have considered solution concepts other than that of Nash.

and proceed to the comparison with the benchmark economy. It is worth noting that the job creation equation (18) is invariant to workers' preferences. For that reason and for parsimony's sake, we impose from the start the free entry condition so that the value of a vacancy is nil, $V = 0$.

5.1 Equilibrium Wages

Workers' preferences are now represented with a concave instantaneous von Neumann and Morgenstern utility function. Risk averse workers want to smooth utility across states. Within the scope of expected utility and when the firm and the worker bargain on wages, any increase in the degree of the worker's risk aversion strengthens the firm's position in the negotiation. Risk aversion reduces the set of feasible allocations Ω and modifies the shape of the Pareto frontier as depicted on the figures below. Note that it is also possible to alter the shape of the Pareto frontier by considering other institutional wage setting agreements. However, assuming risk aversion is more adapted to our model and is of particular relevance when dealing with *e.g.* optimal unemployment insurance policy (see for example Fredriksson and Holmlund, 2001 or Coles and Masters, 2006). In addition and as shown by Burdett and Wright (1998), it should be kept in mind that two-sided search with non-transferable or partially transferable utility may lead to multiple equilibria. However, the simple per period payoff or utility assumed in this paper provides a sufficient condition for uniqueness and rules out the possibility of multiple equilibria (see for instance Burdett and Wright, 1998, Proposition 2, for a formal statement).

Nash: The solution to the bargaining game still satisfies a tangency condition between the Nash curve and the Pareto frontier. Under Nash bargaining with non transferable utility, the aggregate equilibrium wage is implicitly defined by (see Appendix A for details):

$$\frac{v(w) - v(z)}{v'(w)} = y - w + \gamma\theta. \quad (21)$$

Figure 5 about here

The Nash solution with partially transferable utility is represented in Figure 5. The aggregate wage is set so as to equalize the workers' net utility gains (the left hand side) and the firms' profits (the right hand side). Workers' attitude toward risk has a direct impact on the left hand side of the wage equation and reduces the utility gap between wage and real return z . Thus, as the worker's risk aversion increases, the firm is able to hoard a greater share of the surplus and bargains over a smaller wage.

Egalitarian: Under the Egalitarian solution the aggregate equilibrium wage is implicitly defined by (see Appendix A for details):

$$v(w) - v(z) = y - w + \gamma\theta. \quad (22)$$

A straight comparison between (21) and (22) clearly shows the importance of the scale invariance property. Under the Nash solution, the workers' net utility, $v(w) - v(z)$, is normalized (scaled) by the wage's marginal utility, $v'(w)$. This produces scale invariance to affine transformations of utility in the classical Arrow-Pratt sense. Under the egalitarian solution, the workers' net utility is not normalized. Accordingly, the solution depends on the marginal utility of the outcome, *i.e.* on the utility's scaling. It is therefore necessary to assume a given arbitrary normalization for utility. In this case, we assume that the workers agree with the firm on the marginal utility value of labor productivity y , hence $v'(y) = 1$. The following lemma and propositions reassess the equivalence result when workers are risk averse.

Lemma 3. *When utility is partially transferable (workers are risk averse), the Nash and the Egalitarian solutions are no longer equivalent.*

The non equivalence between the two solutions originates from the scale invariance property of the Nash solution. Put differently, when workers are risk neutral, the marginal utility of the wage is equal to unity and the two solutions collapse to a single one. This does not hold true when workers are risk averse.

Kalai-Smorodinsky: Under the KS solution the aggregate equilibrium wage is implicitly defined as a function of the reservation wage rU (see Appendix A for details):

$$\frac{v(w) - rU}{v(y) - rU} (y - v^{-1}(rU)) = y - w \quad (23)$$

Figure 6 about here

The Kalai-Smorodinsky with non transferable utility is represented in Figure 6. It is worth remarking that homogeneity in $v(\cdot)$ of equation (23) ensures that the Kalai-Smorodinsky solution remains scale invariant. In order to compare the Kalai-Smorodinsky and the Nash solutions,¹⁴ it is convenient to rewrite (21) as: $\frac{v(w) - rU}{v'(w)} = y - w$. Comparing the latter expression with (23), it is easy to note that the two solutions differ.

Lemma 4. *When risk-averse workers' utility is partially transferable, the Nash and the Kalai-Smorodinsky solutions are in general no longer equivalent.*

¹⁴Under the Nash solution, when utility is partially transferable, the reservation wage rewrites as $rU = v(z) + \theta\gamma v'(w)$. Using this latter expression together with (21), the expression in the core of the text follows. For further details see Appendix A.

More accurately, the non equivalence originates from the discrepancy between $v'(w)$ and $\frac{v(y)-rU}{y-v^{-1}(rU)}$, and there is, *a priori*, no reason for these two expressions to be identical. Note however that if y and $v^{-1}(rU)$ (which define the upper and lower bounds of feasible wages) are sufficiently close, then $\frac{v(y)-v(v^{-1}(rU))}{y-v^{-1}(rU)}$ is a first order approximation of $v'(x)$ at the reservation wage $x = v^{-1}(rU)$. If the wage under the Nash solution is sufficiently close to the reservation wage, the difference between the two solutions tends to vanish. In other words, the discrepancy between the two solutions is conditioned by their position within the range of feasible wages and the curvature of the utility function. Finally, putting all elements together and remarking that the egalitarian solution can be rewritten as $v(w) - rU = y - w$, it is possible to summarize the preceding results.

Proposition 4. *Equivalence.* *When utility is partially transferable, the Nash, the Egalitarian and the Kalai-Smorodinsky solutions differ and the equivalence result collapses.*

Beyond that we cannot say much more about the differences between the three bargaining solutions from a qualitative standpoint, at least in their general forms. The key point is to evaluate the quantitative relevance of these differences which are themselves determined by the shape of the Pareto frontier. We will discuss these considerations in the next section and return for the moment to the efficiency and comparative statics issues.

5.2 Labor Market Equilibria

In the benchmark model, whatever the bargaining solution, the wage curves all have the same shape, *i.e.* they are upward sloping and linear in the (θ, w) -space. When utility is partially transferable, things turn out to be different. We have already established that the three solutions are no longer equivalent. We can also characterize the difference between the solutions. The shapes of the wage curves are now different. More precisely, when utility is partially transferable, all the wage curves are upward sloping and the Nash, the Egalitarian and the KS wage curves are respectively convex, concave and convex in (θ, w) -space.¹⁵ The Job creation curve is unaffected in our updated framework. A set of possible labor market equilibria when utility is partially transferable is represented in Figure 7.

Insert Figure 7 about Here

Note that with partially transferable utility, it is not possible to establish a systematic hierarchy in the wage or in the labor market tightness. As a consequence, the labor market equilibria may

¹⁵Formal proofs are provided in Appendix C.

be ordered differently than in figure 7, although the current figure is consistent with the main results presented in section 5.4.

5.3 Properties

With partially transferable utility, the production criterion as emphasized in the linear case is non longer equivalent to the Utilitarian criterion. As in the benchmark model, we restrict to the case $r \rightarrow 0$. Following *e.g.* Fredriksson and Holmlund (2001) or Lehmann and van der Linden (2007), we assume that a benevolent social planner will choose the optimal allocation that maximizes an Utilitarian criterion subject to matching constraints and to an aggregate resource constraint. The benevolent social planner controls the tightness, the wages and the unemployment benefits. The social planner's objective simplifies to:

$$\max_{\{\theta, u, w, z\}} \Gamma = (1 - u)v(w) + uv(z) \quad (24)$$

subject to eq. (2) and $(1 - u)w + uz = (1 - u)y - \theta u\gamma$. The welfare function, Γ , is made up of the instantaneous utility of employed and unemployed workers weighted by their respective mass. The first constraint is similar to the one used in the benchmark model and underlines the fact that the planner faces the same matching constraints as the decentralized economy. The last constraint simply states that a feasible allocation must satisfy an aggregate resource constraint.

The planner problem is detailed in Appendix D. A key property of the first-best allocation is perfect insurance. The planner sets wages and unemployment benefits in such a way that $v'(w) = v'(z)$. This perfect insurance property implies that workers are wage takers. Hence, as has already been shown by *e.g.* Lehmann and van der Linden (2007), unless workers have no bargaining power (*i.e.* the workers are paid their reservation wage), the first-best optimum cannot be reached through decentralized bargaining. Of course, none of the wage agreements in this paper satisfies such a rule. The next proposition summarizes the arguments when utility is partially transferable.

Proposition 5. Efficiency. *When utility is partially transferable and the economy is constrained efficient, the Nash, the Egalitarian and the Kalai-Smorodinsky solutions do not maximize social welfare unless workers are assumed to be wage takers. It follows that in the general case, job creation and unemployment are socially inefficient whatever the bargaining solution considered.*

Proof. see Appendix D. □

Propositions 2 and 5 characterize the first-best allocation in the two specifications of the model, although the formal conditions differ significantly. As discussed by Lehmann and van

der Linden (2007), the difference stems from the fact that the planner is not concerned with distributional issues when utility is perfectly transferable. As a consequence, there is a multiplicity of first-best allocations, characterized by a combination of w and z , maximizing total production net of the vacancy costs. The constrained efficient benchmark economy with $z = 0$ corresponds to a particular optimum among all the first-best allocations. Contrary to the transferable case, when utility is partially transferable, the planner pays attention to the manner in which output is distributed among workers and perfectly insures workers against income risk. This perfect insurance property makes bargaining considerations irrelevant from a first-best perspective. Consequently any bargained wage would lead to inefficient outcomes. In what follows, we focus on the second-rank characteristics of the bargaining solutions using the welfare function Γ .

Under Nash or Egalitarian bargaining, it is easy to establish direct comparative statics inferences. The next proposition shows that proposition 3 easily extends to the case of partially transferable utility for both bargaining solutions.

Proposition 6. Comparative statics: Nash and Egalitarian. *Given Nash or Egalitarian bargaining:*

- *The equilibrium wage is increasing in the match productivity y and in the real return z , and decreasing in the vacancy cost γ , the interest rate r , and in the destruction rate s .*
- *The labor market tightness is increasing in the match productivity y , and decreasing in the real return z , in the vacancy cost γ , in the interest rate r , and in the destruction rate s .*

Proof. See Appendix E. □

Hence, it turns out that from a qualitative viewpoint the comparative statics effects are identical for the two solutions considered. Recalling that the unemployment rate is decreasing in the labor market tightness, Proposition 6 shows that equilibrium employment increases in y and decreases in z, γ, r and s . In particular, it is worth noting that a rise in the real return z (which increases the reservation utility rU and subsequently the reservation wage $v^{-1}(rU)$) unambiguously raises the unemployment rate in both cases. This result runs counter to the arguments proposed by Gerber and Upmann (2006) as regards the Egalitarian solution.¹⁶

The comparative statics properties under the KS solution are harder to characterize. As a matter of fact, the equilibrium wage is an implicit function of the reservation utility rU , which

¹⁶For instance, see Gerber and Upmann (2006, pp. 171-172).

is, in itself, a function of the wage and the labor market tightness. Formally, the reservation wage satisfies:

$$rU = \frac{r + s}{r + s + \theta q(\theta)} v(z) + \frac{\theta q(\theta)}{r + s + \theta q(\theta)} v(w) \quad (25)$$

Together with the job creation condition (18) and the wage equation (23), equation (25) defines the labor market equilibrium under the KS solution. Unfortunately, in its general form the KS solution is not easily tractable. To derive analytical results, we proceed to a first-order Taylor expansion in the neighborhood of the linear/transferable case. Consequently, it must be borne in mind that the results apply essentially to a reasonably low degree of worker risk aversion. However, it can be demonstrated that the Nash and the KS solutions are indiscernible in a first-order approximation.¹⁷ We show in Appendix E that the Nash and the KS solutions behave identically under fairly mild conditions. The following proposition summarizes the results:

Proposition 7. Comparative statics: Kalai-Smorodinsky. *Given the Kalai-Smorodinsky solution in the neighborhood of the transferable utility case and under fairly mild conditions:*

- *The equilibrium wage is increasing in the match productivity y and in the real return z , and decreasing in the vacancy cost γ , the interest rate r , and in the destruction rate s .*
- *The labor market tightness is increasing in the match productivity y , and decreasing in the real return z , in the vacancy cost γ , in the interest rate r , and in the destruction rate s .*

Proof. See Appendix E. □

Proposition 7 shows that from a qualitative standpoint the Nash and the KS solutions behave identically in the neighborhood of the linear/transferable case. Again, this contradicts Gerber and Upmann (2006). When utility is non transferable, the Nash and the KS solutions are no longer equivalent however an accumulation of evidence suggests that the discrepancy between the two solutions is small. In the next section, we proceed to a series of numerical exercises in order to further evaluate the generality of our arguments.

5.4 Quantitative Implications

So far it has been established that with partially transferable utility the equivalence result between the three bargaining solutions vanishes. In addition, it has been shown that in general the solutions behave identically at least in the neighborhood of linearity. The purpose here is to determine the qualitative and quantitative implications of the model in its general form when

¹⁷See Appendix B for a formal proof. The two solutions differ however in the second-order. In consequence, the quantitative differences between the two solutions are likely to be second-order differences which is also consistent with the discussion in the previous section.

utility is partially transferable. We proceed in three steps. First, we parameterize the model and compare the three solutions. Second, we search for the degree of risk aversion that maximizes the distance between the solutions, in particular as regards the Nash and the KS solutions. Third, we evaluate the sensitivity of the model to variations in the key exogenous parameters.

Numerical Illustrations: The baseline parameter values are chosen with two criteria in mind: (i) the values of the parameters themselves have to be realistic and coherent with the values usually chosen in the literature; (ii) the values of the endogenous variable that follow from these parameters also have to be reasonable. The parameters used here are in the range of those usually chosen in the matching literature (see *e.g.* Shimer, 2005). We normalize the time period to be a quarter. The discount rate, r , is set to 1.25%. A matching function of the Cobb-Douglas form is assumed such that $M(u, v) = ku^\vartheta v^{1-\vartheta}$, where k is a mismatch parameter, ϑ and $1-\vartheta$ are the elasticities of the matching function with respect to search inputs. We assume ϑ to be equal to $\frac{1}{2}$ (see *e.g.* Petrongolo and Pissarides, 2001). As is traditional in the literature, parameter k may be used as a calibration parameter to pin down the equilibrium unemployment rate. We set this parameter to unity. The values of leisure and the recruiting costs are set to $z = 0.4$ and $\gamma = 0.27$, respectively. The job productivity is normalized to unity. We assume a CRRA utility function of the form:

$$v(x) = \begin{cases} \frac{x^{1-\sigma}}{1-\sigma} & \sigma \geq 0, \sigma \neq 1 \\ \ln x & \sigma = 1 \end{cases} \quad (26)$$

where σ is the coefficient of relative risk aversion in the Arrow-Pratt sense. We take the symmetric Nash bargaining solution under transferable utility as a benchmark. Note that in the benchmark model the economy is constrained efficient since the bargaining power of the workers is equal to the elasticity of the matching function. The baseline parameter values and the resulting endogenous variables are reported in Table 1.

Parameters	y	s	z	γ	r	k
	1	0.1	0.4	0.27	0.0125	1
Endogenous variables	θ		w		u	
	1.91		0.9580		6.75%	

Table 1: Baseline parameter values and equilibrium labor market values with symmetric Nash Bargaining and transferable utility.

Table 2 presents the labor market equilibrium values for the three bargaining solutions. We contrast two cases in terms of risk aversion: low risk aversion ($\sigma = 0.5$) and high risk aversion ($\sigma = 1.5$). In the first case the curvature of the utility function is small whereas in the second the curvature is more pronounced. This range of values is appropriate with the topic at stake here, though much higher values can be found in the macroeconomic literature: in decision

under risk, utility is an interval scale, unique up to unit and level and all information relevant to this interval scale can be captured by the index of absolute risk aversion (Pratt, 1964 and Wakker, 2008). For a CRRA function, this index is equal to σ/x . A consequence is that the empirical meaning of the value of the CRRA coefficient depends on the corresponding outcome domain defined for x . For high amounts of money, large values (up to 50 or 100) are required to generate enough curvature. For small domains, as is the case in this paper, moderate parameter values are sufficient to generate the same absolute degree of risk aversion.

For each solution and each risk level, we report the equilibrium values for the wage, w , the tightness, θ , the unemployment rate, u , and a measure of the Welfare, Γ , which here is similar to an Utilitarian criterion.¹⁸

$\sigma = 0.5$	w	θ	u	Γ
<i>Nash</i>	0.9538	2.3183	6.16%	1.9156
<i>Egalitarian</i>	0.9532	2.3738	6.09%	1.9156
<i>Kalai-Smorodinsky</i>	0.9538	2.3180	6.16%	1.9156
$\sigma = 1.5$	w	θ	u	Γ
<i>Nash</i>	0.9429	3.5281	5.05%	-2.1093
<i>Egalitarian</i>	0.9404	3.8527	4.85%	-2.1095
<i>Kalai-Smorodinsky</i>	0.9430	3.5274	5.06%	-2.1093

Table 2: Equilibrium values for the three bargaining solutions and for two risk aversion's levels ($\sigma = 0.5$ is low risk aversion; $\sigma = 1.5$ is high risk aversion)

The first noticeable result is that for any degree of risk aversion, the quantitative difference between the Nash and the KS solution is immaterial. With low risk aversion, the small curvature of the utility function makes the difference between the two solutions non-existent (for further details see the discussion in subsection 5.1). Therefore when utility is partially transferable and when a low degree of risk aversion is involved, the equivalence result remains. With high risk aversion, the large curvature of the utility function creates a second-order difference between the two solutions. However, for reasonable parameter values, the difference as regards the labor market can be disregarded. Hence, although the two solutions differ (widely) from a theoretical viewpoint when utility is partially transferable, the quantitative discrepancies appear insignificant.

The results diverge more significantly as regards the Egalitarian solution. As detailed in subsection 5.1, the bargaining outcome from the Egalitarian solution depends on marginal utility and differs from the other two scale invariant solutions. For any degree of risk aversion, the wage and the tightness are lower and higher, respectively, under the Egalitarian solution, hence resulting in a lower unemployment rate. Perhaps surprisingly, the welfare tends to be smaller

¹⁸Formally this criterion satisfies: $\Gamma = (1 - u)rE + (1 - u)rJ + vrV + urU$.

despite a lower unemployment rate. For the sake of clarity, let us recall that the instantaneous Utilitarian criterion may be written as $(1 - u)(y - w) + (1 - u)v(w) + uv(z) - u\theta\gamma$. As can be inferred from Table 2, the Egalitarian solution is more responsive in the tightness than the Nash or the KS solutions. It follows that the last term matters more, hence a more pronounced effect on welfare. Finally and perhaps trivially, one can note that whatever the bargaining solution considered, a higher degree of risk aversion implies a lower wage and a higher labor market tightness.

Maximum Difference: We now proceed with the analysis of the model for admittedly plausible values of σ in the range $[0; 2]$. Chetty (2006), for instance, finds bounds on the Arrow-Pratt coefficient of relative risk aversion σ and estimates that this coefficient should be smaller than 2 and is supposedly close to 1. Over the range considered, the difference between the Nash and the Egalitarian solutions is monotonic whereas the difference between the Nash and the KS solutions is non-monotonic.¹⁹ Here we restrict ourselves to the latter difference. It has been established that although these two solutions differ from a theoretical viewpoint they are, under partially transferable utility, arguably close. We search for the maximum discrepancy between the two solutions for the model detailed above. One of the more noteworthy results here is that with a CRRA utility function the Nash and the KS solutions coincide at the upper and the lower bound of the interval considered. The following remark highlights this result.

Remark 1. *With a CRRA utility function, the Nash and the Kalai-Smorodinsky solutions are reduced to a single solution when workers are risk neutral ($\sigma = 0$) or strongly risk averse ($\sigma = 2$).*

Proof. For $\sigma = 0$ the results follows directly by virtue of Proposition 1. For $\sigma = 2$, we get $v(x) = -1/x$, $v'(x) = 1/x^2$ and $v^{-1}(x) = -1/x$. Replacing in the wage equations as defined in Appendix A for partially transferable utility, the solution to the bargaining problem verifies $y = -rUw^2$. □

The wage gaps $w^{KS} - w^N$ (panel a) and $w^E - w^N$ (panel b) are plotted in Figure 8 below for $\sigma \in [0; 2]$. Over the range considered and for plausible parameter values, we have $w^{KS} \geq w^N$ and $w^N \geq w^E$, which is consistent with the labor market equilibria depicted in figures 4 and 7. On the first panel, the wage gap is bell-shaped and maximized for $\sigma = 1.28$. Note that the maximum is quasi-invariant to small changes in the key parameters for the model specification we have chosen. Two remarks can be made here. First, the quantitative parameterizations of the model unambiguously show that the bargained wage under the Kalai-Smorodinsky solution is not always higher than the bargained wage under the Nash solution. This specific ranking

¹⁹See Appendix B for formal details. Numerical illustrations are given in Figure 8 below.

is restricted to a precise range of relative risk aversion coefficients, namely $[0; 2]$. Outside this range, the ranking may be reversed. Second, the deviations from the perfectly transferable case do not yield any systematic or monotonic relationship between risk aversion and the wage gap.

Insert Figure 8 about here

Steady-State Elasticities: The purpose here is to underline the quantitative effects of a change in some key exogenous parameters. In particular, we are interested in the comparative-statics effects of the Kalai-Smorodinsky solution from both a qualitative and quantitative point of view. We evaluate the responsiveness of the wage and the labor market tightness to a variation in: (i) the labor productivity, y ; (ii) the unemployed real return, z ; (iii) the cost of a vacancy, γ ; (iv) the interest rate, r ; and (v) the job separation rate, s . More precisely, we calculate the percentage change in the endogenous variable in response to a 10% increase in the exogenous parameter. It follows that $\varepsilon^{y/x} \equiv \frac{d \log y}{d \log x}$, $\forall x \in \{y, z, \gamma, s, r\}$ and $\forall y \in \{\theta, w\}$. Table 3 summarizes the steady-state elasticities for the wage and the labor market tightness following a change in a key parameter when the difference between the Nash and the KS solutions is maximum for $\sigma = 1.28$ as evaluated above. Note that with these two criteria we stress numerical configurations that should exacerbate the discrepancies between the bargaining solutions especially as regards the Nash and the KS solutions.

$\varepsilon^{w/y}$	y	z	γ	r	s
<i>Nash</i>	0.9866	0.0441	-0.0291	-0.0067	-0.0529
<i>Egalitarian</i>	1.0346	0.0484	-0.0321	-0.0073	-0.0584
<i>Kalai-Smorodinsky</i>	0.9866	0.0441	-0.0291	-0.0066	-0.0529
$\varepsilon^{\theta/y}$	y	z	γ	r	s
<i>Nash</i>	2.4154	-1.4380	-1.1049	-0.0239	-0.1908
<i>Egalitarian</i>	0.9365	-1.5040	-1.0614	-0.0140	-0.1119
<i>Kalai-Smorodinsky</i>	2.4153	-1.4376	-1.1051	-0.0239	-0.1912

Table 3: Steady-State elasticities for a 10% increase in the key exogenous parameters.

From Table 3, it can be pointed out that the comparative statics results of the KS solution in the neighborhood of linearity as presented in Proposition 7 generalize under reasonable parameters values in the neighborhood of linearity. It follows that the three bargaining solutions yield the very same qualitative results. Furthermore, even in a configuration where the discrepancies between the Nash and the KS solutions should be exacerbated, the difference between the two solutions appears to be intangible. This last point corroborates our previous results. Finally, the Egalitarian solution appears to be more responsive to the change of the key parameters. In particular, the elasticities of the wage and the tightness to the labor productivity and the job separation rate are magnified in comparison with the Nash and the KS solutions. Hence from

a macroeconomic viewpoint only the Egalitarian solution is likely to change the labor market responses to aggregate turbulence.

6 Conclusion

In this article we use a stylized model of the labor market to investigate the effects of three alternative and prominent bargaining solutions. We studied the Nash, the Egalitarian and the Kalai-Smorodinsky bargaining solutions in a small firm's matching model of unemployment. Our results are twofold. First, from a theoretical point of view, this paper is to the best of our knowledge the first contribution that attempts to implement and systematically compare the outcome of the different solutions in a search and matching model of the labor market. Second, we establish a series of results arising out of the different bargaining solutions considered. Our results show that even though the traditional results of bargaining theory apply in the context of search-matching economies, they are quantitatively weaker than expected. In particular, we found no difference in labor market outcomes between the Nash and the KS solutions. This suggests that the results advocated by *e.g.* Gerber and Upmann (2006) in a McDonald-Solow efficient contract model do not generalize to dynamic search-matching economies. A main consequence is that the policy implications are not very sensitive to the choice of the bargaining solution used to represent wage bargains on the labor market. A secondary consequence is that the tractability of the Nash solution comes at low cost in the dominant search-matching paradigm.

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Appendix

A Wage equations

A.1 Benchmark model

Nash solution: From (9), the foc verifies $\frac{\partial(E(w)-U)}{E(w)-U} + \frac{\partial(J(w)-V)}{J(w)-V} = 0$ and yields the following sharing rules: $E(w) - U = J(w) - V = \frac{1}{2}S$ where $S \equiv E(w) - U + J(w) - V$ stands for the surplus of a job-worker match. Using (4) and (6) together with the sharing rules, one gets $w - rU = y - w - rV$; eq. (10) follows.

Egalitarian solution: From (11), the Egalitarian solution translates into $E(w) - U = J(w) - V$. Using (4) and (6) together with the latter expression, one gets $w - rU = y - w - rV$; eq. (12) follows.

Kalai-Smorodinsky solution: From (16), it is clear that $\frac{E(w)-U}{J(w)-V} = \frac{\widehat{E}^e(\widehat{w}^e)-U}{\widehat{J}^e(\widehat{w}^e)-V}$. Using (4), (6), (14) and (15) the LHS rewrites as $\frac{w-rU}{y-w-rV}$ and the RHS as $\frac{y-rV-rU}{y-rV-rU}$. Making use of the last two expressions, one gets $\frac{w-rU}{y-w-rV} = 1$; eq. (17) follows.

A.2 Model under partially transferable utility

Nash solution: From (9), the foc verifies $\frac{\partial(E(w)-U)}{E(w)-U} + \frac{\partial(J(w)-V)}{J(w)-V} = 0$ and simplifies to $v'(w)(J(w) - V) = (E(w) - U)$. One gets the following two sharing rules $E(w) - U = \frac{v'(w)}{v'(w)+1}S$ and $J(w) - V = \frac{1}{v'(w)+1}S$ with $S \equiv E(w) - U + J(w) - V$. Using (4) and (6), one gets $J(w) - V = \frac{y-w-rV}{r+s}$ and $S = \frac{y+v(w)-w-r(U+V)}{r+s}$. Combining these two relations with the sharing rules and assuming the free entry condition is satisfied yields, $w = y - \frac{1}{v'(w)}(v(w) - rU)$. A more suitable wage equation can be derived using (3), (5) and the first order condition of the Nash program. Combining these relations and restricting to symmetric equilibria gives $rU = v(z) + \theta\gamma v'(w)$. Replacing this in the previous expression of w , eq. (21) follows.

Egalitarian solution: From (11), the Egalitarian solution translates into $E(w) - U = J(w) - V$. Multiplying both sides by $r + s$, using (4), (6) and assuming that the free-entry condition, $V = 0$, is satisfied, one gets: $y - w = v(w) - rU$. Next, combining (3) and (4), the expected return from unemployment verifies $rU = \frac{(r+s)v(z)+\theta q(\theta)v(w)}{r+s+\theta q(\theta)}$. Finally, combining the last two equations, eq. (22) follows.

Kalai-Smorodinsky solution: From (16), it is clear that $\frac{E(w)-U}{J(w)-V} = \frac{\widehat{E}^e(\widehat{w}^e)-U}{\widehat{J}^e(\widehat{w}^e)-V}$. Using (4), (6), (14) and (15), the LHS satisfies: $\frac{E-U}{J} = \frac{v(w)-rU}{y-w}$, and the RHS satisfies: $\frac{\widehat{E}^e-U}{\widehat{J}^e} = \frac{v(y)-rU}{y-v^{-1}(rU)}$. Combining these relations gives $(v(w) - rU)(y - v^{-1}(rU)) = (v(y) - rU)(y - w)$. Finally, eq. (23) follows as an implicit function of rU : $y - w = \frac{v(w)-rU}{v(y)-rU}(y - v^{-1}(rU))$.

B Equilibrium analysis in the neighborhood of transferability

The purpose of this appendix is to study the properties of the labor market equilibrium in the neighborhood of a linear utility function. This is of particular interest for the Kalai-Smorodinsky solution which is difficult to characterize in its general form. The utility function $v(w, \sigma)$ will be a function of a parameter σ , which will be positively linked to the risk aversion; in particular, $\sigma = 0$ will correspond to the linear utility case: $v(w, 0) = w$, and we require that risk aversion be strictly increasing in σ .

Let ζ be such that $v(\zeta, \sigma) = rU$ with $rU = \frac{(r+s)v(z, \sigma) + \theta q(\theta)v(w, \sigma)}{r+s+\theta q(\theta)}$. We can rewrite this as $v(w, \sigma) - v(\zeta, \sigma) = \frac{r+s}{\theta q(\theta)} (v(\zeta, \sigma) - v(z, \sigma))$. Then, using the Job Creation condition (18), one gets:

$$v(w, \sigma) - v(\zeta, \sigma) = \frac{y - w}{\gamma\theta} (v(\zeta, \sigma) - v(z, \sigma)) \quad (\text{B-1})$$

The wage equations detailed in Appendix A rewrite as:

$$\begin{aligned} y - w &= [v(w, \sigma) - v(\zeta, \sigma)] \frac{1}{v'(w)} \\ y - w &= v(w, \sigma) - v(\zeta, \sigma) \\ y - w &= [v(w, \sigma) - v(\zeta, \sigma)] \frac{y - \zeta}{v(y, \sigma) - v(\zeta, \sigma)} \end{aligned} \quad (\text{B-2})$$

for the Nash, Egalitarian, and Kalai-Smorodinsky solutions, respectively. For consistency and notational convenience, $v'(w)$ corresponds to $\partial v / \partial w$. For every value of the risk aversion parameter, the equilibrium is then given by a triplet (w, θ, ζ) which is implicitly defined by a system $H_i(w, \theta, \zeta, \sigma) = 0$ made up of equations (18), (B-1) and a wage equation (B-2) corresponding to the solution considered. $i \in \{N, E, KS\}$ stands for Nash, Egalitarian and Kalai-Smorodinsky, respectively.

When $\sigma = 0$, the three solutions coincide, and we will write this solution under transferability as (w_0, θ_0, ζ_0) ; we have $H_i(w_0, \theta_0, \zeta_0, 0) = 0$ for all i . Using the implicit function theorem, it is possible to determine the derivative of $Y = (w, \theta, \zeta)$ as a function of σ :

$$\left. \frac{dY}{d\sigma} \right|_i = - \left(\frac{\partial H_i}{\partial Y} \right)^{-1} \frac{\partial H_i}{\partial \sigma}.$$

By using the explicit solution in the case of transferability $\zeta_0 = z + \gamma\theta_0$ and $w_0 = \frac{1}{2}(y + \zeta_0) = \frac{1}{2}(y + z + \gamma\theta_0)$, it is possible to simplify the expressions:

$$\begin{aligned} \left. \frac{dw}{d\sigma} \right|_N &= \frac{(r+s)q'(\theta_0) \left(\frac{\partial v}{\partial \sigma}(w_0, 0) - \frac{\partial v}{\partial \sigma}(z, 0) - (w_0 - z) \frac{\partial^2 v}{\partial \sigma \partial w}(w_0, 0) \right)}{q(\theta_0)^2 - 2(r+s)q'(\theta_0)} \\ \left. \frac{dw}{d\sigma} \right|_E &= \frac{(r+s)q'(\theta_0) \left(\frac{\partial v}{\partial \sigma}(w_0, 0) - \frac{\partial v}{\partial \sigma}(z, 0) \right)}{q(\theta_0)^2 - 2(r+s)q'(\theta_0)} \\ \left. \frac{dw}{d\sigma} \right|_{KS} &= \frac{(r+s)q'(\theta_0) \left(\frac{\partial v}{\partial \sigma}(w_0, 0) - \frac{\partial v}{\partial \sigma}(z, 0) - \frac{w_0 - z}{y - \zeta_0} \left(\frac{\partial v}{\partial \sigma}(y, 0) - \frac{\partial v}{\partial \sigma}(\zeta_0, 0) \right) \right)}{q(\theta_0)^2 - 2(r+s)q'(\theta_0)} \end{aligned} \quad (\text{B-3})$$

The denominator of these three expressions is positive, as long as $q'(\theta) < 0$. With the assumption that $\frac{dv}{d\sigma}(w, \sigma)$ is increasing and concave in w (this assumption is typically satisfied for both the CRRA and CARA utility functions), we have the general results that $\frac{dw}{d\sigma}|_N < 0$, $\frac{dw}{d\sigma}|_E < 0$, and $\frac{dw}{d\sigma}|_{KS} < 0$.

We can thus characterize the equilibrium wage in the neighborhood of linearity with the following Taylor approximation:

$$w_i(\sigma) = w_0 + \sigma \left. \frac{dw(w_0, 0)}{d\sigma} \right|_i + O(\sigma^2) \quad (\text{B-4})$$

Consequently, under partial transferability, the equilibrium wage is lower in the Nash, Egalitarian and Kalai-Smorodinsky cases than in the linear case. We can also compare the relative behavior of Nash and Kalai-Smorodinsky solutions as well as the relative behavior of Nash and Egalitarian solutions. Using (B-3), we find

$$w_{KS} - w_N \equiv \frac{(r+s)q'(\theta_0)(w_0 - z) \left(\frac{\partial^2 v}{\partial \sigma \partial w}(w_0, 0) - \frac{\frac{\partial v}{\partial \sigma}(y, 0) - \frac{\partial v}{\partial \sigma}(\zeta_0, 0)}{y - \zeta_0} \right)}{q(\theta_0)^2 - 2(r+s)q'(\theta_0)} \quad (\text{B-5})$$

However, even with our assumptions on the utility function, it is not possible to establish the sign of this expression, and by using specific examples it can be shown that the difference can go in either directions. Thus there is no general result on the relative behavior of the Nash and Kalai-Smorodinsky solutions that can be established without further hypotheses. Similarly, using (B-3), we find

$$w_E - w_N \equiv \frac{(r+s)q'(\theta_0)(w_0 - z) \frac{\partial^2 v}{\partial \sigma \partial w}(w_0, 0)}{q(\theta_0)^2 - 2(r+s)q'(\theta_0)} \quad (\text{B-6})$$

This expression is of the sign of $q'(\theta)$ and is, in consequence, negative. It follows that the wage gap between the Egalitarian and the Nash solutions is increasing in absolute terms with σ .

C Characterization of the Job Creation and Wage Curves

Job Creation curve: Let $\eta \equiv \eta(\theta) = -\frac{\theta q'(\theta)}{q(\theta)} \in [0, 1]$ be the elasticity of the matching function with respect to search inputs. Differentiation of the job creation equation (18) with respect to the endogenous variables, θ and w , yields:

$$d\theta = -\frac{\theta q(\theta)}{(r+s)\gamma\eta(\theta)} dw \quad (\text{C-7})$$

It follows that the Job Creation curve is decreasing in the (θ, w) -space.

Remark 2. Assuming a Cobb-Douglas matching function, as is traditional in the search-matching literature, the Job creation curve, Jc , is decreasing and convex in the (θ, w) -space.

Proof. from $\frac{d\theta}{dw} \Big|_{Jc} = -\frac{\theta q(\theta)}{(r+s)\gamma\eta} < 0$, one obtains: $\frac{d^2\theta}{dw^2} \Big|_{Jc} = -(1-\eta)q(\theta)\frac{d\theta}{dw} > 0$. \square

Wage curves: First, one must note that contrary to the Job Creation curve, the shape of the wage curves are not invariant to workers' preferences. In consequence, the cases with transferable and partially transferable utility need to be distinguished.

In the former case (*transferable utility*), the three bargaining solutions collapse to a single wage equation. Differentiation of (10), (12) or (17) with respect to the endogenous variables, θ and w , together with the free-entry condition yields:

$$dw^j = \frac{1}{2}\gamma d\theta \quad \text{for } j \in \{N, E, KS\} \quad (\text{C-8})$$

It follows the wage curves are upward-sloping and linear in the (θ, w) -space.

In the latter case (*partially transferable utility*), the shape of the wage curves are not necessarily the same. We proceed in the same manner for the three solutions below.

Remark 3. *When utility is partially transferable, and under the condition that the Arrow-Pratt measure of absolute risk aversion is not too decreasing, the Nash wage curve, wc^N , is increasing and convex in the (θ, w) -space.*

Proof. from $\frac{d\theta}{dw} \Big|_{wc^N} = \frac{1}{\gamma} \left(2 - \frac{v''(w)[v(w)-v(z)]}{[v'(w)]^2} \right) > 0$, one obtains:

$$\frac{d^2\theta}{dw^2} \Big|_{wc^N} = -\frac{1}{\gamma} \left[\frac{(v(w)-v(z))(v'''(w)v'(w)^2 - 2v''(w)v'(w)^2) + v''(w)v'(w)^3}{v'(w)^4} \right].$$

For this to be positive, it is enough that the absolute risk aversion $-v''(w)/v'(w)$ is lower than half the absolute prudence $-v'''(w)/v''(w)$. \square

Remark 4. *When utility is partially transferable, the Egalitarian wage curve, wc^E , is increasing and concave in the (θ, w) -space.*

Proof. from $\frac{d\theta}{dw} \Big|_{wc^E} = \frac{1}{\gamma} (1 + v'(w)) > 0$, one obtains:

$$\frac{d^2\theta}{dw^2} \Big|_{wc^E} = v''(w)/\gamma < 0. \quad \square$$

Remark 5. *When utility is partially transferable, the Kalai-Smorodinsky wage curve, wc^{KS} , is increasing in the (θ, w) -space. The shape of the wage curve is however undetermined as the sign of $\frac{d^2\theta}{dw^2} \Big|_{wc^{KS}}$ is ambiguous.²⁰*

Proof. The equations (23) and (25) implicitly determine w and rU as a function of θ . The system of equations is simpler if we write ζ such that $v(\zeta) = rU$, and $\lambda(\theta) = (r+s)/(r+s+\theta q(\theta))$.

²⁰Note however that under reasonable parameter values, the wage curve is convex in the (θ, w) -space.

By using the Implicit Function theorem, the partial derivative can be calculated and proven to be positive; hence, wc^{KS} is increasing in θ :

$$\frac{d\theta}{dw} \Big|_{wc^{KS}} = - \frac{(v(w) - v(z))(v(w) - v(\zeta) + (w - \zeta)v'(\zeta)) \lambda'(\theta)}{(v(w) - v(\zeta))(-1 + \lambda(\theta))v'(w) + (v(y) - v(\zeta) + (-w + y + (w - \zeta)\lambda(\theta))v'(w))v'(\zeta)}$$

□

Existence and Uniqueness:

When *utility is perfectly transferable*, the wage curves have all the same shape and are all upward sloping whatever, i.e. $\frac{d\theta}{dw} \Big|_{w^j} = \frac{1}{2}\gamma > 0$, and the job creation is curve is downward sloping, i.e. $\frac{d\theta}{dw} \Big|_{J_c} = -\frac{\theta q(\theta)}{(r+s)\gamma\eta} < 0$. Existence and uniqueness of the equilibrium follow.

When *utility is partially transferable*, the wages curves differ but are all upward sloping, i.e. $\frac{d\theta}{dw} \Big|_{wc^N} = \frac{1}{\gamma} \left(2 - \frac{v''(w)[v(w)-v(z)]}{[v'(w)]^2} \right) > 0$, $\frac{d\theta}{dw} \Big|_{wc^E} = \frac{1}{\gamma} (1 + v'(w)) > 0$, $\frac{d\theta}{dw} \Big|_{wc^{KS}} > 0$, and the job creation is curve is downward sloping, i.e. $\frac{d\theta}{dw} \Big|_{J_c} = -\frac{\theta q(\theta)}{(r+s)\gamma\eta} < 0$. Existence and uniqueness of the equilibrium follow from the slopes and the values of the two curves at z and at y .

D Efficiency

D.1 Benchmark model

We restrict ourselves to the case where $r \rightarrow 0$. When utility is perfectly transferable, the social welfare function corresponds to aggregate output. This function satisfies:

$$\Gamma = (1 - u)y - \theta u \gamma \tag{D-9}$$

The program of the benevolent planner consists in maximizing Γ with respect to θ subject to eq. (2). The social planner chooses θ such that:

$$(1 - \eta)y - \frac{s + \eta\theta q(\theta)}{q(\theta)}\gamma = 0 \tag{D-10}$$

where $\eta \equiv \eta(\theta) \in [0, 1]$ is the elasticity of the matching function with respect to the search inputs. By virtue of Proposition 1, whatever the bargaining solution, the wage rule satisfies eq. (19) when utility is transferable. Making use of this relation together with the job creation condition (18), we rewrite the decentralized equilibrium as:

$$\frac{1}{2}(y - z) - \frac{s + \frac{1}{2}\theta q(\theta)}{q(\theta)}\gamma = 0 \tag{D-11}$$

Comparing the social optimum (D-10) with the decentralized equilibrium (D-11), it is simple to establish that the two relationships are equivalent if and only if $\eta = \frac{1}{2}$ and $z = 0$. In other terms, job creation is socially efficient when the elasticity of the matching function is equal to the bargaining power of the workers and unemployment benefits are nil.

D.2 Model under partially transferable utility

We restrict ourselves to the case where $r \rightarrow 0$. When utility is non transferable. The social welfare function writes:

$$\Gamma = (1 - u)v(w) + uv(z) \quad (\text{D-12})$$

The program of the benevolent planner consists in maximizing Γ with respect to θ , w and z subject to eq. (2) and to an aggregate resource constraint, $(1 - u)w + uz = (1 - u)y - \gamma\theta u$. This last constraint determines the set of feasible allocations in the economy. The Lagrangian of the problem writes:

$$L = \frac{\theta q(\theta)}{s + \theta q(\theta)} v(w) + \frac{s}{s + \theta q(\theta)} v(z) + \xi \left[\frac{\theta q(\theta)}{s + \theta q(\theta)} w + \frac{s}{s + \theta q(\theta)} z - \frac{\theta q(\theta)}{s + \theta q(\theta)} y + \frac{s\gamma\theta}{s + \theta q(\theta)} \right]$$

where ξ is the multiplier associated with the second constraint. The first order conditions with respect to θ , w and z are respectively given by:

$$\begin{aligned} \xi [q(\theta)(1 - \eta)w - q(\theta)(1 - \eta)z - q(\theta)(1 - \eta)y + \gamma s + \gamma\theta q(\theta)\eta] &= 0 \\ v'(w) + \xi &= 0 \\ v'(z) + \xi &= 0 \end{aligned}$$

From the last two foci, it's simple to get $v'(w) = v'(z)$. Hence, the planner perfectly insures the workers against unemployment hazard. The first-best allocation is characterized by a perfect insurance property which implies that the workers are wage takers, *i.e.* $w = z$. It follows that any other wage-setting mechanisms are socially inefficient. Generally-speaking, this includes the three bargaining solutions considered in the paper.

E Comparative Statics

As already mentioned, the Job Creation curve does not depend on workers' preferences. Total differentiation of (18) with respect to the endogenous variables and the key exogenous (policy) parameters yields:

$$d\theta = \frac{\theta q(\theta)}{(r + s)\gamma\eta(\theta)} [dy - dw] - \frac{\theta}{\gamma\eta(\theta)} d\gamma - \frac{\theta}{(r + s)\eta(\theta)} [dr + ds] \quad (\text{E-13})$$

where $\eta(\theta) \in [0, 1]$ is the elasticity of the matching function with respect to the search inputs. As regards the wage curves, we once again contrast two cases, *i.e.* transferable and non transferable utility. The wage equations rewrite for $j \in \{N, E, KS\}$ under the following generic form:

$$A^j = y - w + \theta\gamma \quad (\text{E-14})$$

where j stands for the Nash (N), the Egalitarian (E) and the Kalai-Smorodinsky (KS), solutions respectively. Differentiation of (E-14) wage curve with respect to the endogenous variables and the key exogenous (policy) parameters yields:

$$\left[1 + \frac{\partial A^j}{\partial w}\right] dw = \left[\gamma - \frac{\partial A^j}{\partial \theta}\right] d\theta + \left[1 - \frac{\partial A^j}{\partial y}\right] dy - \frac{\partial A^j}{\partial z} dz + \left[\theta - \frac{\partial A^j}{\partial \gamma}\right] d\gamma \quad (\text{E-15})$$

E.1 Benchmark model

When utility is perfectly transferable, the three bargaining solutions collapse to a single wage equation. We have $A^j = w - z$ for $j \in \{N, E, KS\}$ and eq. (E-14) rewrites as $w - z = y - w + \theta\gamma$. For the three solutions, we get $\frac{\partial A^j}{\partial w} = 1$, $\frac{\partial A^j}{\partial z} = -1$ and $\frac{\partial A^j}{\partial \theta} = \frac{\partial A^j}{\partial y} = \frac{\partial A^j}{\partial \gamma} = 0$. Using the partial derivatives together with (E-13) and (E-15), the comparative-statics results follow. The table below summarizes the results.

<i>Endogenous variable</i>	<i>Bargaining solution</i>	y	z	γ	r	s
θ (tightness)	Nash	+	-	-	-	-
	Egalitarian	+	-	-	-	-
	Kalai-Smorodinsky	+	-	-	-	-
w (wage)	Nash	+	+	-	-	-
	Egalitarian	+	+	-	-	-
	Kalai-Smorodinsky	+	+	-	-	-

Table 4: Comparative statics results when utility is perfectly transferable.

E.2 Model under partially transferrable utility

When utility is partially transferable, the three bargaining solutions are no longer equivalent. We proceed with the analysis of the Nash, the Egalitarian and the Kalai-Smorodinsky solutions separately. In the first two cases, the comparative statics results established when utility is perfectly transferable readily extend to the case of partially transferable utility. Things turn out to be a little more complicated as regards the KS solution which depends on rU (see below).

Nash solution: Under the Nash solution, we have $A^N = (v(w) - v(z)) / v'(w)$ and eq. (E-14) rewrites as $(v(w) - v(z)) / v'(w) = y - w + \gamma\theta$. We get $\frac{\partial A^N}{\partial w} = \left[1 - \frac{v''(w)v(w) - v'(w)^2}{v'(w)^2}\right] > 0$, $\frac{\partial A^N}{\partial z} = -\frac{v'(z)}{v'(w)} < 0$ and $\frac{\partial A^N}{\partial \theta} = \frac{\partial A^N}{\partial y} = \frac{\partial A^N}{\partial \gamma} = 0$. Using the partial derivatives together with (E-13) and (E-15), the comparative-statics results follow. Table 5 summarizes the results.

Egalitarian solution: Under the Egalitarian solution we have $A^E = (v(w) - v(z))$ and eq. (E-14) rewrites as $(v(w) - v(z)) = y - w + \gamma\theta$. We get $\frac{\partial A^E}{\partial w} = v'(w) > 0$, $\frac{\partial A^E}{\partial z} = -v'(z) < 0$ and $\frac{\partial A^E}{\partial \theta} = \frac{\partial A^E}{\partial y} = \frac{\partial A^E}{\partial \gamma} = 0$. Using the partial derivatives together with (E-13) and (E-15), the comparative-statics results follow. Table 5 summarizes the results.

Kalai-Smorodinsky solution: Under the KS solution we have $A^{KS} = \frac{(v(w)-rU)(y-v^{-1}(rU))+\theta\gamma(v(y)-rU)}{v(y)-rU}$ and eq. (E-14) rewrites as $\frac{(v(w)-rU)(y-v^{-1}(rU))+\theta\gamma(v(y)-rU)}{v(y)-rU} = y - w + \gamma\theta$. In its general form, the KS solution is not tractable. In order to derive analytical results, we restrict our analysis to the neighborhood of the linearity which make the results appropriate for a low degree of risk aversion. We consider now the following approximation of the utility function: $v(w) = w - \sigma w^2 + o(\sigma)$. Solving for ζ in (B-1), then for w in (B-2), and taking the Taylor series expansion for σ small, we get:

$$H(w, \theta, y, \gamma, z, r, s, \sigma) = \left\{ w - \left(\frac{1}{2}(y + z + \gamma\theta) - \frac{\sigma}{8}(y - z + \gamma\theta)^2 \right) \frac{r+s}{q(\theta)} - \frac{y-w}{\gamma} \right\} \quad (E-16)$$

Using the implicit function theorem and matrix notations with $x = (y, \gamma, z, r, s)$ et $X = (w, \theta)$, we have:

$$\frac{dX}{dx} = - \left(\frac{\partial H}{\partial X} \right)^{-1} \frac{\partial H}{\partial x}.$$

Let $\Delta = - \left[(-2 + \sigma(y + \gamma\theta - z))q(\theta)^2 + 4rq'(\theta) \right]$, then we get:

$$\begin{aligned} \frac{dw}{dy} &= \frac{1}{\Delta}(\gamma\theta + \eta(y - w))(2 - \sigma(y + \gamma\theta - z)) \\ \frac{dw}{dz} &= \frac{1}{\Delta}\eta(y - w)(2 - \sigma(y + \gamma\theta - z)) \\ \frac{dw}{d\gamma} &= \frac{1}{\Delta}(\eta - 1)(y - w)(2 - \sigma(y + \gamma\theta - z)) \\ \frac{dw}{dr} = \frac{dw}{ds} &= -\frac{1}{r\Delta}\gamma\theta(y - w)(2 - \sigma(y + \gamma\theta - z)) \end{aligned}$$

$$\begin{aligned} \frac{d\theta}{dy} &= \frac{1}{\Delta}\theta(2 + \sigma(y + \gamma\theta - z)) \\ \frac{d\theta}{dz} &= -\frac{1}{\gamma\Delta}\theta(1 - \eta)(y - w)(2 - \sigma(y + \gamma\theta - z)) \\ \frac{d\theta}{d\gamma} &= -\frac{1}{\Delta}\theta(2\gamma\theta + 4(y - w) + \sigma\gamma\theta(y + \gamma\theta - z)) \\ \frac{d\theta}{dr} = \frac{d\theta}{ds} &= -\frac{1}{r\Delta}\theta(y - w) \end{aligned}$$

Hence if $\frac{\sigma}{2}(y + \gamma\theta - z) < 1$, we know that $\Delta > 0$ since $q'(\theta) < 0$ and we have $\frac{dw}{dy} > 0$, $\frac{dw}{dz} > 0$, $\frac{dw}{d\gamma} < 0$, $\frac{dw}{dr} < 0$ and $\frac{dw}{ds} < 0$, as well as $\frac{d\theta}{dy} > 0$, $\frac{d\theta}{dz} < 0$, $\frac{d\theta}{d\gamma} < 0$, $\frac{d\theta}{dr} < 0$ and $\frac{d\theta}{ds} < 0$. As the derivation requires $\sigma \simeq 0$, the condition will hold for σ that is small enough.

<i>Endogenous variable</i>	<i>Bargaining solution</i>	<i>y</i>	<i>z</i>	γ	<i>r</i>	<i>s</i>
θ (tightness)	Nash	+	-	-	-	-
	Egalitarian	+	-	-	-	-
	Kalai-Smorodinsky	+	-	-	-	-
w (wage)	Nash	+	+	-	-	-
	Egalitarian	+	+	-	-	-
	Kalai-Smorodinsky	+	+	-	-	-

Table 5: Comparative statics results when utility is partially transferable utility; in the neighborhood of linearity for the Kalai-Smorodinsky solution.

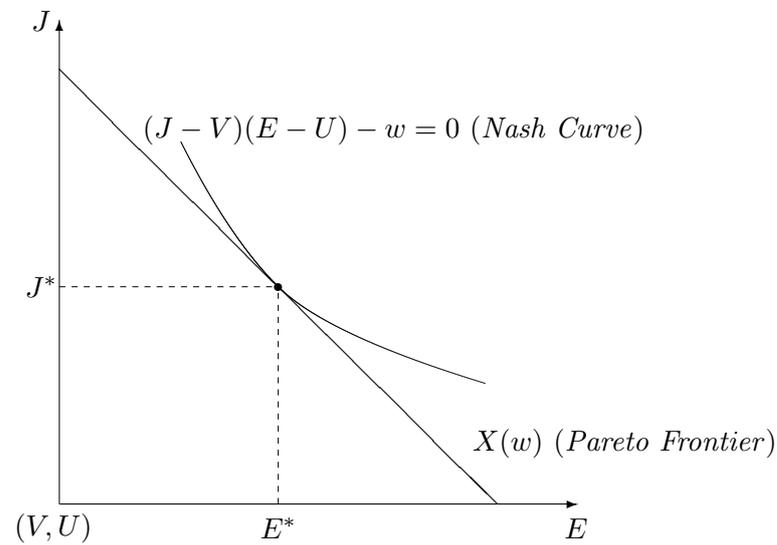


Figure 1: Nash solution with transferable utility.

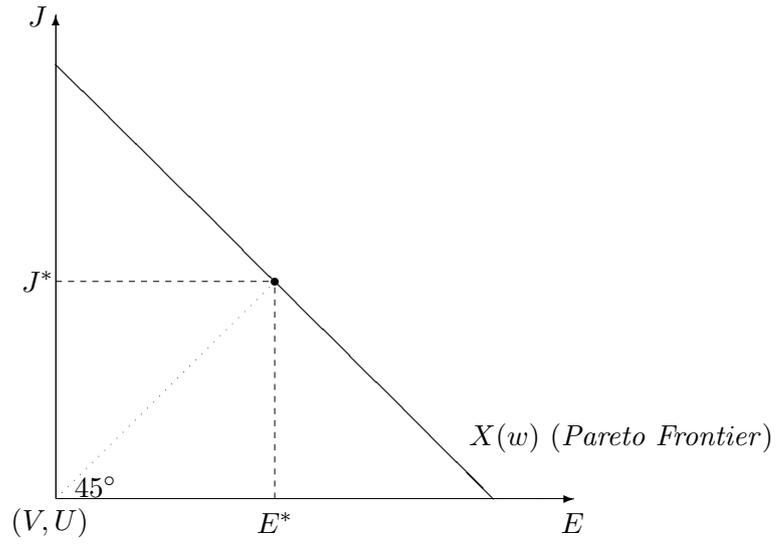


Figure 2: Egalitarian solution with transferable utility.

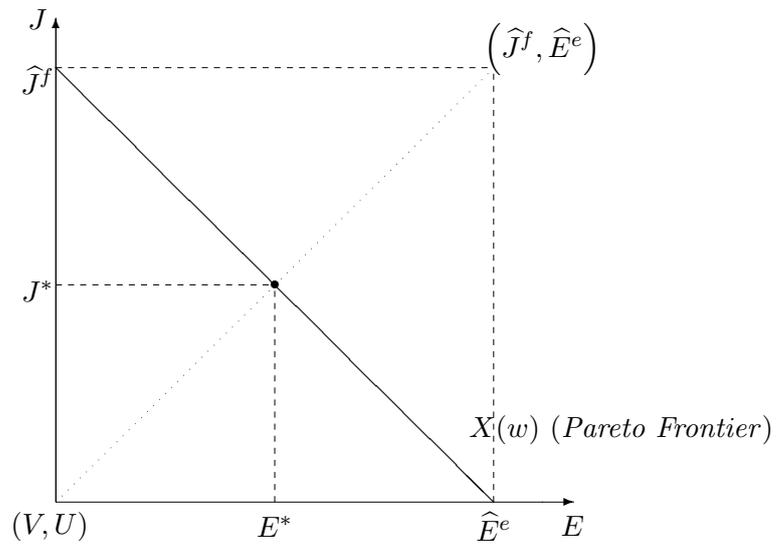


Figure 3: Kalai-Smorodinsky solution with transferable utility.

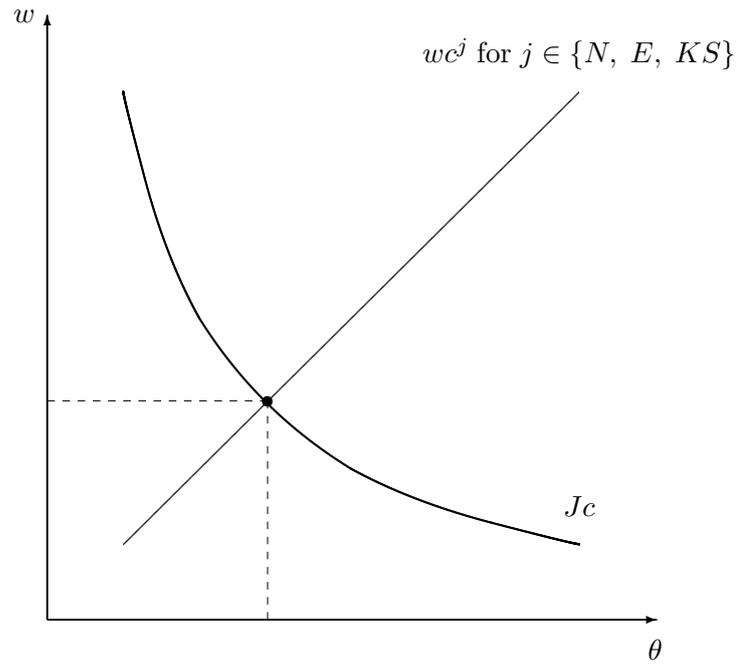


Figure 4: Labor market equilibria with transferable utility; N for Nash, E for Egalitarian, KS for Kalai-Smorodinsky.

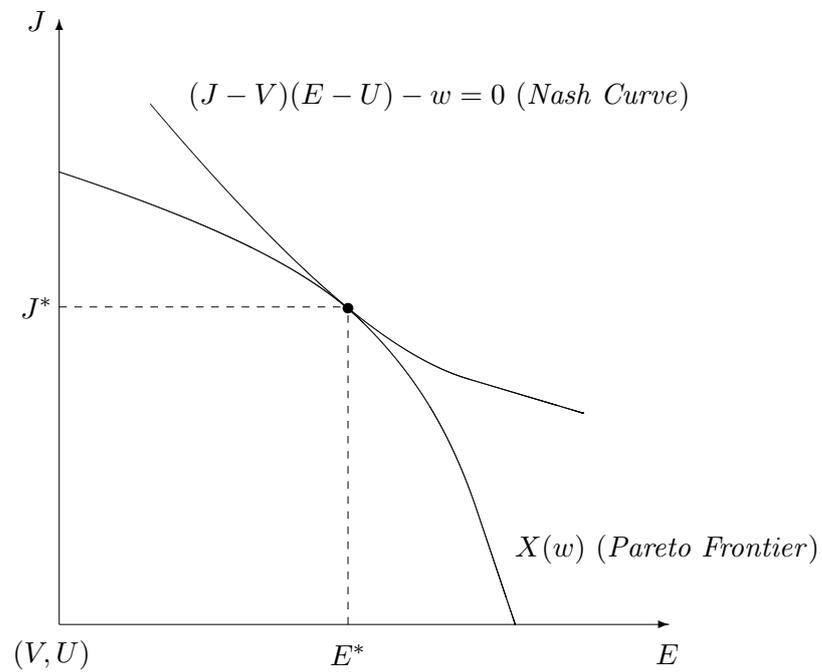


Figure 5: Nash solution with partially transferable utility.

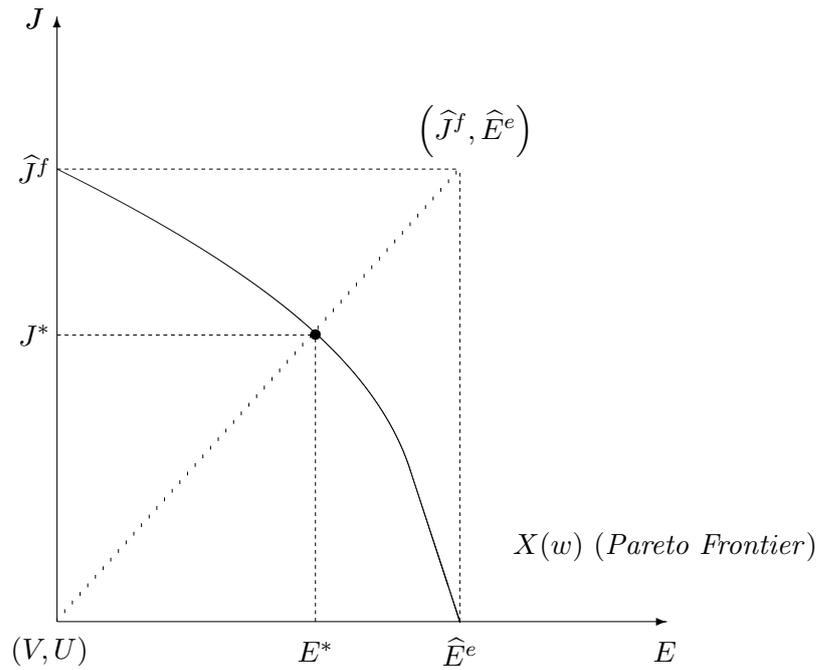


Figure 6: Kalai-Smorodinsky solution with partially transferable utility.

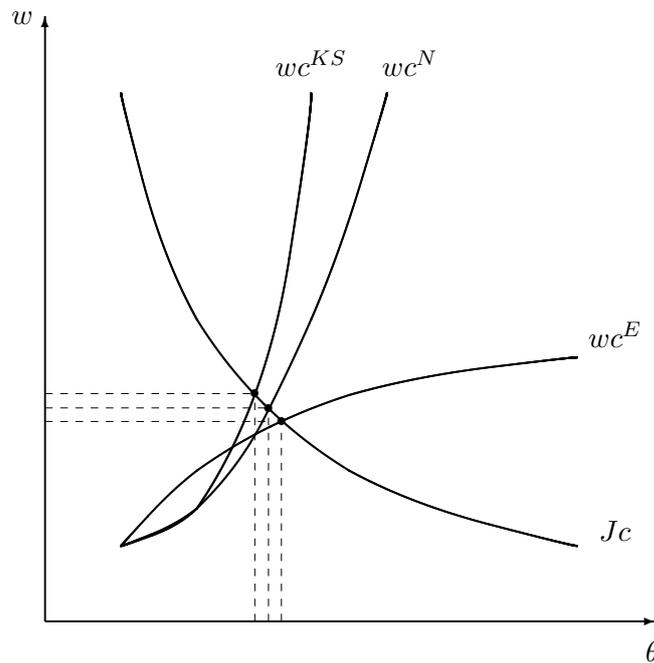


Figure 7: Labor market equilibria with partially transferable utility; N for Nash, E for Egalitarian, KS for Kalai-Smorodinsky; Wages and Tightness are ranked as shown in Table 2; Hierarchy in the endogenous variables for the three solutions may be mathematically established in the neighborhood of linearity. Proofs are available from the authors upon request.

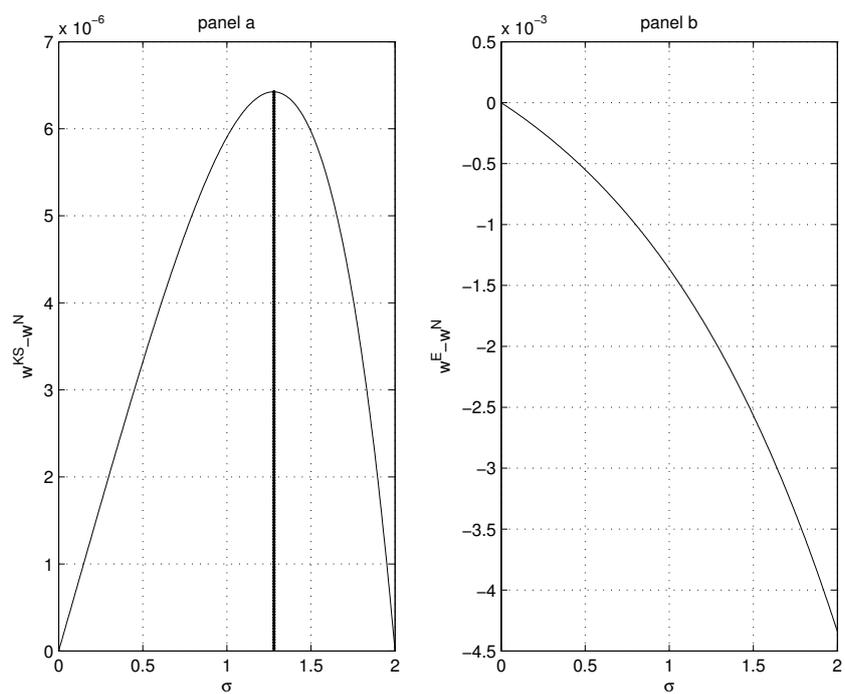


Figure 8: Difference between the Nash and the Kalai-Smorodinsky solutions (panel a) and the Nash and the Egalitarian solutions (panel b) under CRRA specification over the range $\sigma \in [0, 2]$. On the first panel, the difference is maximized for $\sigma = 1.28$.

