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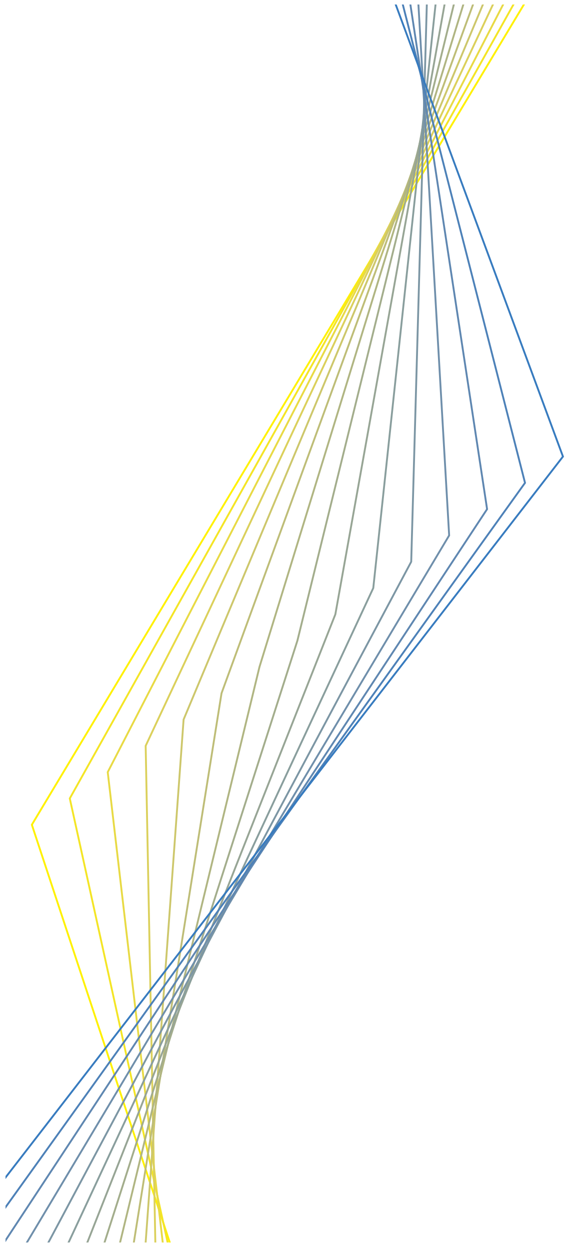

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PUBLIC PENSIONS AND GROWTH*

BY STÉPHANE LAMBRECHT[1], PHILIPPE MICHEL[2] AND JEAN-PIERRE VIDAL[3]

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Abstract

This paper investigates the relationship between the size of an unfunded public pension system and economic growth in an overlapping generation economy, in which altruistic parents finance the education of their children and leave bequests. Unlike the existing literature, we model intergenerational altruism by assuming that children's income during adulthood is an argument of parental utility. Unfunded public pensions can promote growth when families face liquidity constraints preventing them from investing optimally in the education of their children. We consider two alternative ways of financing a public pension system, either by levying social contributions in a lump-sum manner or in proportion to labour income. We find that there is no case for unfunded public pensions in economies where bequests are operative. By contrast, there exists a growth-maximising size of the public pension system in economies where bequests are not operative and individuals are sufficiently patient.

JEL classification system: H55, I20, D91

Keywords: Public pension; Education; Growth

Non-technical summary

Unfunded public pension benefits have a negative impact on private savings, since individuals partially rely on these to finance their retirement. The size of a public pension scheme also affects the tax burden, since benefits are typically financed by levying taxes on labour (social security contributions). In addition, unfunded public pensions modify the allocation of resources between generations, since today's benefits are financed by current social contributions. The overall impact of such a redistributive scheme on economic activity mainly depends on individuals' reactions to taxes and benefits.

Our main argument is that publicly-provided transfers between generations have an impact on private transfers between parents and children. Parents finance the education of their children and leave them a bequest, provided that they are sufficiently wealthy. In poor families, however, parents leave no bequests and cannot optimally finance the education of their children owing to liquidity constraints. Poor parents cannot borrow to invest more in the education of their children, since they are forbidden by law to force their children to reimburse them. By transferring resources from children to parents, public intergenerational transfers alleviate such liquidity constraints, thereby increasing the level of education. This is good for growth, provided that public pensions do not offset the favourable impact on human capital formation by crowding out savings.

In this paper, we investigate the relationship between the size of an unfunded public pension system and economic growth in an overlapping generation model, in which altruistic parents may affect the income of their children through bequests and education. We find that there is no case for public pensions if transfers of both physical (bequests) and human (education) capital are operative in the family. Public pensions do not affect the economic equilibrium if they are financed by lump-sum social contributions, but are bad for growth if they are financed by distortionary taxation. In contrast, public pensions can be good for growth if parents leave no bequests. In such a situation, public pensions make parents increase their consumption and educational spending. One has then to balance the positive growth effects of enhanced human capital formation with the negative effects of lower savings and physical capital. We show that, unless individuals are sufficiently impatient, there exists a growth-maximising size of the public pension system.

1 Introduction

This paper investigates the relationship between the size of a public pension system and economic growth in an overlapping generation economy, in which altruistic parents may affect their children's income through education and bequests.

Individuals' reaction to fiscal policy, which determines the effectiveness of policy making in stimulating growth[1], mainly depends on the span of their forecasting horizon. The current income-driven consumer of the Keynesian paradigm is typically short-sighted, which contrasts with the far-sighted Ricardian individual, who is able to see through the government's intertemporal budget constraint and to counter the effects of fiscal policy. Whereas pay-as-you-go public pensions fully crowd out private savings in the textbook version of overlapping generation models[2], where individuals' horizons are limited to their own life-cycle, successive generations of altruistic individuals are nested through a chain of bequests, thereby making fiscal policies ineffective (Barro 1974).

A large body of literature has examined and qualified the conditions under which Ricardian equivalence holds in altruistic overlapping generation models (e.g., Abel 1987; Weil 1987; Thibault 2000). If bequests are operative both before and after a policy change, fiscal policy is then ineffective. This literature is based on Barro's seminal assumption that individual preferences are defined by a recursive relation (a Bellman equation), which extends the planning horizon of economic agents to infinity in spite of a finite lifetime. Regardless of bequests, significant altruistically-motivated transfers of human capital take place in the family, as children are not capable of caring for themselves or making contractual arrangements to self-finance education (Becker 1991). Drazen (1978), extending Barro's model to transfers of both human capital (education) and physical capital (bequests), assumes that altruistic parents face a trade-off between education and bequests, which is determined by returns on investment in each type of capital. Parents invest in the education of their children as long as the return on education is higher than the rate of interest. The main result of this approach (see also Becker and Tomes 1986) is that households who cannot leave bequests underinvest in the formation of their children's human capital, inasmuch as the rate of return on education in these households remains above the rate of interest. Parents cannot borrow to invest more in the education of

[1]For a review of the effectiveness of fiscal policy in stimulating economic activity, see Hemming *et al.* (2000).

[2]See Blanchard and Fisher (1989).

their children, since they are forbidden by law to force their children to reimburse them. Obviously, publicly-provided intergenerational transfers such as unfunded public pension benefits can alleviate such a liquidity constraint by transfering resources from children to parents, thereby increasing the level of education. This could enhance growth, provided that public pensions do not offset the favourable impact on human capital formation by crowding out savings. The question addressed in this paper is, under which conditions of preferences and technology can an increase in the size of a public pension system be good for growth?

Our approach departs from much of the literature on public pensions and growth, which assumes either that individuals have an infinite planning horizon and face binding bequest constraints (Caballé 1995) or that intergenerational transfers are motivated by joy-of-giving altruism (Kaganovich and Zilcha 1999; Sanchez-Losada 2000). Kaganovich and Zilcha consider only transfers of human capital and analyse the role of government's allocation of revenue between public spending on education and social security benefits, in line with the literature on private versus public education (see Glomm and Ravikumar 1992). Our study however considers transfers of both human and physical capital, focusing on the impact of public pensions on the allocation of private resources between education, consumption, savings and bequests. In contrast to the existing literature, Sanchez-Losada derives a new result that unfunded public pensions can increase growth in an economy in which bequests are operative. However, this result is driven by the assumption of joy-of-giving altruism, which bears two caveats: first, it does not allow for inoperative bequests; second, the return on human capital is not related to the return on physical capital. In our model we assume a form of altruism, adapted from Becker (1991), that allows for a trade-off between bequests and education driven by relative returns. Unlike joy-of-giving or recursive altruism, this form of altruism posits that parents are concerned for the income of their adult children (not for their utility), which they can affect by either educating them or leaving a bequest. This intuitive assumption makes altruistic models more tractable from a technical viewpoint than the recursive form of altruism advocated by Barro and allows us to provide a full characterisation of the effect of public pensions on growth.

In this framework we find that there is no case for public pensions if transfers of both physical and human capital are operative in the family. The Ricardian equivalence theorem applies to our model if bequests are operative and public pension benefits are financed by lump-sum contributions. When contributions are proportional to labour income, public pensions are bad for growth in an economy with operative bequests, since their only effect is to distort parental choice between bequests and education. These first results are in line with most of the literature. However, there is a stronger case for public pensions if parents leave no bequests. Providing publicly-financed old age support makes individuals increase not only their own consumption but also their educational spending. One has to balance the positive effects of enhanced human capital formation with the negative effects of lower savings and physical capital. We show that, unless individuals are sufficiently impatient, there exists a growth-maximising size of the public pension system. Interestingly, this result applies to both lump-sum and proportional contributions. Finally, it should be made clear that our results are positive, not normative, since we only look at the growth effect of public pensions and do not assess their welfare effect.

The paper is organised as follows. In section 2, we set up the basic model. In section 3, we study the balanced growth path, which is characterised by either operative or inoperative bequests. In section 4, we examine the growth effect of an increase in the scale of a public pension programme in an economy where bequests are inoperative. We set out our concluding remarks in section 5.

2 The Model

The basic framework is an overlapping generation model *à la* Allais (1947)-Samuelson (1958)-Diamond (1965), in which parents are altruistically concerned for the well-being of their children. This concern is expressed by providing children with a disposable income later on in life and departs from both joy-of-giving and recursive altruism. We borrow this formulation from Becker (1991), who points out that wealth of children differs from expenditure on children. A parent faces a trade-off between transfers of human capital (education) and physical capital (bequests) to enhance the disposable income of his children.

2.1 Individuals

The economy consists of a sequence of individuals who live for three periods: childhood, adulthood and old-age. In the second period of their life, each individual gives birth to $1 + n$ children, so that the population grows at a constant rate n. In each period

t, a new cohort consisting of $N_t (= (1+n) N_{t-1})$ identical individuals is born. During childhood individuals do not make any decisions, as it is assumed that their consumption is included in their parents' consumption. They are reared by their parents, who finance their education. When adult, they work and receive the market wage, consume, save and rear their own children. When old, they are retired and leave a bequest x_{t+1} to each of their $1+n$ children.

During adulthood each individual born at $t-1$ supplies inelastically h_t efficiency units of labour, his level of human capital depending on his parents' spending on education. In addition, each adult inherits x_t from his parents so that his current income is:

$$\omega_t \equiv w_t h_t + x_t \tag{1}$$

where w_t is the wage rate per efficiency unit of labour. Adults distribute their income among own consumption, c_t, spending on their children's education, $(1+n) e_t$, and savings, s_t:

$$\omega_t = c_t + (1+n) e_t + s_t \tag{2}$$

During old-age individuals are retired and receive the proceeds of their savings. Each individual leaves a bequest x_{t+1} to each of his $1+n$ children. Old-age consumption is simply equal to the proceeds of savings minus bequests:

$$d_{t+1} = R_{t+1} s_t - (1+n) x_{t+1} \tag{3}$$

Bequests are assumed to be non-negative:

$$x_{t+1} \geq 0 \tag{4}$$

The human capital of each individual born at t, h_{t+1}, is a function of his parents' private educational spending, e_t, and his parents' human capital, h_t:

$$h_{t+1} = D e_t^{\delta} h_t^{1-\delta} \tag{5}$$

where $D > 0$ is a scale factor and $\delta \in \,]0,1[$ denotes the elasticity of the technology of education with respect to educational spending. Since an individual's level of human capital only depends on his own parents' human capital and spending on education, there is no intragenerational externality in human capital. Education is privately financed and a family's decision on children's education does not impact on other families' human capital formation. The model, therefore, does not address the issue of public versus private education and focuses on the impact of public pensions on privately-financed human capital formation.

An adult (in period $t+1$) receives an income that consists of labour earnings and inheritance:

$$\omega_{t+1} = w_{t+1}h_{t+1} + x_{t+1} = w_{t+1}De_t^{\delta}h_t^{1-\delta} + x_{t+1} \tag{6}$$

We assume that parents are altruistic towards their children and model this by making the disposable income of their adult children, ω_{t+1}, an argument of their utility function. Each individual born at $t-1$ has the following lifetime utility function:

$$U_t = (1-\beta)\ln c_t + \beta \ln d_{t+1} + \gamma \ln \omega_{t+1} \tag{7}$$

where $0 < \beta < 1$ and $\gamma > 0$ is the intergenerational degree of altruism. Children are educated during chilhood (period t) and inherit during adulthood.

We now add public pension contributions and benefits to an individual's budget constraints. Contributions can be levied either as lump-sum taxes (η_t) or as proportional labour income taxes (τ), whereas public pension benefits are distributed in a lump-sum way. We modify the budget constraints (1), (3) and (6) accordingly:

$$\omega_t = (1-\tau)\, w_t h_t - \eta_t + x_t \tag{8}$$

$$d_{t+1} = R_{t+1}s_t + \theta_{t+1} - (1+n)\, x_{t+1} \tag{9}$$

$$\omega_{t+1} = (1-\tau)\, w_{t+1}h_{t+1} - \eta_{t+1} + x_{t+1} = (1-\tau)\, w_{t+1}De_t^{\delta}h_t^{1-\delta} - \eta_{t+1} + x_{t+1} \tag{10}$$

An individual born in period $t-1$ is endowed with h_t units of human capital at the outset of adulthood and chooses e_t, s_t, c_t, d_{t+1}, x_{t+1} and ω_{t+1} so as to maximise his life-cycle utility (7) under constraints (2), (8), (9), (4) and (10). An individual's optimal choice is characterised by the first order conditions:

$$\frac{\partial U_t}{\partial s_t} = -\frac{1-\beta}{c_t} + \frac{\beta R_{t+1}}{d_{t+1}} = 0 \tag{11}$$

$$\frac{\partial U_t}{\partial e_t} = -\frac{(1+n)\,(1-\beta)}{c_t} + \frac{(1-\tau)\,\gamma w_{t+1}D\delta e_t^{\delta-1}h_t^{1-\delta}}{\omega_{t+1}} = 0 \tag{12}$$

$$\frac{\partial U_t}{\partial x_{t+1}} = -\frac{(1+n)\,\beta}{d_{t+1}} + \frac{\gamma}{\omega_{t+1}} \leq 0 \tag{13}$$

If bequests are operative, the expression (13) holds with equality:

$$\frac{\partial U_t}{\partial x_{t+1}} = -\frac{(1+n)\,\beta}{d_{t+1}} + \frac{\gamma}{\omega_{t+1}} = 0 \tag{14}$$

Plugging (11) and (12) into (13) gives:

$$(1-\tau)\, w_{t+1} D\delta e_t^{\delta-1} h_t^{1-\delta} \geq R_{t+1} \tag{15}$$

The rate of return on education is greater than (or equal to) the rate of interest. The rate of return on education is modified by proportional contributions, but not by lump-sum contributions. Both rates are equal when bequests are operative. In such a case we say that private educational spending is optimal. When bequests are inoperative, the rate of return on education is strictly higher than the rate of interest and the level of private educational spending is suboptimal.

2.2 Firms

In each period t, production occurs according to a Cobb-Douglas technology using two inputs, physical capital K_t and human capital H_t. Output is given by:

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} \tag{16}$$

where $A > 0$ is a scale parameter and $\alpha \in \,]0,1[$ denotes the capital share. In each period the stock of capital results from individuals' savings in the preceding period. In any period $t \geq 1$ we have:

$$K_t = N_{t-2} s_{t-1} \tag{17}$$

The initial stock of capital (K_0) is given, and belongs to the N_{-2} individuals, who are old in period 0; each of them owns $s_{-1} = K_0/N_{-2}$.

The demand for labour (human capital) maximises profit:

$$\Pi_t = \max_{H_t} \left(AK_t^{\alpha} H_t^{1-\alpha} - w_t H_t \right)$$

This implies:

$$w_t = (1-\alpha)\, AK_t^{\alpha} H_t^{-\alpha} = (1-\alpha)\, Ak_t^{\alpha} \tag{18}$$

where $k_t = K_t/H_t$ is the physical to human capital ratio. Profits are rebated to capital owners, the old in period t. The return on savings is therefore given by:

$$R_t = \frac{\Pi_t}{K_t} = \alpha AK_t^{\alpha-1} H_t^{1-\alpha} = \alpha Ak_t^{\alpha-1} \tag{19}$$

2.3 Intertemporal equilibrium

The public pension scheme works as follows. The government in period t raises $N_{t-1}(\eta_t + \tau w_t h_t)$ in contributions from the adult and rebates the revenue to the current old. Each of the N_{t-2} retirees in period t receives a lump-sum benefit, θ_t. The government's budget is balanced in each period t:

$$\theta_t = (1+n)(\eta_t + \tau w_t h_t) \tag{20}$$

To obtain a balanced growth path (along which all individual variables grow at the same rate) we further assume that lump-sum social contributions are a fraction $\eta \in \,]0,1[$ of total labour income, $w_t H_t$:

$$N_{t-1}\eta_t = \eta w_t H_t$$

This implies:

$$\theta_t = (1+n)(\eta + \tau) w_t h_t$$

and

$$\omega_t \equiv (1 - \tau - \eta) w_t h_t + x_t$$

In the remainder of this paper we refer to $\eta + \tau$ as the public pension ratio. Given the initial capital stock, k_0 $(= K_0/N_{-2} = s_{-1})$, the initial level of human capital, h_0, and a public pension ratio $\eta + \tau$, a perfect foresight intertemporal equilibrium is a sequence of quantities and prices $\{c_t, d_t, k_t, e_t, s_t, x_t; w_t, R_t\}$ such that individuals maximise utility, factor markets are competitive and all markets clear. The labour and the good market clear:

$$H_t = N_{t-1} h_t \tag{21}$$

$$Y_t = N_{t-1}(c_t + s_t + (1+n) e_t) + N_{t-2} d_t \tag{22}$$

According to Walras' law in period t, the equilibrium of the labour market implies that of the good market. In each period we can substitute the old's budget constraint for the equilibrium condition of the good market. Using (2), the expression (22) becomes:

$$d_t + (1+n)\omega_t = (1+n) A k_t^{\alpha} h_t \tag{23}$$

The intertemporal equilibrium is parameterised by the public pension ratio (η and τ), thereby allowing for an analysis of the growth effects of unfunded public pensions financed by either lump-sum or proportional contributions. In the next section we further analyse the intertemporal equilibrium and show that, given values for the parameters, it is characterised by either operative or inoperative bequests.

3 Dynamics

3.1 Operative bequests

When bequests are operative in period $t+1$, the expressions (14) and (23) give:

$$\omega_{t+1} = \frac{1}{1+\beta/\gamma} A k_{t+1}^{\alpha} h_{t+1} \tag{24}$$

$$d_{t+1} = \frac{(1+n)\,\beta}{\beta+\gamma} A k_{t+1}^{\alpha} h_{t+1} \tag{25}$$

The non-negative bequest condition, $x_{t+1} = \omega_{t+1} - (1-\eta-\tau)\, w_{t+1} h_{t+1} \geq 0$, defines a lower bound (χ) on the public pension ratio:

$$\eta + \tau \geq 1 - \frac{\gamma}{(1-\alpha)\,(\beta+\gamma)} \equiv \chi \tag{26}$$

When the degree of altruism is sufficiently high, i.e. $\gamma \geq \frac{1-\alpha}{\alpha}\beta$, this lower bound is negative ($\chi \leq 0$) and bequests are operative, regardless of the public pension ratio. When the condition (26) is satisfied, (15) holds with equality. The combination of (15), (18) and (19) gives a simple expression for the educational spending per unit of human capital, $\overline{e}_t \equiv e_t/h_t$:

$$\overline{e}_t^{1-\delta} = \frac{(1-\tau)\,\delta D\,(1-\alpha)}{\alpha} k_{t+1} \tag{27}$$

The expression (5) can be re-written as follows: $h_{t+1} = D\overline{e}_t^{\delta} h_t$, which implies together with (27):

$$h_{t+1} k_{t+1} = \frac{\alpha}{(1-\tau)\,\delta\,(1-\alpha)} \overline{e}_t h_t$$

We also obtain simple expressions for savings (from $N_{t-1} s_t = K_{t+1}$) and consumption (from (11) and (25)):

$$s_t = (1+n)\, k_{t+1} h_{t+1} = \frac{(1+n)\,\alpha}{(1-\tau)\,\delta\,(1-\alpha)} \overline{e}_t h_t \tag{28}$$

$$c_t = \frac{(1+n)\,(1-\beta)}{\alpha\,(\beta+\gamma)} k_{t+1} h_{t+1} = \frac{(1+n)\,(1-\beta)}{(1-\tau)\,\delta\,(\beta+\gamma)\,(1-\alpha)} \overline{e}_t h_t \tag{29}$$

We can therefore re-write the budget constraint of an adult:

$$\omega_t = B\,(\tau)\, \overline{e}_t h_t \tag{30}$$

where

$$B(\tau) = (1+n)\left(1 + \frac{1-\beta}{(1-\tau)(\beta+\gamma)\delta(1-\alpha)} + \frac{\alpha}{(1-\tau)\delta(1-\alpha)}\right) \tag{31}$$

$B(\tau)$ is increasing, while $(1-\tau)B(\tau)$ is decreasing. Given stocks of physical and human capital, k_t and h_t, the educational spending per unit of human capital

$$\overline{e}_t = \frac{1}{B(\tau)}\frac{\omega_t}{h_t} = \frac{1}{B(\tau)}\frac{1}{1+\beta/\gamma}Ak_t^{\alpha} \tag{32}$$

decreases with τ, whereas consumption and savings, which are proportional to $\overline{e}_t/(1-\tau)$, increase. Next period's capital stock k_{t+1} is proportional to $\overline{e}_t^{1-\delta}/(1-\tau)$, and increases with τ, since $B(\tau)(1-\tau)^{\frac{1}{1-\delta}}$ is increasing. There are two effects at work; the direct negative effect of a proportional tax reducing the perceived return on education combines with a positive effect on savings and the accumulation of physical capital.

The educational spending per unit of human capital, $\overline{e}_t$, is increasing with respect to γ and β, as:

$$\begin{aligned}\frac{\partial((1+\beta/\gamma)B(\tau))}{\partial\beta} &= -\frac{(1+n)(1-(1-\tau)\delta)}{(1-\tau)\delta\gamma} < 0 \\ \frac{\partial((1+\beta/\gamma)B(\tau))}{\partial\gamma} &= -\frac{\beta B(\tau)}{\gamma^2} - \frac{(1+n)(1-\beta)}{\gamma(\beta+\gamma)(1-\tau)\delta(1-\alpha)} < 0\end{aligned}$$

More altruistic parents devote more resources to the education of their children. Patience is also a positive determinant of parental spending on education.

The following proposition characterises the intertemporal equilibrium path with operative bequests and establishes that lump-sum taxation is neutral and that proportional taxation reduces the long-run growth rate.

Proposition 1 *When the public pension ratio is not too low*[3]*, $\eta+\tau \geq \chi$, there exists a unique intertemporal equilibrium with operative bequests in each period, given initial values for physical, $k_0 > 0$, and human capital, $h_0 > 0$. This equilibrium is characterised by a sequence $(k_t, \overline{e}_t)$ defined by:*

$$\left(\frac{(1-\tau)\delta D(1-\alpha)}{\alpha}k_{t+1}\right)^{\frac{1}{1-\delta}} = \overline{e}_t = \frac{Ak_t^{\alpha}}{B(\tau)(1+\beta/\gamma)} \tag{33}$$

This sequence converges monotonically towards a steady state $(k, \overline{e})$ that defines a balanced growth path. Along the balanced growth path, all individual variables $(\omega_t, c_t, s_t, e_t, d_t, x_t)$ grow at the same rate as human capital[4]*: $h_{t+1}/h_t = D\overline{e}^{\delta}$. The intertemporal equilibrium is independent from the lump-sum contribution η, whereas the long-run growth rate decreases with the proportional contribution τ.*

[3] This ratio only needs to be positive when $\chi \leq 0$, i.e. $\gamma \geq \frac{1-\alpha}{\alpha}\beta$.
[4] Hence the growth rate is given by $D\overline{e}^{\delta} - 1$.

Proof: The condition $\eta+\tau \geq \chi$ guarantees that there exists a temporary equilibrium with operative bequests in each period $t \geq 0$. In the initial period $t = 0$, the first old allocates the proceeds of his savings, $R_0 s_{-1}$, between old-age consumption, d_0, and bequests, x_0, to his $1+n$ children; (24) and (25) give: $\omega_0 = \frac{1}{1+\beta/\gamma} A k_0^{\alpha} h_0$ and $d_0 = \frac{(1+n)\beta}{\beta+\gamma} A k_0^{\alpha} h_0$. (24), (25), (27) and (29) then imply that (33) holds in any period $t \geq 0$. A sequence $(k_t, \overline{e}_t)$ that satisfies (33) together with $h_{t+1} = D\overline{e}_t^{\delta} h_t$ determines all variables uniquely. These variables do not depend on η. The sequence (k_t) solution to (33) converges monotonically towards a steady state k; the sequence $(\overline{e}_t)$ converges likewise towards $\overline{e} = \frac{Ak^{\alpha}}{B(\tau)(1+\beta/\gamma)}$.

The educational spending per unit of human capital is given by:

$$\overline{e}^{1-\alpha(1-\delta)} = \frac{A}{B(\tau)(1+\beta/\gamma)} \left(\frac{\alpha}{(1-\tau)\delta D(1-\alpha)} \right)^{\alpha}$$

where $B(\tau) = (1+n)\left(1 + \frac{1-\beta+\alpha(\beta+\gamma)}{\delta(\beta+\gamma)(1-\alpha)} \frac{1}{1-\tau}\right)$. The growth effect of an increase in proportional contributions can be easily obtained by studying the function $\left[(1-\tau)^{\alpha} B(\tau)\right]^{-1}$. One easily sees that the derivative of this function is always negative, since for any $\tau \geq 0$:

$$\alpha(1-\tau) - \frac{1-\beta}{\delta(\beta+\gamma)} - \frac{\alpha}{\delta} < 0$$

In the long run the economy grows along a balanced growth path. All individual variables grow at the same rate $h_{t+1}/h_t = D\overline{e}^{\delta}$, while the capital stock, the wage rate and the rate of interest are constant.■

Public pensions do not foster growth when intergenerational transfers of both physical and human capital are operative in the family. In the case of lump-sum contributions, public pensions are neutral. In the case of proportional contributions, an increase in the scale of a public pension system slows down growth, since taxes levied to finance the programme distort the educational choices of parents.

3.2 Inoperative bequests

When bequests are inoperative in period $t+1$ ($x_{t+1} = 0$), (10) and (23) become:

$$\omega_{t+1} = (1-\eta-\tau) w_{t+1} h_{t+1} = (1-\eta-\tau)(1-\alpha) A k_{t+1}^{\alpha} h_{t+1} \tag{34}$$

$$d_{t+1} = (1+n)(1-(1-\eta-\tau)(1-\alpha)) A k_{t+1}^{\alpha} h_{t+1} \tag{35}$$

The educational spending per unit of human capital, $\overline{e}_t \equiv e_t/h_t$, satisfies (12). Using (11), (34) and (35), we obtain:

$$\overline{e}_t^{1-\delta} = \frac{(1-\tau)\,\gamma\delta D}{\alpha\beta}\left(\frac{1}{1-\eta-\tau}-1+\alpha\right)k_{t+1} \tag{36}$$

This expression shows that, for k_{t+1} given, an increase in either proportional or lump-sum contributions enhances educational spending. On the other hand, an increase in contributions has a negative impact on k_{t+1} *via* savings. There is, therefore, the question whether unfunded public pensions speed up or slow down growth in an economy where bequests are inoperative. In this section we characterise the intertemporal equilibrium of an economy with inoperative bequests, and we come to grips with this question in the next.

We re-write the non-negative bequest condition (15) as follows:

$$\overline{e}_t^{1-\delta} \leq \frac{(1-\tau)\,\delta D\,(1-\alpha)}{\alpha}k_{t+1}$$

This and (36) define an upper bound on the public pension ratio:

$$\eta+\tau \leq 1-\frac{\gamma}{(1-\alpha)\,(\beta+\gamma)} \equiv \chi \tag{37}$$

Bequests are inoperative when this inequality holds. Using $h_{t+1} = D\overline{e}_t^{\delta}h_t$ and (36) we obtain:

$$(1-\tau)\left(\frac{1}{1-\eta-\tau}-1+\alpha\right)k_{t+1}h_{t+1} = \frac{\alpha\beta}{\gamma\delta}\overline{e}_t h_t$$

We then obtain simple expressions for savings and consumption (using (11), (35) and $N_{t-1}s_t = K_{t+1}$):

$$s_t = (1+n)\,k_{t+1}h_{t+1} = \frac{(1+n)\,\alpha\beta\,(1-\eta-\tau)}{\gamma\delta\,(1-\tau)\,(\alpha+(\tau+\eta)\,(1-\alpha))}\overline{e}_t h_t$$

$$c_t = \frac{(1-\beta)\,(1+n)\,(\alpha+(\tau+\eta)\,(1-\alpha))}{\alpha\beta}k_{t+1}h_{t+1} = \frac{(1-\beta)\,(1+n)\,(1-\eta-\tau)}{\gamma\delta\,(1-\tau)}\overline{e}_t h_t$$

The budget constraint of an adult (2) becomes:

$$\omega_t = \widetilde{B}\,(\eta,\tau)\,\overline{e}_t h_t \tag{38}$$

where

$$\widetilde{B}\,(\eta,\tau) = (1+n)\left(1+\frac{\alpha\beta\,(1-\eta-\tau)}{\gamma\delta\,(1-\tau)\,(\alpha+(\tau+\eta)\,(1-\alpha))}+\frac{(1-\beta)\,(1-\eta-\tau)}{\gamma\delta\,(1-\tau)}\right)$$

The next proposition characterises the intertemporal equilibrium of an economy with inoperative bequests.

Proposition 2 *When the public pension ratio satifies $\eta + \tau < \chi$, there exists a unique intertemporal equilibrium with inoperative bequests in each period, given initial values for physical capital, $k_0 > 0$, and human capital, $h_0 > 0$. This equilibrium is characterised by a sequence $(k_t, \overline{e}_t)$ defined by:*

$$\frac{(1-\eta-\tau)(1-\alpha) A k_t^{\alpha}}{\widetilde{B}(\eta,\tau)} = \overline{e}_t = \left(\frac{(1-\tau)\gamma\delta D}{\alpha\beta} \left(\frac{1}{1-\eta-\tau} - 1 + \alpha \right) k_{t+1} \right)^{\frac{1}{1-\delta}} \tag{39}$$

This sequence converges monotonically towards a steady state $(k, \overline{e})$ that defines a balanced growth path.

The proof of proposition 2 follows the same lines as that of proposition 1.

Our main purpose is now to assess the growth effect of a change in the size of a public pension system when bequests are inoperative.

4 Public pensions and long-run growth

When bequests are operative, the intertemporal equilibrium is independent from the public pension ratio in the case of lump-sum contributions. Bequests are operative and the public pension programme is neutral, in line with Barro's (1974) Ricardian result. Individuals are able to see through the government's intertemporal budget constraint and to counter public transfers between generations. There is therefore no case for a public pension system financed by lump-sum contributions when individuals are sufficiently altruistic ($\gamma \geq (1-\alpha)\beta/\alpha$). In the case of proportional contributions, increasing the size of a public pension system reduces the return on human capital, therefore harming growth. It is well known that the Ricardian result of fiscal policy neutrality only applies to lump-sum transfers between generations.

When bequests are inoperative ($\chi > 0$ or $\gamma < (1-\alpha)\beta/\alpha$), public pensions modify the educational choices of parents even in the case of lump-sum contributions. In the absence of public pensions ($\eta = \tau = 0$), parents' educational spending is suboptimal. Increasing the public pension ratio alleviates the suboptimality of educational choices and is likely to foster growth. Pension benefits, all other things being equal, increase parents' lifetime income and allow them to spend more on both their own lifetime consumption and the education of their children. However, there are two effects working in opposite directions. The first one is that, in an intertemporal framework, individuals receive social transfers in old-age but pay social contributions, which impact negatively on their savings, thereby reducing the accumulation of physical capital. The second is a general equilibrium effect: lower levels of physical capital result in lower wages per unit of human capital, therefore lowering the return on education. There is the question whether the positive effect can dominate the negative effects of public pensions. In the case of proportional contributions there is one additional negative effect of public pensions, since proportional taxes on labour income reduce the return on education.

4.1 Lump-sum contributions ($\chi > \eta > 0$, $\tau = 0$)

We first consider the case of lump-sum contributions. Rearranging (39) in steady state gives the following expression for the long-run educational spending per unit of human capital, which takes account of all positive and negative effects of public pensions on growth:

$$\overline{e}^{1-\alpha(1-\delta)} = \frac{(1-\eta)(1-\alpha)A}{\widetilde{B}(\eta,0)} \left(\frac{\alpha\beta(1-\eta)}{\gamma\delta D(1-(1-\eta)(1-\alpha))} \right)^{\alpha} \tag{40}$$

To assess the growth effect of an increase in the public pension ratio η, one must characterise the sign of $\frac{\partial \overline{e}}{\partial \eta}$. We pursue the study of this derivative in appendix I, where we show that $\frac{\partial \overline{e}}{\partial \eta}$ has the same sign as a function $\Phi(\eta)$, which decreases from

$$\Phi(0) = \frac{\gamma}{\beta}(-2\alpha\delta\gamma - \alpha + \beta(1-\alpha))$$

to

$$\Phi(\chi) = \frac{1-\alpha}{\alpha}\big((1-\delta)\alpha(1-\alpha)\gamma - \alpha(1-\beta) - \delta\beta - \alpha^2\beta(1-\delta)\big)$$

Three cases therefore are possible:

Case A (Public pensions bad for growth): If $\Phi(0) \leq 0$, i.e. $\beta \leq \frac{\alpha(2\delta\gamma+1)}{1-\alpha}$, $\frac{\partial \overline{e}}{\partial \eta}$ is negative for all $\eta \in [0,\chi]$. The negative effects of public pensions dominate the positive effect for all levels of the public pension ratio. This case arises when individuals are sufficiently impatient, i.e. they do not save much. The crowding out effect of public pensions on savings dominates and public pensions harm growth.

Case B (Growth-maximising public pension ratio): If $\Phi(0) > 0 > \Phi(\chi)$, the introduction of a public pension programme is good for growth. $\frac{\partial \overline{e}}{\partial \eta}$ is positive as long as $\Phi(\eta)$ is positive. There exists a threshold $\widehat{\eta}$ $(= \Phi^{-1}(0))$ above which a further increase in the public pension ratio reduces growth. The public pension ratio $\widehat{\eta}$ corresponds to the growth-maximising size of a public pension system.

Case C (Public pensions good for growth): If $\Phi(\chi) \geq 0$, $\frac{\partial \overline{e}}{\partial \eta}$ is positive for all $\eta \in [0, \chi]$, i.e. increasing the size of the public pension system promotes growth. This case obtains for more restrictive values of the parameters.

In the proof of proposition 3 below, we show that the case C cannot obtain when the elasticity of the technology of education with respect to educational spending (δ) is above a threshold, $\overline{\delta} = \frac{1-2\alpha}{2(1-\alpha)}$. To illustrate how the pattern of public pensions and growth depends on patience and altruism we resort to a diagrammatic representation. The effect of public pensions on growth depends on two key parameters characterising households' behaviours, namely β, which represents thrift, and γ, which indicates the strength of parental concern for children. Figure 1 depicts the pattern of public pensions and growth, when the elasticity of the technology of education is larger than $\overline{\delta} = \frac{1-2\alpha}{2(1-\alpha)}$. Figure 2 depicts this pattern in the case of a low elasticity of the technology of education ($\delta < \overline{\delta}$). The three cases A, B and C are then possible, depending on the values taken by γ and β.

Insert figures 1 and 2 here

The main results of this section are summarised in the following proposition.

Proposition 3 *If parents are sufficiently altruistic towards their children, i.e.* $\gamma \geq \frac{1-\alpha}{\alpha}\beta$, *bequests are operative and a public pension programme financed by lump-sum contributions does not affect growth. If parents are not sufficiently altruistic towards their children, i.e.* $\gamma < \frac{1-\alpha}{\alpha}\beta$, *bequests are inoperative and the introduction of a public pension programme financed by lump-sum contributions promotes growth if and only if individuals are sufficiently patient, i.e.:*

$$\beta > \frac{\alpha(1+2\delta\gamma)}{1-\alpha} \tag{41}$$

There then exists a growth-maximising size of the public pension programme, $\widehat{\eta} \leq \chi$. *When the elasticity of the technology of education with respect to educational spending is sufficiently high*[5], *i.e.* $\delta > \overline{\delta} = \frac{1-2\alpha}{2(1-\alpha)}$, *this maximum is interior* ($\widehat{\eta} < \chi$) *and the growth rate decreases with the public pension ratio if the size of the system is larger than* $\widehat{\eta}$.

Proof: The positive effect of public pensions on growth arises in the cases B and C. This requires that $\chi > 0$ and $\Phi(0) > 0$, i.e.:

$$\beta > \frac{\alpha\gamma}{1-\alpha} \text{ and } \beta > \frac{\alpha(1+2\delta\gamma)}{1-\alpha}$$

[5] For a capital share α equal to 1/3 condition, $\overline{\delta}$ is equal to 1/4 and (41) requires $\beta > 1/2$.

We have $\widehat{\eta} < \chi$ in the case B and $\widehat{\eta} = \chi$ in the case C.

Moreover, we have $\Phi(\chi) \geq 0$ in the case C, which is equivalent to:

$$(1-\delta)\gamma \geq \frac{1}{1-\alpha} - \beta + \beta\delta\left(\frac{1+\alpha}{\alpha}\right)$$

Using this and the condition $\chi > 0$ ($\Leftrightarrow \gamma < \frac{1-\alpha}{\alpha}\beta$) we obtain:

$$2\delta < 1 - \frac{\alpha}{(1-\alpha)\beta} \leq \frac{1-2\alpha}{1-\alpha}, \text{ since } \beta \leq 1$$

This implies: $\delta < \frac{1-2\alpha}{2(1-\alpha)} = \overline{\delta}$. The case C is therefore excluded for $\delta > \overline{\delta}$.■

In the case A, increasing the size of a public pension system bears a negative effect on growth, since it makes individuals save less, thereby decreasing the accumulation of physical capital and growth. This negative effect on growth dominates the positive effect on human capital formation.

In the case B, increasing the size of a public pension system has a positive effect on growth until the system has reached its growth-maximising size. Increasing further the size of the system has a negative impact on growth. Typically, this is an example of non-linearities in the pattern of public pensions and growth

In the case C, increasing the size of a public pension system has always a positive impact on growth until bequests become operative. Individuals must be sufficiently patient and have a degree of altruism sufficiently close to the level that would make them leave bequests (i.e., $\frac{1-\alpha}{\alpha}\beta$). In addition, this case is ruled out if the elasticity of the technology of education with respect to educational spending is sufficiently high. For instance, if the physical capital share is larger than 1/2, this case is excluded. For a capital share of 1/3, this case is excluded if the elasticity of the technology of education is larger than 1/4.

4.2 Proportional contributions ($\eta = 0$, $\chi > \tau > 0$)

When bequests are operative, increasing the size of a public pension system financed by proportional contributions is bad for growth, since it reduces the incentives for human capital formation. By contrast, this section shows that there is a case for a public pension programme financed by proportional contributions when bequests are not operative.

Bequests are inoperative as long as the tax rate does not exceed an upper bound:

$$\tau < 1 - \frac{\gamma}{(1-\alpha)(\beta+\gamma)} \equiv \chi$$

By assumption, individuals are not too altruistic, i.e. $\chi > 0 \Leftrightarrow \gamma < \beta(1-\alpha)/\alpha$. We have a new expression for the long-run educational spending per unit of human capital (see proposition 2):

$$\overline{e}^{1-\alpha(1-\delta)} = (1-\alpha) A \left(\frac{\alpha\beta}{\gamma\delta D}\right)^{\alpha} F(\tau)$$

where

$$F(\tau) = \frac{1-\tau}{\widetilde{B}(0,\tau)\left(1-(1-\alpha)(1-\tau)\right)^{\alpha}}$$

and

$$\widetilde{B}(0,\tau) = (1+n)\left(1 + \frac{1-\beta}{\gamma\delta} + \frac{\alpha\beta}{\gamma\delta\left(1-(1-\alpha)(1-\tau)\right)}\right)$$

The study of the first derivative of $\overline{e}$ with respect to τ is carried out in appendix II, where we establish that the derivative $\frac{\partial \overline{e}}{\partial \tau}$ has the same sign as a function $\Psi(\tau)$, which decreases from

$$\Psi(0) = \alpha\left(\beta(1-\alpha) - \alpha(2-\alpha)(\gamma\delta+1)\right)$$

to

$$\Psi(\chi) = \frac{\beta}{(\beta+\gamma)^2}\left(\alpha\gamma^2(1-\alpha-\delta) + \gamma\beta(\alpha(1-\alpha)-\delta) + (1-\alpha)\beta^2 - \beta - \alpha\gamma\right)$$

We again find three possible cases:

Case A' (Public pensions bad for growth): If $\Psi(0) \leq 0$, i.e. $\beta \leq \frac{\alpha(2-\alpha)(1+\gamma\delta)}{1-\alpha}$, increasing the size of a public pension system slows down growth for all $\tau \in \left]0,\chi\right[$.

Case B' (Growth-maximising public pension ratio): If $\Psi(0) > 0 > \Psi(\chi)$, increasing the size of a public pension system promotes growth for $\tau \leq \widehat{\tau}$ and reduces growth for $\tau > \widehat{\tau}$, where $\widehat{\tau}$ is the solution to $\Psi(\tau) = 0$.

Case C' (Public pensions good for growth): If $\Psi(\chi) > 0$, an increase in the size of a public pension system promotes growth for all $\tau \in \left]0,\chi\right[$.

In the proof of proposition 4 below we show that the case C' obtains only if the elasticity of the technology of education with respect to educational spending (δ) is lower than $\overline{\delta}' = \frac{1-3\alpha+\alpha^2}{(2-\alpha)(1-\alpha)}$. It is worth noting that $\overline{\delta}' < \overline{\delta}$. Another difference with the lump-sum case is that the frontier between C' and B' is not a line, but a curve defined by $\Psi(\chi) = 0$. Apart from that, the diagrammatic representation of the pattern of public pensions and growth is qualitatively similar to that obtained in the case of lump-sum social contributions.

The next proposition summarises the main results of this section.

Proposition 4 *If parents are sufficiently altruistic towards their children, i.e.* $\gamma \geq \frac{1-\alpha}{\alpha}\beta$, *bequests are operative and a public pension programme financed by proportional contributions has a negative effect on growth. If parents are not sufficiently altruistic towards their children, i.e.* $\gamma < \frac{1-\alpha}{\alpha}\beta$, *bequests are inoperative and the introduction of a public pension programme promotes growth if and only if individuals are sufficiently patient, i.e.:*

$$\beta > \frac{\alpha\,(1+\delta\gamma)\,(2-\alpha)}{1-\alpha} \tag{42}$$

There then exists a growth-maximising size of the public pension system ($\widehat{\tau} \leq \chi$)*. If the elasticity of the technology of education is sufficiently high, i.e.* $\delta > \overline{\delta}' = \frac{1-3\alpha+\alpha^2}{(2-\alpha)(1-\alpha)}$, *this maximum is interior,* ($\widehat{\tau} < \chi$) *and the growth rate decreases with the size of the system if the public pension ratio is larger than* $\widehat{\tau}$.

Proof: The cases B' or C' arise if and only if $\chi > 0$ and $\Psi(0) > 0$, or equivalently:

$$\beta > \frac{\alpha\gamma}{1-\alpha} \text{ and } \beta > \frac{\alpha\,(2-\alpha)\,(1+\gamma\delta)}{1-\alpha}$$

The case C' arises if and only if:

$$\Psi(\chi) \geq 0 \Leftrightarrow \alpha\gamma\,(1-\alpha-\delta) - \frac{\beta}{\gamma}\,(1-(1-\alpha)\,\beta) \geq \alpha + \beta\,(\delta - \alpha\,(1-\alpha)) \tag{43}$$

This cannot occur if $\delta > 1-\alpha$.

Let us now deal with the case $\delta < 1-\alpha$. If $\delta < 1-\alpha$, the LHS of (43) is increasing with γ. Since $\gamma < \frac{1-\alpha}{\alpha}\beta$ ($\Leftrightarrow \chi > 0$), (43) implies:

$$\beta\,(1-\alpha)\,(1-\alpha-\delta) - \frac{\alpha}{(1-\alpha)}\,(1-(1-\alpha)\,\beta) > \alpha + \beta\,(\delta - \alpha\,(1-\alpha))$$

$$\Longrightarrow \delta < \frac{1}{2-\alpha} - \frac{\alpha}{(1-\alpha)\,\beta}$$

Since $\beta < 1$, this inequality further implies:

$$\delta < \frac{1-3\alpha+\alpha^2}{(1-\alpha)\,(2-\alpha)} = \overline{\delta}',$$

The case C' cannot therefore occur if $\delta > \overline{\delta}'$.■

As far as the assessment of a public pension programme is concerned, the case of proportional contributions is qualitatively similar to the case of lump-sum contributions, although there is one additional negative effect of public pensions on growth owing to distortionary taxation.

5 Conclusion

A substantial body of empirical literature investigates the relation between taxation, public spending and growth. Barro (1991) and Leibfritz *et al.* (1997) point to a negative impact of taxation on growth. Kneller *et al.* (1999) find that distortionary taxation reduces growth, whereas productive government expenditure enhances growth. According to this body of literature public pensions, which are financed by distortionary contributions and consist of transfers between individuals, would tend to have a negative impact on growth. Empirical evidence, however, is inconclusive and many researchers have suggested that there may be non-linearities in the pattern of taxation, public expenditure and growth. Increasing public expenditure or taxes may be good for growth when starting from a low level and bad for growth when starting from a high level. Our model provides a theoretical case of non-linearities in the pattern of unfunded public pensions.

From a theoretical viewpoint, there is a case for public pensions only if individuals initially leave no bequests. When bequests are not operative, the family fails to reap the full gain of investment in human capital, which then offers a higher return than physical capital. Pension benefits may alleviate this market failure by giving parents more resources to invest in the human capital of their children. Two opposing effects are nevertheless at work, since public pensions increase investment in human capital (a positive effect on growth) but reduce savings (a negative effect on growth). In an economy with inoperative bequests, an unfunded public pension system enhances growth if parents are sufficiently patient (given their degree of altruism), i.e they save enough for providing finances for their old-age. A sufficient degree of patience moderates the adverse effect of public pensions on savings and physical capital.

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7 Appendix

I- Lump-sum contributions:

We define a new variable, which varies from $\frac{\gamma}{\beta}$ to $\frac{1-\alpha}{\alpha}$ when η varies from 0 to χ:

$$v = \frac{\gamma}{\alpha\beta}\left(\frac{1}{1-\eta} - 1 + \alpha\right) \equiv v(\eta)$$

We then re-write (40):

$$\overline{e}^{1-\alpha(1-\delta)} = C\frac{v^{-\alpha}}{m + \frac{\gamma(1-\alpha)}{v} + \delta\alpha\beta v} \equiv H(v)$$

where $m = \gamma\delta(1-\alpha) + 1 - \beta + \alpha\beta$ and $C = \frac{(1-\alpha)A\gamma\delta}{(\delta D)^{\alpha}(1+n)}$. Clearly, $\frac{\partial\overline{e}}{\partial\eta}$ is of the same sign as $H'(v)$. Taking the logarithmic derivative of $H(v)$, we find that $\frac{\partial\overline{e}}{\partial\eta}$ is of the same sign as:

$$P(v) = -\delta(1+\alpha)\alpha\beta v^2 - \alpha m v + (1-\alpha)^2\gamma$$

The domain of $H(v)$ and $P(v)$ is $\left[\frac{\gamma}{\beta}, \frac{1-\alpha}{\alpha}\right]$. $\Phi(\eta) = P(v(\eta))$ decreases from

$$\Phi(0) \equiv P\left(\frac{\gamma}{\beta}\right) = \frac{\gamma}{\beta}(-2\alpha\delta\gamma - \alpha + \beta(1-\alpha))$$

to

$$\Phi(\chi) \equiv P\left(\frac{1-\alpha}{\alpha}\right) = \frac{1-\alpha}{\alpha}\left((1-\delta)\alpha(1-\alpha)\gamma - \alpha(1-\beta) - \delta\beta - \alpha^2\beta(1-\delta)\right)$$

II- Proportional contributions:

To analyse the impact of an increase in the scale of public pension we study the function.

$$F(\tau) = \frac{1-\tau}{\widetilde{B}(0,\tau)\left(1-(1-\alpha)(1-\tau)\right)^{\alpha}}$$

We introduce a new variable: $x = 1-(1-\alpha)(1-\tau) \equiv x(\tau)$, the range of which is $\left[\alpha, \frac{\beta}{\beta+\gamma}\right]$, since the contribution rate vary, by assumption, between 0 and $\chi = 1 - \frac{\gamma}{(1-\alpha)(\beta+\gamma)}$. We can then study a simpler function:

$$G(x) = \frac{1-x}{(1+n)(1-\alpha)x^{\alpha}\left(1+\frac{1-\beta}{\gamma\delta}+\frac{\alpha\beta}{\gamma\delta x}\right)}$$

The logarithmic derivative of G has the same sign as:

$$\Gamma(x) = -(1-\alpha)(\gamma\delta+1-\beta)x^2 - \alpha(\beta(1-\alpha)+\gamma\delta+1)x + \alpha\beta(1-\alpha)$$

The function $\Psi(\tau) \equiv \Gamma(x(\tau))$ decreases from:

$$\Psi(0) \equiv \Gamma(\alpha) = \alpha(\beta(1-\alpha) - \alpha(2-\alpha)(\gamma\delta+1))$$

to

$$\Psi(\chi) \equiv \Gamma\left(\frac{\beta}{\beta+\gamma}\right) = \frac{\beta}{(\beta+\gamma)^2}\left(\alpha\gamma^2(1-\alpha-\delta) + \gamma\beta(\alpha(1-\alpha)-\delta) + (1-\alpha)\beta^2 - \beta - \alpha\gamma\right)$$

when τ varies from 0 to χ.

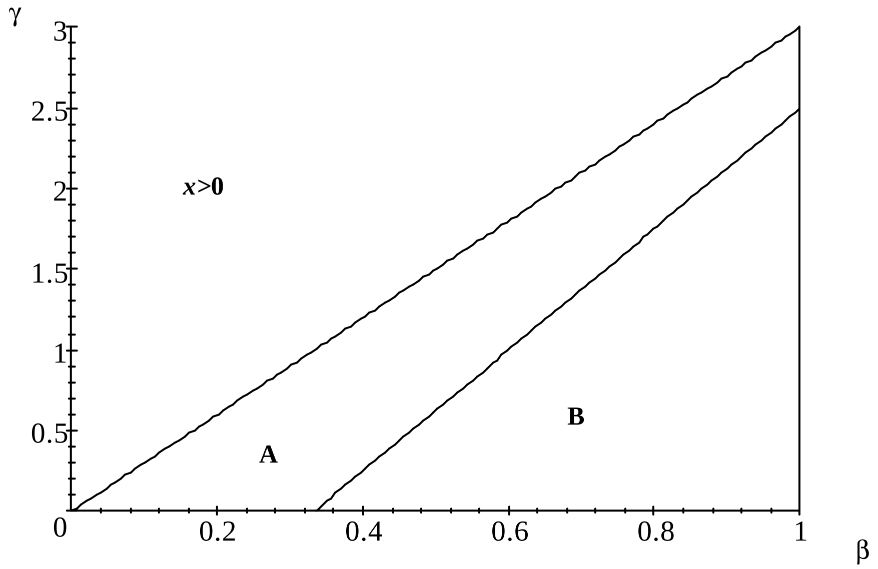

Figure 1

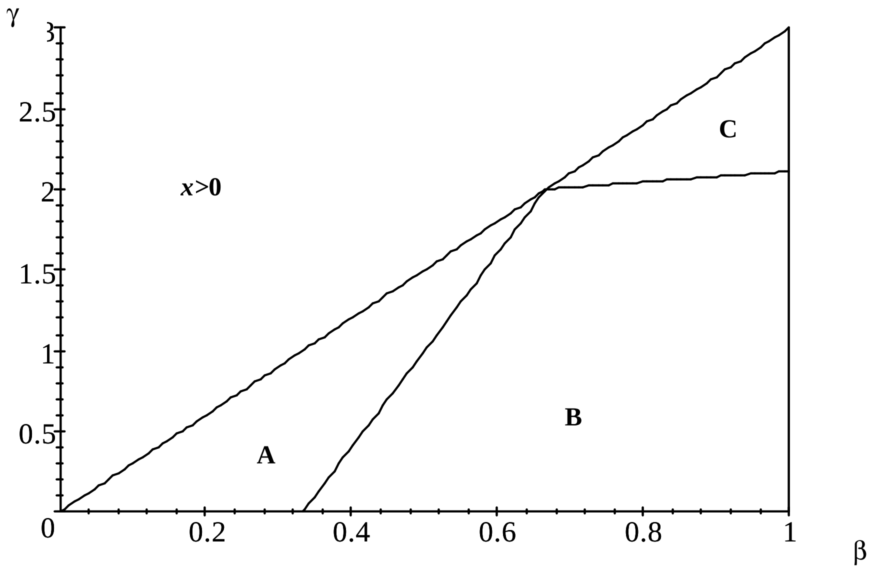

Figure 2

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