Technological Changes, the Labor Market, and Schooling—A General equilibrium Model with Multidimensional Individual Skills

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December, 2019

Motivation

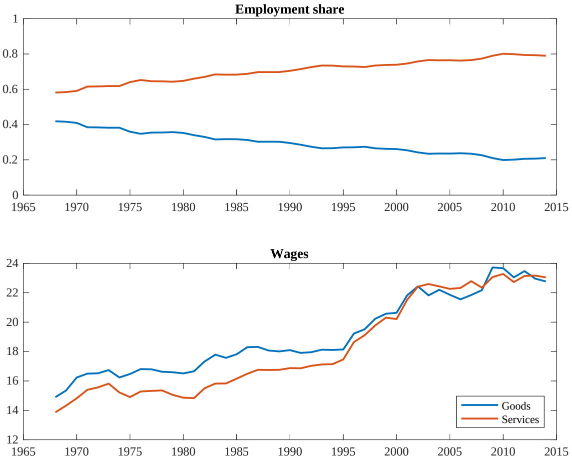
- What is the "future of work"?
- Rapidly changing technologies are believed to be the main driving forces behind recent changes in the labor market.
- In most developed countries:
  - Substantial employment shifts across sectors and occupations
  - Huge implications on inequality
  - Changing incentives for human capital accumulation
- We develop and estimate a general equilibrium (GE) model
- Evaluate the full impact of various policies

Contribution of this Paper

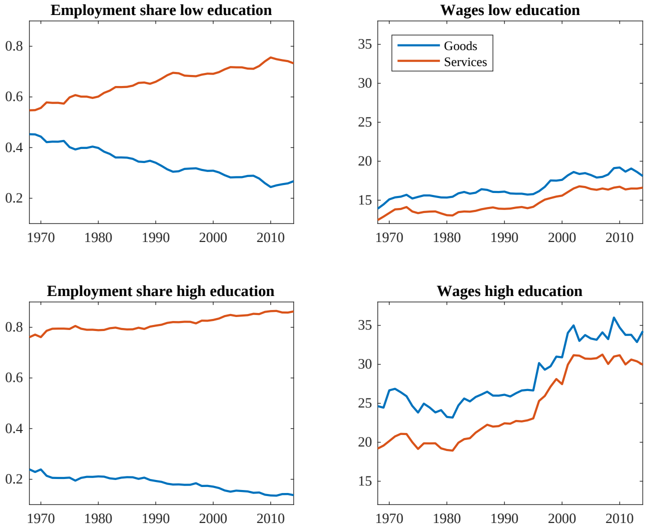
- Multi-sector, multi-occupation general equilibrium model
- Exogenous changes in sector-occupation specific skill augmenting technologies
- Non-homothetic preferences
- Overlapping generation model with human capital accumulation
- Multi-dimensional skills in a Roy-type selection model, where we price the bundles of skills used in production
- The above imply transitions from existing steady state
- We structurally estimate the model in transition from one steady state to another
- For that we need to use data from multiple sources

Why do we do things this way?

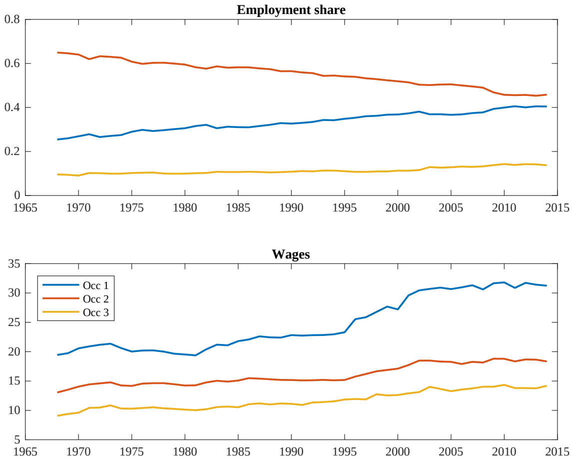
Employment and Wage, by Sector



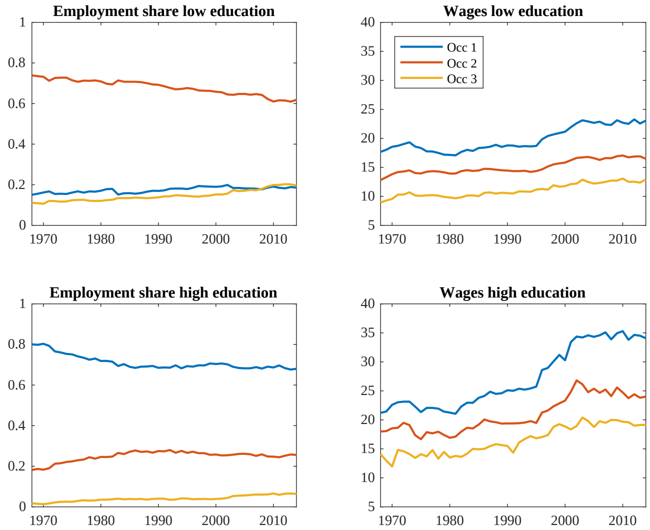
Employment and Wage, by Sector and Education



Employment and Wages, by Occupation



Employment and Wages, by Occupation and Education



Data Sources

- National Longitudinal Survey of Youth 1979 (NLSY79)
- Current Population Survey (CPS) 1964-2014
- O*NET
- The Bureau of Economic Analysis (BEA)
  - Provide prices and quantities by sector: Goods, c^G, and Services, c^S
  - Overall output growth by sector

NLSY79

- We use data from the NLSY79 for the year 1979 through 2014
- Annual data from 1979 to 1994; bi-annual data from 1996 onward
- We use the NLSY79 for two closely related purposes
- First, we construct a multi-dimensional vector of initial ability: (1) *abstract*; (2) *manual*, and (3) *inter-personal*
- Second, we construct a panel data for the individuals who were 14–22 in 1979 with detailed employment and wage history

NLSY79—Raw Statistics

Variables	Year										
	1979	**1981**	**1983**	**1985**	**1987**	**1989**	**1991**	**1994**	**1998**	**2006**	**2014**
Observations	4,601	4,601	4,574	4,404	4,174	4,006	3,883	3,726	3,505	2,976	2,560
Age	17.9	19.9	21.9	23.8	25.9	27.9	29.9	32.9	34.9	38.9	42.8
	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2
Highest grade completed	10.5	11.6	12.2	12.6	12.7	12.8	12.8	12.8	12.8	12.9	12.9
	1.9	1.8	2.0	2.3	2.5	2.6	2.7	2.7	2.7	2.7	2.7
Weeks worked	33.2	36.5	38.2	42.1	44.4	45.9	46.2	46.9	48.5	49.1	49.2
	16.6	15.9	16.0	14.2	12.8	11.9	11.5	11.1	9.4	8.5	8.7
Weeks unemployed	9.6	10.6	12.7	11.5	12.5	10.8	10.2	12.6	15.6	17.6	23.5
	11.1	11.1	12.8	11.6	12.3	10.4	10.1	11.4	15.3	15.0	18.2
Weeks OLF	30.4	24.4	25.3	24.2	24.8	25.2	26.0	25.5	29.5	35.5	39.3
	16.7	17.6	18.2	18.8	19.4	19.9	20.0	20.0	20.7	20.0	18.7
Annual hours worked	1059.1	1207.1	1387.4	1664.7	1836.3	1943.5	1972.8	2022.2	2098.4	2132.9	2129.3
	763.3	759.9	805.4	790.1	770.2	749.9	750.6	771.0	837.2	771.4	763.2
Hourly wage	3.52	4.23	5.21	6.17	8.39	10.16	11.79	13.07	20.38	25.10	34.57
	6.27	5.65	9.53	4.31	10.91	13.86	13.05	10.70	130.31	31.45	69.00
In school	0.70	0.48	0.26	0.14	0.06	0.02	0.01	0.00	0.00	0.00	0.00

NLSY79—Raw Statistics

Variables	Year										
	1979	**1981**	**1983**	**1985**	**1987**	**1989**	**1991**	**1994**	**1998**	**2006**	**2014**
Percents in:											
Good, 1	0.37	0.83	1.53	2.91	3.93	5.02	5.43	5.42	6.73	6.96	5.74
Services, 1	2.43	5.00	7.50	12.83	19.00	18.72	22.33	22.11	26.22	26.31	28.59
Good, 2	10.22	11.45	15.17	17.76	17.20	19.00	18.65	16.85	16.12	13.84	11.60
Services, 2	14.39	19.08	28.42	27.82	27.55	27.06	25.16	24.85	24.94	25.24	23.16
Goods, 3	0.17	0.24	0.39	0.39	0.10	0.12	0.18	0.21	0.34	0.50	0.35
Services, 3	12.15	12.63	14.23	12.47	10.18	9.44	9.76	9.74	8.93	8.90	8.01
Experience in:											
Goods, 1	1.00	1.26	1.67	2.12	2.24	2.95	3.79	4.99	5.36	7.34	10.01
	0.00	0.55	0.90	1.26	1.65	2.10	2.52	3.25	3.72	4.70	5.76
Services, 1	1.00	1.51	1.89	2.40	2.98	3.94	4.86	6.75	7.19	9.72	12.13
	0.00	0.72	1.09	1.46	1.88	2.29	2.76	3.48	4.24	5.10	6.03
Goods, 2	1.00	2.01	2.71	3.54	4.40	5.27	6.49	8.37	9.01	11.71	14.58
	0.00	0.83	1.41	1.96	2.51	2.95	3.29	4.07	4.81	5.66	6.53
Services, 2	1.00	1.86	2.63	3.53	4.25	5.23	6.29	8.05	8.31	10.49	12.78
	0.00	0.79	1.28	1.78	2.32	2.73	3.25	3.78	4.52	5.36	6.21
Goods, 3	1.00	1.09	1.50	1.71	1.25	1.80	2.00	1.88	1.17	2.40	5.67
	0.00	0.30	0.86	0.85	0.50	0.84	1.73	0.99	0.39	1.84	3.08
Services, 3	1.00	1.80	2.45	3.18	3.96	4.67	5.46	6.49	6.73	8.25	9.92
	0.00	0.79	1.26	1.71	2.29	2.79	3.09	3.84	4.50	5.40	6.18

NLSY79—Raw Statistics

Variables	Year										
	1979	**1981**	**1983**	**1985**	**1987**	**1989**	**1991**	**1994**	**1998**	**2006**	**2014**
Hourly wage in:											
Goods, 1	3.63	5.59	8.13	9.35	10.40	15.57	16.65	18.71	22.61	31.42	51.19
	1.87	2.32	3.77	3.89	4.33	21.58	9.39	10.65	11.99	21.35	42.60
Services, 1	3.21	4.90	7.74	7.41	9.13	12.61	14.32	15.72	20.82	31.93	40.94
	2.01	2.94	29.07	3.86	5.23	16.87	12.24	9.89	17.83	31.61	43.77
Goods, 2	4.69	4.82	5.55	6.52	7.93	9.45	10.62	11.81	14.73	20.87	26.42
	10.47	2.49	3.32	3.80	4.76	9.11	6.74	6.52	11.01	18.34	22.01
Services, 2	3.14	4.24	4.97	5.90	7.42	8.72	10.51	11.76	14.11	21.39	23.43
	1.84	2.36	2.96	3.42	4.21	4.65	8.13	7.43	16.33	29.16	22.58
Goods, 3	2.05	3.40	4.86	7.15	6.59	6.96	8.14	8.54	13.83	16.98	18.28
	1.31	1.53	2.85	2.34	1.11	5.72	4.83	6.58	6.11	9.86	14.38
Services, 3	3.14	3.54	3.90	4.78	6.10	6.72	8.27	8.53	10.72	14.86	16.84
	4.41	2.54	2.47	2.94	3.77	4.18	5.60	5.52	7.33	12.68	15.53

NLSY79—Sector Transitions

Year 2 to 9

Time t-1 \ Time t	G	S	U
G	0.68	0.27	0.05
S	0.10	0.83	0.07
U	0.09	0.33	0.58

Year 19 to 26

Time t-1 \ Time t	G	S	U
G	0.77	0.2	0.03
S	0.07	0.88	0.05
U	0.07	0.29	0.64

NLSY79—Occupation Transitions

Year 2 to 9

Time t-1	Time t: 1	2	3	U
1	0.64	0.25	0.07	0.04
2	0.09	0.76	0.08	0.08
3	0.08	0.27	0.53	0.12
U	0.04	0.25	0.14	0.57

Year 19 to 26

Time t-1	Time t: 1	2	3	U
1	0.77	0.16	0.04	0.03
2	0.10	0.80	0.05	0.05
3	0.09	0.21	0.61	0.09
U	0.05	0.21	0.11	0.62

NLSY79—Sector-Occupation Transitions

Years 2 to 9

Time t (columns); Time t-1 (rows)

	G1	S1	G2	S2	G3	S3	U
G1	0.44	0.17	0.21	0.10	0.00	0.02	0.06
S1	0.04	0.59	0.03	0.19	0.00	0.07	0.08
G2	0.04	0.02	0.63	0.16	0.01	0.03	0.12
S2	0.01	0.09	0.10	0.60	0.00	0.07	0.13
G3	0.01	0.02	0.23	0.20	0.21	0.16	0.17
S3	0.01	0.07	0.06	0.17	0.00	0.51	0.18
U	0.01	0.04	0.06	0.13	0.00	0.09	0.67

Years 19 to 26

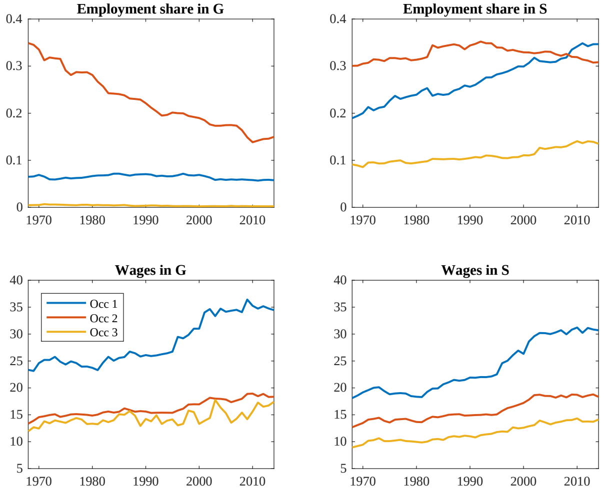
Time t (columns); Time t-1 (rows)

	G1	S1	G2	S2	G3	S3	U
G1	0.64	0.10	0.14	0.06	0.00	0.01	0.04
S1	0.03	0.74	0.02	0.12	0.00	0.04	0.06
G2	0.05	0.02	0.70	0.11	0.00	0.02	0.09
S2	0.01	0.11	0.08	0.66	0.00	0.04	0.10
G3	0.03	0.02	0.20	0.11	0.38	0.14	0.13
S3	0.01	0.08	0.04	0.13	0.00	0.59	0.15
U	0.01	0.06	0.06	0.12	0.00	0.08	0.67

CPS

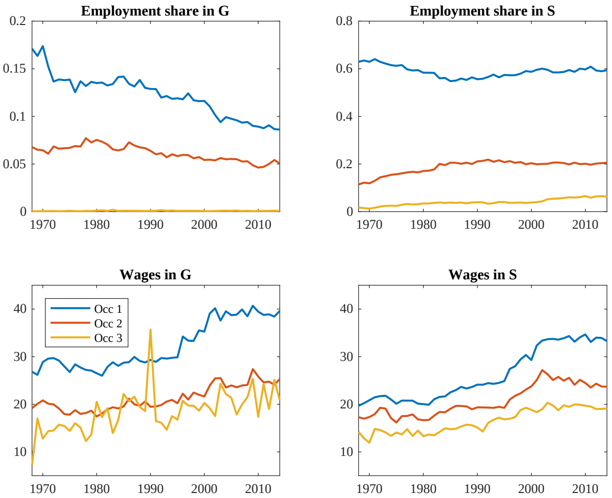
- The NLSY79 provide data on, essentially, one cohort
- The CPS provide us with that necessary information to compute the aggregate measures in all six sector-occupation, (J, o), cells in each year
- We match the simulated decisions provided by the model against the analogous empirical information from the CPS

CPS—Employment and Wages, by Sector-Occupation



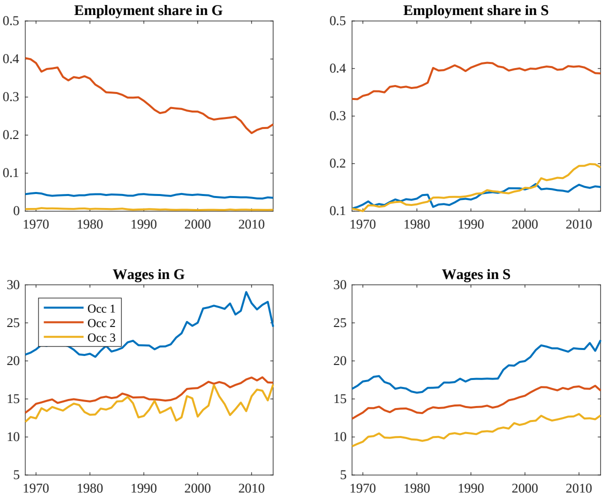
CPS—Employment and Wages, by Sector-Occupation

Educated Individuals

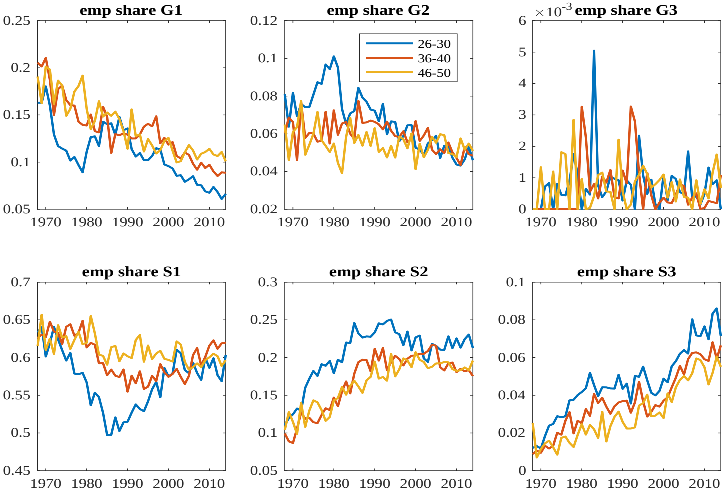


CPS—Employment and Wages, by Sector-Occupation

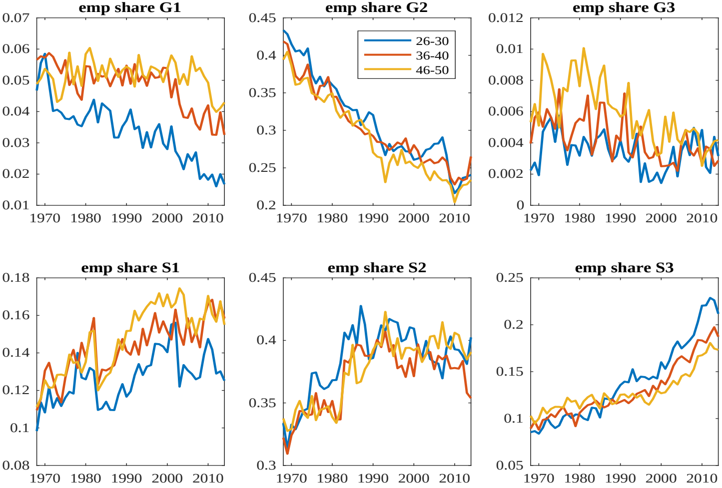
Non-Educated Individuals



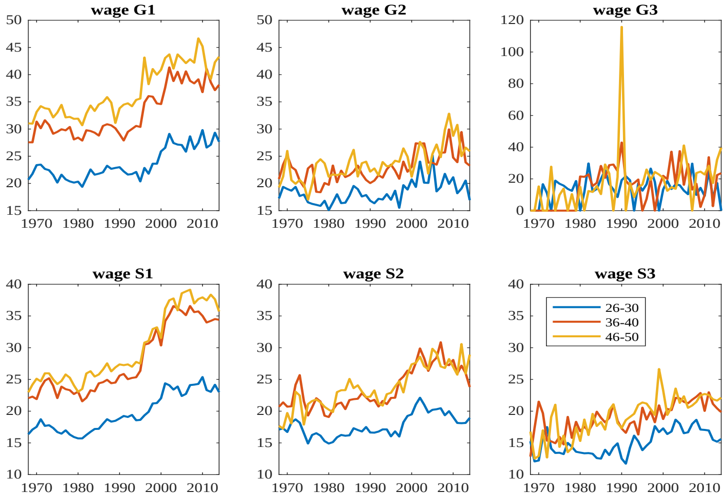
CPS—Employment Shares for the Educated, by (J, o)



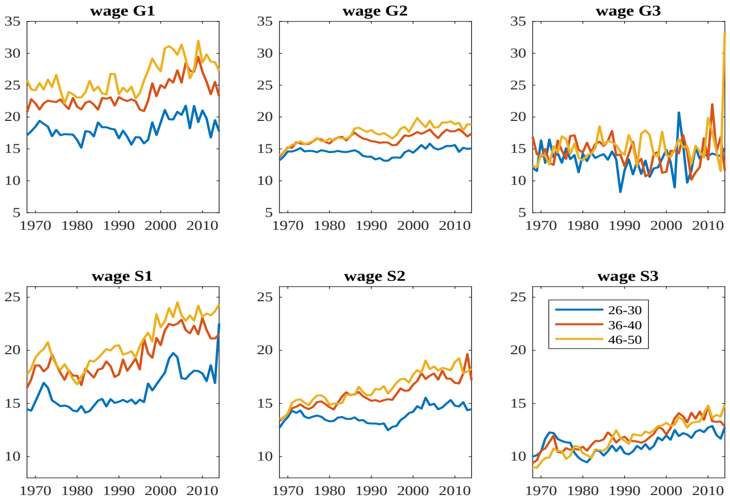
CPS—Employment Shares for the Uneducated, by (J, o)



CPS—Wages for the Educated, by (J, o)



CPS—Wages for the Uneducated, by (J, o)



O*NET

- The O*NET Program is the primary source of occupational information in the U.S.
- The O*NET database contains hundreds of standardized and occupation-specific descriptors on almost 1,000 occupations covering the entire U.S. economy
- From the O*NET we extract skill requirements in all sector-occupation cells

What Do We Do with the O*NET?

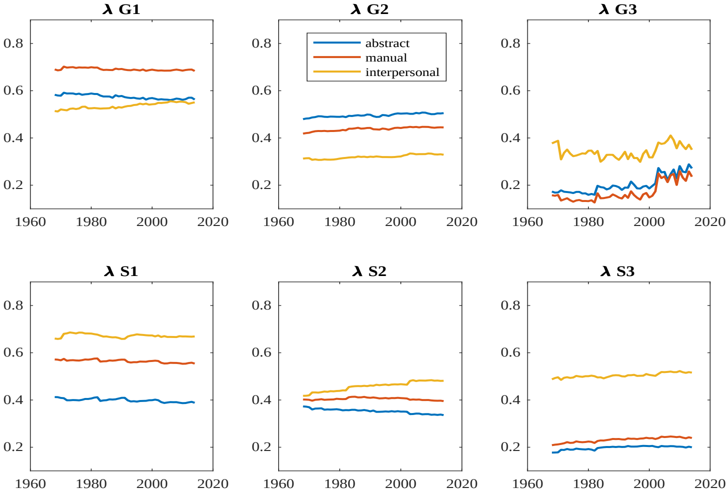
- We extract skill requirements for fine O*NET occupations, similarly to Lise and Postel-Vinay (2016)
- We retain the importance value of all descriptors in Abilities, Knowledge, Skills and Work Activities, and all Context (CX) and Context Category (CTP) descriptors in Work Context
- This gives 199 descriptors for each of the 954 occupational categories of the O*NET
- We order these such that the first three descriptors are:
  - mathematical knowledge;
  - mechanical knowledge; and
  - social perceptiveness skill

For greater details see: O*NET Use

What Do We Do with the O*NET?

- Classify each *Occ1990* category into one of three groups, following the same method as in Acemoglu and Autor (2011)
- Use this classification to compute Sector-Occupation (J, o) specific λ_{Jo}
  (weighted average labor supply of the skill requirements obtained above from the CPS)
  [For details see: O*NET Lambda]

λ's by Sectors and Occupations



Multidimensional Skills a_0

NLSY79 First Step

- Identify as many variables as possible in the NLSY79 that are correlated with the individual innate ability a_0.
- Use principal component analysis (PCA) to reduce the dimension to only three elements.
- Impose exclusion restrictions, on the link between the components of the innate ability and the observed variables, to identify $a_{0i}' = (a_{01i}, a_{02i}, a_{03i})\,,\ \text{for}\ \ i = 1, ..., N.$: (1) *abstract*; (2) *manual*, and (3) *inter-personal*.

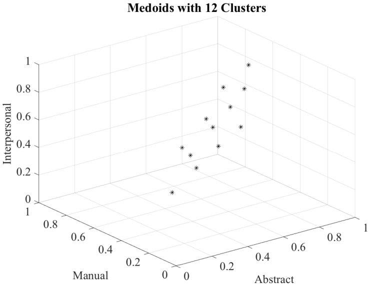
Multidimensional Skills a_0

NLSY79 First Step

- We approximate the (almost) continuous distribution of a_0 by a discrete version, with using 12 points of support
- We pick the points using machine learning algorithm for cluster analysis
- Assign each individual in the sample the appropriate point of support estimated from the cluster analysis
- For details see: Multi-dim-a

Estimated Points of Support for a_0

NLSY79 First Step



Group	a_0 Abstract	a_0 Manual	a_0 Inter-personal	Probability	% with higher education
1.	0.148	0.221	0.380	0.076	2.3
2.	0.193	0.207	0.694	0.056	4.4
3.	0.305	0.294	0.559	0.085	4.4
4.	0.391	0.362	0.407	0.086	9.0
5.	0.406	0.309	0.781	0.081	14.9
6.	0.521	0.410	0.634	0.092	18.2
7.	0.582	0.415	0.899	0.060	29.5
8.	0.590	0.460	0.450	0.093	19.5
9.	0.686	0.499	0.682	0.106	37.3
10.	0.804	0.576	0.460	0.087	49.4
11.	0.824	0.547	0.915	0.082	60.1
12.	0.862	0.626	0.694	0.095	64.4
All	0.545	0.423	0.621	1	27.0

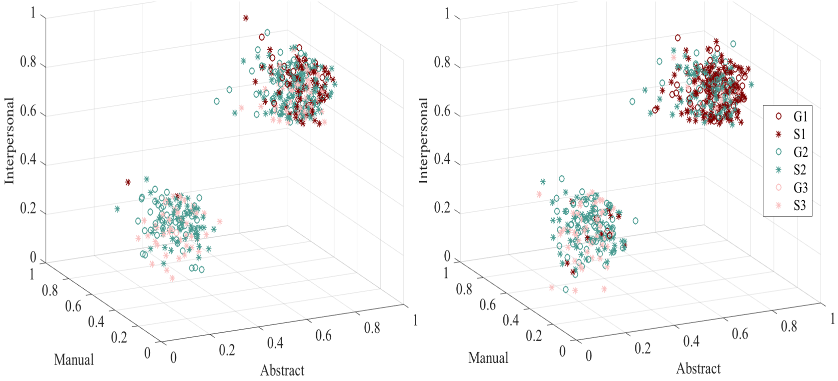
Estimated Points of Support for a_0

NLSY79 First Step

- Very different combinations of the three skills for the 12 points
- $Corr(a_{01}, a_{02}) = .80$, $Corr(a_{01}, a_{03}) = .30$, and $Corr(a_{02}, a_{03}) = .18$
- Strong indication that a one-factor model (a-la Becker) will not work because
  - Overall 27% acquire education, but ...
  - Individuals with higher abstract ability are more likely to acquire higher education
- Also, individuals with different initial skill sets follow different career paths
- They will end-up in the long run with different sector-occupation choices

Sector-Occupation Choice over Time

Type 1 vs. type 12—Choices after 5 years (left) and 15 years (right)



Estimated Points of Support for a_0

Sector-Occupation Choice over Time—Type 1 vs. type 12

- The cluster of points at the bottom left corner are all classified as type 1
- The cluster of points at the top right corner are all of type 12
- Type 1 individuals are almost exclusively employed in occupation groups 2 (blue) and 3 (pink)
- Type 12 individuals have a larger occurrence of occupation 1 (red) employment
- Also, a larger career progression in 10 years for type 12 individuals than for type 1 individuals

The Model

Individuals and the markets

- Infinite, discrete time horizon, OLG model
- Individuals are born with multi-dimensional ability (skills) $\underline{a}_0$
- Individuals make educational decisions once at the beginning of their life, which increases $\underline{a}_0$, at some costs
- Also, those who work acquire skills on the job in all three dimensions of ability: The speed is determined by the sector-occupation, and the specific skill
- Each period an individual makes sector-occupation choices, to maximize expected lifetime utility
- Individuals derive utility from consuming services and goods
- Preferences are non-homothetic
- They allocate all their income, in each period, optimally between services and goods

The Model

Individuals and the markets

- The values of the different abilities, $\underline{a}_t$, can differ across (J, o) cells (i.e., their efficiency labor units differ)
- Unit wages across (J, o) also differ
- One starts in the labor market after getting education (or right away if does not) as unemployed
- Jobs arrive and are destroyed at some exogenous rates, that (potentially) are sector-occupation specific
- An unemployed individual conducts *directed search* in a specific sector-occupation
- An individual must accept a job offer from the specific (J, o) in which he/she searches
- There is also on-the-job search, at some (utility) cost

The Model

Sectors and Occupations

- There are two sectors Goods (G) and Services (S)
- Both sectors employ labor from all occupations, given the unit wages
- Unit wages are equal to the marginal product of an efficiency unit of labor in each sector-occupation cell
- The economy is in a perfectly competitive equilibrium
- The unit wages clear all sector-occupation markets
- Prices clear the Goods and Services markets
- Structural change is induced by differential productivity growth across sector-occupation cells

Sectors and Production Functions

- Production in each sector is the CES:
- $$Y^J = \left(\sum_{o=1}^{O} \alpha^{Jo} (L^{Jo})^{\frac{\rho^J - 1}{\rho^J}} \right)^{\frac{\rho^J}{\rho^J - 1}} \quad \text{for } J = G, S, \tag{1}$$
- $\sum_{o=1}^{O} \alpha_{Jo} = 1$
- L^{Jo} = total amount of efficiency units of labor in (J, o)
- $\rho_J \in [0, \infty)$ = elasticity of substitution between workers in different occupations in sector J
- w^{Jo} = sector-occupation wage per efficiency unit

Production

Efficiency Units and Aggregate Labor

- Effective ability vector, $\underline{h}^{Jo}$, of individuals with the skill vector $\underline{a}$, in (J, o), is the (Hadamard) product

$$\underline{h}^{Jo} = \underline{\lambda}^{Jo} \circ \underline{a} \quad \text{where}$$

- The three elements of both $\underline{\lambda}^{Jo}$ and $\underline{a}$ are: abstract, manual, and interpersonal
- $\underline{\lambda}^{Jo}$ represents the skill requirements in (J, o)
- $\underline{\gamma}^{Jo} \in \mathbb{R}^3$ is the sector-occupation specific task augmenting technology
- $l^{Jo} = (\underline{\gamma}^{Jo})' \left(\underline{\lambda}^{Jo} \circ \underline{a}\right) = (\underline{\gamma}^{Jo})' \underline{h}^{Jo}$ Efficiency units of labor of a worker in (J, o)
- $L^{Jo} = \sum l^{Jo}$ Total efficiency units of labor in (J, o)
- $w^{Jo} l^{Jo} =$ Worker's earnings

Human Capital Accumulation

- With $e_H > 0$ periods of education ability is $\underline{a}_1 = f_H(\underline{a}_0)$
- Experience also increases abilities in (J, o) by

$$\Delta^{Jo} = \underline{\varphi} \circ \underline{\lambda}^{Jo}$$

- The vector of increases, $\underline{\varphi}$, differs across skills
- $\underline{\lambda}^{Jo}$ capture the fact that the more intensively used skills are also accumulated faster
- Our prior: $\varphi_2 > \varphi_1 > \varphi_3 \geq 0$, manual skill grows at the fastest rate, while interpersonal skill grows at the slowest rate
- For the unemployed individual $\underline{a}_{it} = \underline{a}_{i,t-1}$

Preferences

- Individual maximizes the non-homothetic per-period utility

$$\max_{c^G, c^S} \left(\theta_G^{\frac{1}{\rho}} (c^G)^{\frac{\rho-1}{\rho}} + \theta_S^{\frac{1}{\rho}} (c^S + \overline{c}^S)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$
$$\text{s.t.} \quad p^G c^G + p^S c^S \leq m$$
$$0 \leq c^G, \; 0 \leq c^S,$$

where $\theta_G + \theta_S = 1$, $\rho \in [0, \infty)$, $\overline{c}^S > 0$

- He/she spends all current period disposable income
- The data suggest that with a homothetic utility it is not possible to match relative sectoral prices and value added shares

Demand for Consumption Goods

- The demands for G and S are:

$$c^G = \frac{m + p^S \bar{c}^S}{p^G + p^S \frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^\rho}, \tag{2}$$

$$c^S = \frac{\frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^\rho m - p^G \bar{c}^S}{p^G + p^S \frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^\rho}. \tag{3}$$

- With sufficiently low income, i.e., $\frac{m}{p^G} < \frac{\theta_G}{\theta_S} \left(\frac{p^S}{p^G}\right)^\rho \bar{c}^S$, all income, $c^G = m/p^G$, is spent on goods
- The unemployed, with no income, and has zero utility

Choice of the Sector-Occupation of Work

- Jobs arrival rates are: (i) η^{Jo} for the unemployed; and (ii) $\zeta\eta^{Jo}$ for employed
- Job are destroyed at rate δ^{Jo} in cell (J, o)
- Unemployed agent decides at $t-1$ in which (J, o) to search for period t
- With probability η^{Jo} a job will arrive for period t employment
- With probability $1-\eta^{Jo}$ (realized at t) he/she remains unemployed
- Searching individual incurs a stochastic *i.i.d.* preference shock of $\varepsilon_{it}^{Jo} \cdot \mathcal{U}_t$, the utility/disutility of searching in (J, o) cell
- $\mathcal{U}_t$ is a utility factor that scales all non-pecuniary costs (Normalization)

Choice of the Sector-Occupation of Work

- A $t-1$ year old unemployed worker with ability $\underline{a}_{t-1}$, searching in (J,o) has:
  - An expected value from working in (J,o) at t is $W_t^{Jo}(\underline{a}_t)$
  - Similarly the value of unemployment at t is $U_t(\underline{a}_t)$
- $W_t^{Jo}(\underline{a}_t)$ and $U_t(\underline{a}_t)$ depend on all, current and future, wage rates
- This is because wages determine the earnings in each (J,o) and the prices of G and S in each period

Choice for an Unemployed Agent

- The worker will search in the (J, o) cell that solves

$$\max_{(J,o)} \left\{(1-\eta^{Jo})U_t(\underline{a}_t) + \eta^{Jo} W_t^{Jo}(\underline{a}_t) + \varepsilon_{it}^{Jo} \cdot \mathcal{U}_t\right\}$$

$$= U_t(\underline{a}_t) + \max_{(J,o)} \left\{ \underbrace{\eta^{Jo}\left[W_t^{Jo}(\underline{a}_t) - U_t(\underline{a}_t)\right]}_{\equiv V_t^{Jo}(U,\underline{a}_t)} + \varepsilon_{it}^{Jo} \cdot \mathcal{U}_t \right\}. \quad (4)$$

Unemployed

Choice for an Employed Agent in (J, o)

- A $t-1$ years old, employed in (J, o), become unemployed at rate δ^{Jo}
- If he/she does not become unemployed, he/she will search in (J', o') that solves

$$\max_{(J',o')}\left\{(1-\zeta\eta^{J'o'})W_t^{Jo}(\underline{a}_t)+\zeta\eta^{J'o'}W_t^{J'o'}(\underline{a}_t)+\varepsilon_{it}^{J'o'}\mathcal{U}_t-\chi\mathcal{U}_t\mathbf{I}((J',o')\neq(J,o))\right\}$$

$$=W_t^{Jo}(\underline{a}_t)+\max_{(J',o')}\left\{\underbrace{\zeta\eta^{J'o'}\left(W_t^{J'o'}(\underline{a}_t)-W_t^{Jo}(\underline{a}_t)\right)-\chi\mathcal{U}_t\mathbf{I}((J',o')\neq(J,o))}_{\equiv V_t^{J'o'}((J,o),\underline{a}_t)}+\varepsilon_{it}^{J'o'}\mathcal{U}_t\right\}. \tag{5}$$

Employed-(J,o)

The New entrants Problem

- A new entrant (age 0) decides whether or not to acquire education
- In his/her first period on the labor he/she decides in which (J, o) to search
- This is identical to the problem of an unemployed at
  - age $t = 1$ and $\underline{a} = \underline{a}_0$ if uneducated; or
  - age $t = e_H + 1$ and $\underline{a} = f_H(\underline{a}_0)$ if educated

New Entrants

Competitive Equilibrium

- A competitive equilibrium is:
  (i) A sequence of cutoff education costs $\{\overline{\varepsilon}_{\kappa,s}\}_{s=1}^{\infty}$;
  (ii) Probabilities $\{\pi_s^{NE}, \pi_s^{ED}, \{\pi_{st}^{U}, \pi_{st}^{Jo}\}_{t=1}^{T}\}_{s=1}^{\infty}$;
  (iii) Wages $\{w_s^{Jo}\}_{s=1}^{\infty}$; and
  (iv) Prices $\{p_s^G, p_s^S\}_{s=1}^{\infty}$, given the path of productivities $\{\underline{\gamma}_s^G, \underline{\gamma}_s^S\}_{s=1}^{\infty}$;
  that clear all segments of the market
- That means:
  (i) Education cost cutoffs are optimal;
  (ii) Search probabilities arise as the result of maximizing expected lifetime utility;
  (iIi) Unit wage rates clear the labor markets in all (J, o) cells; and
  (iv) p^G and p^S clear the markets for G and S

Thanks!

Multidimensional Skills—Construction

- The objective is to construct an empirical distribution for the endowments' vector a_0 (i.e., of abstract, manual and inter-personal skills) for all individual in the original sample, without the supplement and military samples, of the NLSY79.
- We adopt quite closely Lise and Postel-Vinay (2019).
- This is also identical to the procedure we follow for extracting skill requirements of different occupations from the O*NET.

return

Multidimensional Skills—Construction

- We select a number of variable that are presumed to be highly correlated with one's initial ability vector a_0, as follows.
- From the SF-12 Health Survey: Physical component summary score, mental component summary score, and assessment of the respondent general health.
- Family background variables:
  - The highest grade completed for the father and the mother of the respondent;
  - The highest grade completed of the respondent himself/herself;
  - A set of dummy variables for the race (white, Hispanic, African American, and others) and gender of the individuals;
  - height, weight and Body-Mass-Index (BMI) of the respondent;

return

Multidimensional Skills—Construction

- Rotter locus-of-control scale score;
- Rosenberg self-esteem test score;
- Measures of criminal and anti-social behavior.
- Measure from the Armed Services Vocational Aptitude Battery (ASVAB):
  - General knowledge (section 1);
  - arithmetic reasoning (section 2);
  - word knowledge (section 3);
  - paragraph comprehension (section 4);
  - numerical operations (section 5);
  - coding speed (section 6);
  - auto and shop information (section 7);
  - mathematics knowledge (section 8);
  - mechanical comprehension (section 9);
  - and electronics information (section 10).

return

Multidimensional Skills—Identification

- For identification of the three components of the individual's vector of skills, assume that:
  - mathematical knowledge (ASVAB section 8) only affects the abstract skill;
  - the auto and shop information (ASVAB section 7) only affects the manual skill;
  - the Rosenberg self-esteem test score only affect an individual's inter-personal skill.

return

Multidimensional Skills—Clustering

- The PCA reduces the very large dimension of the data to only three dimension, the adjusted three principal components.
- The first three principal components account for 99.9% of the variation in the data.
- We approximate the distribution of a_0 observed in the data by a discrete distribution of individuals "types".
- We use the well known k-medoids algorithm of cluster analysis to obtain the k clusters based on the estimated individual ability vectors a_0 obtained from the PCA.

return

What Do We Do with the O*NET?

- We do that in order to be able to impose the exclusion restrictions that:
  - Mathematical knowledge only reflects abstract requirements;
  - Mechanical knowledge only captures manual requirements; and
  - social perceptiveness skill only reflects interpersonal requirements
- Denote this matrix by D
- Using principal component analysis (PCA) on D
- Pick the first three principal components and impose the exclusion restrictions to get X_0
- Normalize X_0 so each columns entries are between 0 and 1 to get the normalized matrix X

What Do We Do with the O*NET?

- The three columns of X give the abstract, manual and interpersonal requirement of each occupation for each O*NET occupational category.
- Use a crosswalk between the O*NET occupations and the *1990* occupational categories of the CPS (Occ1990) to assign a requirement vector to each *Occ1990* category (as in Acemoglu and Autor (2011))
  [For details see: O*NET Xwalk]

What Do We Do with the O*NET?

Step 1: PCA

- Denote this matrix by D
- We run PCA on the 199 descriptors in D to get

$$D = F\Gamma,$$

  where
  - each row of Γ contains the coefficients for one principal components
  - F contains the representation of data D in the principal component space
- Let F_3 be a matrix containing the first three columns of F (the first three principal components)

What Do We Do with the O*NET?

Step 1: PCA

- Let Γ_{33} be the first 3-by-3 matrix of Γ
- This yield a representation of all skill requirements in the space of the first three PCs, that is

$$D \approx X_0 = F_3 \Gamma_{33}.$$

- We normalize all elements (i.e. each column of X_0) such that all values lie between 0 and 1
- $X =$ the normalized matrix
- The three columns of X give the abstract, manual and interpersonal requirement of each occupation for each O*NET occupational category.

return

What Do We Do with the O*NET?

Step 3: Classification of Occ1990

- We use a crosswalk between the O*NET occupations and the *1990* occupational categories of the CPS (Occ1990) to assign a requirement vector to each *Occ1990* occupational category Similarly to Acemoglu and Autor (2011)
- We use information from a cluster-analysis combined with broader occupational categories in the CPS (one-digit and two-digit occupational codes)
- We classify each *Occ1990* occupational category into one of three groups, following the same method as in Acemoglu and Autor (2011)

return

What Do We Do with the O*NET?

Step 4: Generating (J, o) cell-level λ_{Jo}

- We merge these broad occupational group ID with the CPS data
- In each sector-occupation cell we take the weighted average labor supply of the skill requirements obtained in the step 2 above
- These average skill requirements constitute our (J, o) cell-level λ_{Jo}s
- Since the labor supply weight of each *Occ1990* category changes within each cell over time, the λ_{Jo}s can change over time
- In our case they are remarkably stable
- We take the values from year 1990 as our baseline.

return

Sectors and Production Functions

- Firms always use the labor that is matched with them
- Wages per efficiency units of labor have to satisfy:

$$w^{Jo} = \frac{\partial p^J Y^J}{\partial L^{Jo}} = p^J \left(\sum_{o=1}^{O} \alpha^{Jo} (L^{Jo})^{\frac{\rho^J - 1}{\rho^J}} \right)^{\frac{1}{\rho^J - 1}} \alpha^{Jo} (L^{Jo})^{-\frac{1}{\rho^J}}, \tag{6}$$

  for $J = G, S$ and $o = 1, ..., O$.
- Within sector, optimal occupation labor use is:

$$\frac{L^{Jo}}{L^{Jo'}} = \left(\frac{\alpha^{Jo}}{\alpha^{Jo'}} \frac{w^{Jo'}}{w^{Jo}} \right)^{\rho^J}. \tag{7}$$

return

Sectors and Production Functions

- So, we can express the price of sector J output as:

$$p^J = \left(\sum_{o=1}^{O} (\alpha^{Jo})^{\rho^J} (w^{Jo})^{(1-\rho^J)} \right)^{\frac{1}{1-\rho^J}}. \qquad (8)$$

- From (1), (7) and (8), the optimal labor use of occupation o in sector J is:

$$L^{Jo} = \left(\frac{p^J \alpha^{Jo}}{w^{Jo}} \right)^{\rho^J} Y^J. \qquad (9)$$

- The earnings for a worker with l^{Jo} efficiency units is

$$w^{Jo} l^{Jo}.$$

return

Choice for an Unemployed Agent

- Assume ε^{Jo} are drawn from a mean zero Gumbel distribution with a scale parameter σ.
- Then the probability that (J, o) is the best option for a the worker is

$$\pi_t^U((J,o),\underline{a}_t) \equiv \Pr((J,o) \text{ is better than } (J',o')) \quad (10)$$
$$= \frac{\exp\left(\frac{V_t^{Jo}(U,\underline{a}_t)}{\sigma \mathcal{U}_t}\right)}{\sum_{(J',o')} \exp\left(\frac{V_t^{J'o'}(U,\underline{a}_t)}{\sigma \mathcal{U}_t}\right)}.$$

return

Choice for an Unemployed Agent

- Using the probabilities in (10), the expected value of unemployment for individual with ability $\underline{a}_{t-1}$ at the beginning of period $t-1$ is

$$
\begin{aligned}
U_{t-1}(\underline{a}_{t-1}) &= \underline{u} + \beta E\left[U_t(\underline{a}_t) + \max_{(J,o)}\left(\eta^{Jo}(W_t^{Jo}(\underline{a}_t) - U_t(\underline{a}_t) + \varepsilon_t^{Jo}\cdot \mathcal{U}_t\right)\right] \\
&= \underline{u} + \beta U_t(\underline{a}_t) + \beta E\left[\mathcal{U}_t \max_{(J,o)}\left(\frac{V_t^{Jo}(U,\underline{a}_t)}{\mathcal{U}_t} + \varepsilon_t^{Jo}\right)\right] \\
&= \underline{u} + \beta U_t(\underline{a}_t) + \beta \mathcal{U}_t \sigma \log \sum_{(J,o)} \exp\left(\frac{V_t^{Jo}(U,\underline{a}_t)}{\sigma \mathcal{U}_t}\right). \qquad (11)
\end{aligned}
$$

- Thus $U_{t-1}(\underline{a}_{t-1})$ is expressed as a function of $U_t(\underline{a}_t)$ and $W_t^{Jo}(\underline{a}_t)$ for all $(J,o) \in \{\{G,S\} \times \{1,...,O\}\}$, and the model's parameters.

return

Necessary Normalization

- There are a number of required restrictions for the model to be internally consistent:
  - The model has to be homogenous of degree 0 in wages
  - Thus, $\mathcal{U}_t$ has to be homogenous of degree 0 in wages (and the implied prices)
  - The model has to be homogenous of degree 0 in $\underline{\gamma}^G, \underline{\gamma}^S$ and $\overline{c}^S$
  - Thus $\mathcal{U}_t$ has to be homogenous of degree 1 in $\underline{\gamma}^G, \underline{\gamma}^S, \overline{c}^S$,jointly

Necessary Normalization

- These requirements are satisfied if (for example)

$$\mathcal{U}_t = v(w_t^{G1}(\underline{\gamma}_t^G \circ \underline{\lambda}^{G1}); \mathbf{p}_t)$$

- In other words, the utility factor which scales the non-pecuniary costs of the model is equal to
  - the indirect utility from working in sector-occupation $(G, 1)$,
  - with unit ability in each skill (i.e. $\underline{a} = [1, 1, 1]$).

return

Choice for an Employed Agent in (J, o)

- An employed worker will search in (J', o') for which he/she has the largest

$$V_t^{J'o'}((J,o),\underline{a}_t) + \varepsilon_t^{J'o'}\mathcal{U}_t$$

- $V_t^{J'o'}((J,o),\underline{a}_t)$ denotes the expected gain from searching in (J', o') relative to staying in cell (J, o).
  [For $(J', o') = (J, o)$, this is zero by construction.]
- The probability that searching in (J', o') is the best option is

$$\pi_t^{Jo}((J',o'),\underline{a}_t) = \frac{\exp\left(\frac{V_t^{J'o'}((J,o),\underline{a}_t)}{\sigma\mathcal{U}_t}\right)}{\sum_{(\widehat{J},\widehat{o})}\exp\left(\frac{V_t^{\widehat{J}\widehat{o}}((J,o),\underline{a}_t)}{\sigma\mathcal{U}_t}\right)}. \tag{12}$$

Choice for an Employed Agent in (J, o)

- The expected value for working in (J, o) for a $t-1$ years old with ability $\underline{a}_{t-1}$

$$
\begin{aligned}
& W_{t-1}^{Jo}(\underline{a}_{t-1}) = u^{Jo}(\underline{a}_{t-1}) + \beta\delta^{Jo} U_t(\underline{a}_t) + \\
& + \beta(1-\delta^{Jo}) E\left[W_t^{Jo}(\underline{a}_t) + \max_{(J',o')} \left(\zeta\eta^{J'o'}(W_t^{J'o'}(\underline{a}_t) - W_t^{Jo}(\underline{a}_t)) - \chi \mathcal{U}_t \mathbf{I}((J',o') \neq (J,o)) + \varepsilon_t^{J'o'} \mathcal{U}_t \right) \right] \\
& = u^{Jo}(\underline{a}_{t-1}) + \beta\delta^{Jo} U_t(\underline{a}_t) + \beta(1-\delta^{Jo}) \left(W_t^{Jo}(\underline{a}_t) + E\left[\mathcal{U}_t \max_{(J',o')} \left(\frac{V_t^{J'o'}((J,o),\underline{a}_t)}{\mathcal{U}_t} + \varepsilon_t^{J'o'} \right) \right] \right) \\
& = u^{Jo}(\underline{a}_{t-1}) + \beta\delta^{Jo} U_t(\underline{a}_t) + \beta(1-\delta^{Jo}) \left[W_t^{Jo}(\underline{a}_t) + \mathcal{U}_t \sigma \log \sum_{(J',o')} \exp\left(\frac{V_t^{J'o'}((J,o),\underline{a}_t)}{\sigma \mathcal{U}_t} \right) \right],
\end{aligned} \tag{13}
$$

Choice for an Employed Agent in (J, o)

- $W^{Jo}_{t-1}(\underline{a}_{t-1})$ is a function of $U_t(\underline{a}_t)$ and $W^{J'o'}_t(\underline{a}_t)$ and all future prices, for all (J', o'), and the model's parameters
- An individual lives for T years

$$U_{T+1}(\underline{a}) = W^{J'o'}_{T+1}(\underline{a}) = 0 \text{ for all } (J', o')$$

- Value functions are calculate backwards, from age T to age 0

return

The New entrants Problem

The education decision

- The lifetime value without and with education are given, similarly to ((11)), by

$$V^{NE}(\underline{a}_0) = U_1(\underline{a}_0) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V_1^{Jo}(U, \underline{a}_0)}{\sigma}\right), \tag{14}$$

$$V^{ED}(\underline{a}_0) = -\varepsilon_\kappa \sum_{t=1}^{e_H} \beta^{t-1} \mathcal{U}_t + U_{e_H+1}(f(\underline{a}_0)) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V_{e_H+1}^{Jo}(U, f_H(\underline{a}_0))}{\sigma}\right), \tag{15}$$

- Note that V^{ED} incorporate the time cost e_H and a non-pecuniary cost $\varepsilon_\kappa \cdot \mathcal{U}_t$ in every period while studying
- The search probabilities $\pi^{NE}((J, o), \underline{a}_0)$ and $\pi^{ED}((J, o), f_H(\underline{a}_0))$ are analogous to that for the other choices

The New entrants Problem

The education decision

- ε_κ is drawn from a distribution $F(\cdot)$ with mean μ_κ and standard deviation σ_κ
- Because $\varepsilon_\kappa \cdot \mathcal{U}_t$ is stochastic, only a fraction those with innate ability $\underline{a}_0$ will get education
- The cutoff stochastic disutility multiplier for type $\underline{a}_0$ is

$$\bar{\varepsilon}_\kappa(\underline{a}_0) = \max\left\{ \frac{U_{e_H+1}(f(\underline{a}_0)) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V^{Jo}_{e_H+1}(U, f_H(\underline{a}_0))}{\sigma}\right) - V^{NE}(\underline{a}_0)}{\sum_{t=1}^{e_H} \beta^{t-1} \mathcal{U}_t}, 0 \right\}. \tag{16}$$

- The fraction of individuals that get education is then $F(\bar{\varepsilon}_\kappa(\underline{a}_0))$

return

Title

- Technological Changes, the Labor Market, and Schooling—A General equilibrium Model with Multidimensional Individual Skills
- Zsofia Barany (SciencesPo)
- Moshe Buchinsky (UCLA)
- Tamas K. Papp (Institute for Advanced Studies)

Preliminary Results

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