A Technical Supplement to Exploring the International Linkages of the Euro Area: a Global VAR Analysis

Stephane Dees
European Central Bank

Filippo di Mauro
European Central Bank

M. Hashem Pesaran
University of Cambridge and USC

L. Vanessa Smith
University of Cambridge

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1 Bootstrapping the GVAR and Tests of Structural Stability

To derive the empirical distribution of the structural stability tests and impulse response functions we employ the sieve bootstrap. The sieve bootstrap has been studied by Kreiss (1992), Bühlmann (1997) and Bickel and Bühlmann (1999) among others and has now become a standard tool when bootstrapping time series models.[1] The method rests on the assumption that the precise form of the parametric model generating the data is not known and that the true model belongs to the class of linear processes having an autoregressive representation of infinite order. Taking the estimated finite order vector autoregressive process that describes in our case the GVAR model to be an approximation to the underlying infinite order vector autoregressive process, we can use the sieve bootstrap for the basis of deriving critical values for the structural stability tests and for constructing bootstrap confidence regions.

In the case of stationary multivariate models, the sieve bootstrap has been used successfully to handle parameter estimation (Paparoditis, 1996). In the context of non-stationary time series, Park (2002) established an invariance principle applicable for the asymptotic analysis of the sieve bootstrap, which led Chang and Park (2003) to establish its asymptotic validity in the case of ADF unit root tests. Subsequently, Chang, Park and Song (2005) established the consistency of the sieve bootstrap for the OLS estimates of the cointegrating parameters assuming there exists one cointegrating relation amongst the variables under consideration. In what follows we consider the sieve bootstrap approach by resampling the residuals of the finite order global vector autoregressive process.

When bootstrapping unit root tests based on first order autoregressions, Basawa et al. (1991) show that the bootstrap samples need to be generated with the unit root imposed in order to achieve consistency for the bootstrap unit root tests. While our focus is not on bootstrapping unit root or cointegration tests, it seems natural to impose the unit root and cointegrating properties of the model when bootstrapping the statistics of interest. See also Li and Maddala (1997) who study the bootstrap cointegrating regression by means of simulation.

We begin by estimating the individual country VARX*$(\hat{p}_i, \hat{q}_i)$ models

[1] Another popular method is the block bootstrap by Künsch (1989). Choi and Hall (2000) discuss the substantial advantages of the sieve bootstrap over the block bootstrap for linear time series.

in their error correction form subject to reduced rank restrictions, for $i = 0, 1, 2, ..., N$ and $t = 1, 2, ..., T$, where $\hat{p}_i$ and $\hat{q}_i$ are the estimated lag orders of the endogenous and foreign variables respectively based on the AIC. The estimated VARX*$(\hat{p}_i, \hat{q}_i)$ are given by

$$\begin{aligned}\mathbf{x}_{it} &= \hat{\mathbf{a}}_{i0} + \hat{\mathbf{a}}_{i1}t + \hat{\mathbf{\Phi}}_{i1}\mathbf{x}_{i,t-1} + ... + \hat{\mathbf{\Phi}}_{i\hat{p}_i}\mathbf{x}_{i,t-\hat{p}_i} \\ &+ \hat{\mathbf{\Psi}}_{i0}\mathbf{x}^*_{it} + \hat{\mathbf{\Psi}}_{i1}\mathbf{x}^*_{i,t-1} + ... + \hat{\mathbf{\Psi}}_{i\hat{q}_i}\mathbf{x}^*_{i,t-\hat{q}_i} + \hat{\mathbf{u}}_{it},\end{aligned} \tag{1}$$

where we denote by $\hat{r}_i$ the estimated number of cointegrating relations for country i. In estimating the cointegrating rank we entertain the case of an unrestriced intercept and restricted trend, the latter restsricted to lie in the cointegrating space so as to avoid giving rise to quadratic trends in the level of the process.

The country specific models (1) are then combined via the link matrices W_i as described in Section 2, giving rise to the GVAR$(\hat{p})$ model expressed in terms of the global variables vector $\mathbf{x}_t$ as

$$\hat{\mathbf{G}}\mathbf{x}_t = \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 t + \hat{\mathbf{H}}_1\mathbf{x}_{t-1} + ... + \hat{\mathbf{H}}_{\hat{p}}\mathbf{x}_{t-\hat{p}} + \hat{\mathbf{u}}_t \tag{2}$$

with $\hat{p} = max(\hat{p}_i, \hat{q}_i)$, or alternatively,

$$\mathbf{x}_t = \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_1 t + \hat{\mathbf{F}}_1\mathbf{x}_{t-1} + ... + \hat{\mathbf{F}}_{\hat{p}}\mathbf{x}_{t-\hat{p}} + \hat{\varepsilon}_t \tag{3}$$

where $\hat{\mathbf{F}}_j = \hat{\mathbf{G}}^{-1}\hat{\mathbf{H}}_j$, $\hat{\mathbf{b}}_j = \hat{\mathbf{G}}^{-1}\hat{\mathbf{a}}_j$, for $j = 0, 1, ..., \hat{p}$, $\hat{\varepsilon}_t = \hat{\mathbf{G}}^{-1}\hat{\mathbf{u}}_t$ and $\hat{\mathbf{\Sigma}}_\varepsilon = \sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_t'/T$. The total number of variables in the model is given by $k = \sum_{i=0}^{N} k_i$ where k_i is the number of endogenous regressors in country i, $i = 0, 1, ..., N$.

Using the estimates from the fitted model (3) obtained from the observed data for $\hat{p} = 2$, we generate B bootstrap samples denoted by $\mathbf{x}_t^{(b)}$, $b = 1, ..., B$, from the process

$$\mathbf{x}_t^{(b)} = \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_1 t + \hat{\mathbf{F}}_1\mathbf{x}_{t-1}^{(b)} + \hat{\mathbf{F}}_2\mathbf{x}_{t-2}^{(b)} + \varepsilon_t^{(b)},\ t = 1, 2, ..., T, \tag{4}$$

by resampling the residuals $\hat{\varepsilon}_t$ of the fitted model, with $\mathbf{x}_0^{(b)} = \mathbf{x}_0$, and $\mathbf{x}_{-1}^{(b)} = \mathbf{x}_{-1}$, where $\mathbf{x}_0$ and $\mathbf{x}_{-1}$ are the the actual initial data vectors. Prior to any resampling the residuals $\hat{\varepsilon}_t$ are recentered to ensure that their bootstrap population mean is zero. The sieve bootstrap effectively reinterprets the familiar parametric AR model as a device for nonparametric estimation. The errors $\varepsilon_t^{(b)}$ could also be drawn by parametric methods. Both these methods will be described in what follows. Simulating the GVAR model is clearly preferable to simulating the country specific models separately. The latter requires that the country specific foreign variables, $\mathbf{x}_{it}^{*}$, and their lagged values are treated as strictly exogeneous which might not be appropriate and could lead to unstable outcomes for $\mathbf{x}_t$.

It should be noted that the GVAR model given in (4) contains among others the inflation and real exchange rate variables. The choice of these variables rather than the price level and the nominal exchange rate was dictated by the results of the unit root tests. Once a set of $\mathbf{x}_t^{(b)}$, $b = 1, 2, ..., B$ are generated, the price level and the nominal exchange rate are easily recovered, the foreign star variables are constructed using the weights in Table 2 and the inflation and real exchange rate variables are recreated.[2] Alternatively, the foreign (star) counterparts of these variables could be directly constructed, although it should be noted that this option is only valid in the case of fixed weights.

For each replication b, the individual country models are estimated in their error correction form, where p_i, q_i and the number of the cointegrating relations, r_i, are fixed over all replications at the estimated values $\hat{p}_i$, $\hat{q}_i$ and $\hat{r}_i$ obtained from the observed data, and a new set of VARX* estimates are computed from

$$\begin{aligned} \mathbf{x}_{it}^{(b)} &= \hat{\mathbf{a}}_{i0}^{(b)} + \hat{\mathbf{a}}_{i1}^{(b)} t + \hat{\boldsymbol{\Phi}}_{i1}^{(b)} \mathbf{x}_{i,t-1}^{(b)} + ... + \hat{\boldsymbol{\Phi}}_{i\hat{p}_i}^{(b)} \mathbf{x}_{i,t-\hat{p}_i}^{(b)} \\ &+ \hat{\boldsymbol{\Psi}}_{i0}^{(b)} \mathbf{x}_{it}^{*(b)} + \hat{\boldsymbol{\Psi}}_{i1}^{(b)} \mathbf{x}_{i,t-1}^{*(b)} + ... + \hat{\boldsymbol{\Psi}}_{i\hat{q}_i}^{(b)} \mathbf{x}_{i,t-\hat{q}_i}^{*(b)} + \hat{\mathbf{u}}_{it}^{(b)}. \end{aligned} \tag{5}$$

We denote by $E\hat{C}M_{ij,t-1}^{(r)}$ the estimated error correction terms that correspond to the $\hat{r}_i$ cointegrating relationships for country i, where $i = 0, 1, ..., N$ and $j = 1, 2, ..., \hat{r}_i$.

1.1 Structural Stability Tests

For the structural stability tests consider the ℓ^{th} equation of the estimated i^{th} country error correction model given by

$$\Delta x_{it,\ell} = \hat{\mu}_{i\ell} + \sum\nolimits_{j=1}^{\hat{r}_i} \hat{\gamma}_{ij,\ell} E\hat{C}M_{ij,t-1} + \sum\nolimits_{n=1}^{\hat{p}_i} \hat{\varphi}_{in,\ell}' \Delta \mathbf{x}_{i,t-n} + \sum\nolimits_{s=0}^{\hat{q}_i} \hat{\vartheta}_{is,\ell}' \Delta \mathbf{x}_{i,t-s}^{*} + e_{it,\ell}, \tag{6}$$

which can be written more compactly as

[2] As the maximum order of the GVAR model is 2 in the current application, the actual data is used for the first two observations following such a transformation.

$$y_{it,\ell} = \hat{\theta}'_{it,\ell}\mathbf{z}_{it} + e_{it,\ell}, \tag{7}$$

where $y_{it,\ell} = \Delta x_{it,\ell}$, $\mathbf{z}_{it} = (1, E\hat{C}M'_{ij,t-1}, \Delta\mathbf{x}'_{i,t-n}, \Delta\mathbf{x}'^{*}_{i,t-s})'$ for $j = 1, ..., \hat{r}_i$, $n = 1, ..., \hat{p}_i$ and $s = 0, ..., \hat{q}_i$ and $\hat{\theta}_{it,\ell} = (\hat{\mu}_{i\ell}, \hat{\gamma}_{ij,\ell}, \hat{\varphi}'_{in,\ell}, \hat{\vartheta}'_{is,\ell})'$. Let $e_{it,\ell}$ be the residuals from the estimated model (7) and $\hat{\sigma}^2_{i\ell} = T^{-1}\sum_{t=1}^{T} e^2_{it,\ell}$ the corresponding estimated error variance.

We consider a number of structural stability tests similar to those considered by Stock and Watson (1996). The null hypothesis for all the tests is that of parameter constancy, that is $\theta_{\ell t} = \theta_\ell$. The alternative varies depending on the test from non-stationarity, e.g random walks, to a one time change at an unknown break point for the sequential Wald type statistics, or some systematic movement in the parameters which we consider all to be subject to change. For expositional purposes we abstract from the index i.

1.1.1 1. Tests Based on the Cumulative Sum of OLS Residuals

The maximal OLS CUSUM statistic proposed by Ploberger and Krämer (1992) is similar to Brown, Durbin and Evans' (1975) CUSUM statistic although it is computed using OLS rather than recursive residuals. The mean square version of this test is also considered. Let $\zeta_T(\delta) = \hat{\sigma}_\ell^{-1} T^{-1/2} \sum_{s=1}^{[T\delta]} e_{\ell s}$, where $[\cdot]$ is the greatest integer function, then

$$PK_{\sup} = \sup_{\delta \in [0,1]} |\zeta_{\ell T}(\delta)| \tag{8}$$

$$PK_{msq} = \int_0^1 \zeta_{\ell T}(\delta)^2 d\delta \tag{9}$$

1.1.2 2. Random Walk Alternatives

Nyblom (1989) specifies as the alternative that $\theta_{\ell t}$ follows a random walk, that is, $\theta_{\ell t} = \theta_{\ell,t-1} + \eta_{\ell t}$, where $\eta_{\ell t}$ is i.i.d. and uncorrelated with error term corresponding to equation (7) and proposed the following statistic

$$\mathfrak{N}_\ell = T^{-2} \sum_{t=1}^{T} S'_{\ell t} \hat{\mathbf{V}}_\ell^{-1} S_{\ell t}, \tag{10}$$

where $S_{\ell t} = \sum_{s=1}^{t} \mathbf{z}_s e_{\ell s}$ and $\hat{\mathbf{V}}_\ell = (T^{-1}\sum_{t=1}^{T} \mathbf{z}_t \mathbf{z}'_t)\hat{\sigma}^2_\ell$. The heteroskedasticity-robust version of the $\mathfrak{N}_\ell$ statistic is obtained by replacing $\hat{\mathbf{V}}_\ell$ in (10) with $\tilde{\mathbf{V}}_\ell = T^{-1}\sum_{t=1}^{T} e^2_{\ell t}\mathbf{z}_t\mathbf{z}'_t$.

1.1.3 3. Sequential Wald Statistics

(i) Quandt (1960) likelihood ratio (QLR) statistic, in Wald form

$$QLR = \sup_{\delta \in (\delta_0, \delta_1)} F_{\ell T}(\delta)$$

(ii) The mean Wald statistic (Hansen (1992), Andrews and Ploberger (1994))

$$MW = \int_{\delta_0}^{\delta_1} F_{\ell T}(\delta) d\delta$$

(iii) The exponential average Wald statistic by Andrews and Ploberger (1994)

$$APW = \ln\{\int_{\delta_0}^{\delta_1} \exp(F_{\ell T}(\delta)/2) d\delta\}.$$

To obtain the Wald statistic $F_{\ell T}(\delta)$ for a break at $t = m$, where $\delta = m/T$ or $m = [T\delta]$ in all the above tests, equation (7) is initially estimated under the null of no structural change and the resulting sum of squares are defined as $\mathcal{R}_\ell = \mathbf{e}'_\ell \mathbf{e}_\ell$, where $\mathbf{e}_\ell = (e_{\ell 1,} e_{\ell 2,} ..., e_{\ell T})'$. The model with a one time break at $t = m$ is given by

$$\text{Subsample 1:} \quad y_{\ell t} = \theta'_{1\ell} \mathbf{z}_t + \varepsilon_{1\ell t}, \; t = 1, 2, ..., m \tag{11}$$

$$\text{Subsample 2:} \quad y_{\ell t} = \theta'_{2\ell} \mathbf{z}_t + \varepsilon_{2\ell t}, \; t = m + 1, ..., T. \tag{12}$$

Let $e_{1\ell t}$ and $e_{2\ell t}$ be the residuals from the OLS estimation of (11) and (12) respectively. Define $\mathcal{R}_{1\ell} = \mathbf{e}'_{1\ell} \mathbf{e}_{1\ell}$ and $\mathcal{R}_{2\ell} = \mathbf{e}'_{2\ell} \mathbf{e}_{2\ell}$.

Then,

$$F_{\ell T}(\delta) = (T - 2\kappa) \frac{\mathcal{R}_\ell - \mathcal{R}_{\ell 1} - \mathcal{R}_{\ell 2}}{\mathcal{R}_{\ell 1} + \mathcal{R}_{\ell 2}}$$

where κ is the dimension of $\theta_{it,\ell}$, and $\delta \in [\delta_0, \delta_1]$ with $\delta_1 = 1 - \delta_0$. The value for δ_0 was set to 0.25 and was chosen based on the maximum number of regressors over the individual VARX* models.

The heteroskedasticity-robust version of the sequential Wald tests is given by

$$F_T(\delta) = (\hat{\mathbf{b}}_1 - \hat{\mathbf{b}}_2)' \mathbf{Q}_\ell^{-1} (\hat{\mathbf{b}}_1 - \hat{\mathbf{b}}_2)$$

where

$$\hat{\mathbf{b}}_1 = (\mathbf{Z}_1'\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\mathbf{Y}_{1\ell}$$
$$\hat{\mathbf{b}}_2 = (\mathbf{Z}_2'\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\mathbf{Y}_{2\ell}$$

and

$$\mathbf{Q}_\ell = (\mathbf{Z}_1'\mathbf{Z}_1)^{-1}(\sum\nolimits_{t=1}^{m} e_{\ell t}^2 \mathbf{z}_t\mathbf{z}_t')(\mathbf{Z}_1'\mathbf{Z}_1)^{-1} +$$
$$(\mathbf{Z}_2'\mathbf{Z}_2)^{-1}[(\sum\nolimits_{t=1}^{T} e_{\ell t}^2 \mathbf{z}_t\mathbf{z}_t') - (\sum\nolimits_{t=1}^{m} e_{\ell t}^2 \mathbf{z}_t\mathbf{z}_t')](\mathbf{Z}_2'\mathbf{Z}_2)^{-1}$$

with $\mathbf{Z} = (\mathbf{z}_1', ..., \mathbf{z}_T')'$ and $\mathbf{Y}_\ell = (y_{\ell 1}, ..., y_{\ell T})'$. Subscripts 1 and 2 refer to equations (11) and (12), respectively.

For each replication b, we consider the ℓth equation of the country-specific error correction models of $\mathbf{x}_{it}^{(b)}$ given in (13)

$$\Delta x_{it,\ell}^{(b)} = \mu_{i\ell}^{(b)} + \sum\nolimits_{j=1}^{\hat{r}_i} \gamma_{ij,\ell}^{(b)} ECM_{ij,t-1}^{(b)} + \sum\nolimits_{n=1}^{\hat{p}_i} \hat{\varphi}_{in,\ell}^{\prime(b)} \Delta\mathbf{x}_{i,t-n}^{(b)} + \sum\nolimits_{s=0}^{\hat{q}_i} \vartheta_{is,\ell}^{\prime(b)} \Delta\mathbf{x}_{i,t-s}^{*(b)} + e_{it,\ell}^{(b)}, \tag{13}$$

where $ECM_{ij,t-1}^{(b)}$, $j = 1, 2, ..., \hat{r}_i$ are the estimated error correction terms corresponding to the $\hat{r}_i$ cointegrating relations found for the i^{th} country based on the observed data and $\ell = 1, ..., k_i$, and compute the above statistics $W_{\ell s}^{(b)}$, $WW_{\ell s}^{(b)}$, $PK_{\sup}^{(b)}$, $PK_{msq}^{(b)}$, $\mathfrak{N}_\ell^{(b)}, QLR^{(b)}, MW^{(b)}$ and $APW^{(b)}$. These statistics are then sorted into ascending order and their value which exceeds 95% of the observed statistics represents the appropriate 95% critical value for the structural stability tests.

1.2 Bootstraping of Impulse Response Functions

On the assumption that the error term $\mathbf{u}_t$ associated with equation (2) has a multivariate normal distribution, the $k \times 1$ vector of the generalized impulse response functions in the case of a one standard error shock to the j^{th} equation corresponding to a particular shock in a particular country on $\mathbf{x}_{t+n}$ is given by

$$GI_{j,n} = \frac{\tilde{\mathbf{F}}^n \mathbf{G}^{-1} \mathbf{\Sigma}_u \mathfrak{e}_j}{\sqrt{\mathfrak{e}_j' \mathbf{\Sigma}_u \mathfrak{e}_j}}, \; n = 0, 1, 2, ... \tag{14}$$

where $\mathfrak{e}_j$ is a $k \times 1$ selection vector with unity as its j^{th} element, $\mathbf{\Sigma}_u$ is the covariance matrix of $\mathbf{u}_t$, and $\tilde{\mathbf{F}} = \mathbf{E}_1\mathbf{F}\mathbf{E}_1'$,with $\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix}$ and

$\mathbf{E}_1 = \begin{pmatrix} \mathbf{I}_k & \mathbf{0}_{k\times k} \end{pmatrix}$, which follows from re-writing (3) in its companion form.[3] This result also holds in non-Gaussian but linear settings where the conditional expectations can be assumed to be linear.

In the case of a structural shock the corresponding generalized impulse response function is given by

$$SI_{j,n} = \frac{\tilde{\mathbf{F}}^n (\mathbf{P}_G^0 \mathbf{G})^{-1} \mathbf{\Sigma}_u \mathfrak{e}_j}{\sqrt{\mathfrak{e}_j' \mathbf{\Sigma}_u \mathfrak{e}_j}}, \ n = 0, 1, 2, ... \tag{15}$$

where $\mathbf{P}_G^0$ is defined in Section 7.

For each bootstrap replication, having estimated the individual country models using the simulated data $\mathbf{x}_t^{(b)}$, the GVAR is reconstructed as described above and the impulse responses are calculated based on the formulas (14) and (15) as $GI_{j,n}^{(b)}$, $SI_{j,n}^{(b)}$ $\forall n$. These statistics are then sorted into ascending order $\forall n$ and the $(1-\gamma)100\%$ confidence interval is calculated by using the $\gamma/2$ and $(1-\gamma/2)$ quantiles, say $s_{\gamma/2}$ and $s_{(1-\gamma/2)}$, respectively of the bootstrap distribution of $GI_{j,n}$ and $SI_{j,n}$.

1.3 Generating the Simulated Errors

1.3.1 Parametric Approach

Under the parametric approach the errors are generated from a multivariate distribution with zero means and covariance matrix $\hat{\mathbf{\Sigma}}_\varepsilon$ given by $\hat{\mathbf{\Sigma}}_\varepsilon = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t'$. To obtain the simulated errors for the k variables in the GVAR model we first generate kT draws from an i.i.d distribution which we denote by $\mathbf{v}_t^{(b)}$, $t = 1, 2, ..., T$. In our application we generate $\mathbf{v}_t^{(b)}$ as $IIN(0, \mathbf{I}_k)$ although other parametric distributions could also be entertained. Invoking the spectral decomposition, the variance-covariance matrix of the estimated GVAR residuals are decomposed as $\hat{\mathbf{\Sigma}}_\varepsilon = \hat{\mathbf{P}} \hat{\mathbf{\Lambda}} \hat{\mathbf{P}}'$, where $\hat{\mathbf{\Lambda}}$ is a diagonal matrix containing the eigenvalues of $\hat{\mathbf{\Sigma}}_\varepsilon$ on its diagonal and $\hat{\mathbf{P}}$ is an orthogonal matrix consisting of its eigenvectors. Note that the Choleski decomposition of $\hat{\mathbf{\Sigma}}_\varepsilon$ is not applicable in this case due to the semi-positive definite nature of this matrix that follows from the underlying common factor structure of the GVAR. The errors $\varepsilon_t^{(b)}$, $t = 1, 2, ..., T$, are then computed as $\varepsilon_t^{(b)} = \hat{\mathbf{A}} \mathbf{v}_t^{(b)}$, where $\hat{\mathbf{A}} = \hat{\mathbf{P}} \hat{\mathbf{\Lambda}}^{1/2}$.

[3] See Pesaran and Shin (1998) for further details.

1.3.2 Non-Parametric Approach

To obtain a bootstrap sample for the k variables in the GVAR model, we initially pre-whiten the residuals $\hat{\eta}_t$ by using the generalized inverse of $\hat{\mathbf{A}}$ as given above, denoted $\hat{\mathbf{A}}_g^-$, so that $\hat{\eta}_t = \hat{\mathbf{A}}_g^- \, \hat{\varepsilon}_t$. The generalized inverse of $\hat{\mathbf{A}}$ is required due to the semi-positive definite nature of this matrix as was pointed out earlier. We then resample with replacement from the kT elements of the matrix obtained from stacking of the vectors $\hat{\eta}_t$, for $t = 1, 2, ..., T$. This is done in order to reduce the repetition of the bootstrap samples. The bootstrap error vector is then obtained as $\varepsilon_t^{(b)} = \hat{\mathbf{A}}\hat{\eta}_t^{(b)}$, where $\hat{\mathbf{A}}$ is the same as above, and $\hat{\eta}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $(\hat{\eta}_1, \hat{\eta}_2, ..., \hat{\eta}_T)$.

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