The Flattening of the Phillips Curve and the Learning Problem of the Central Bank

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Abstract

We illustrate an intuitive channel through which price stickiness *limits* the ability of a central bank to improve welfare through stabilization policy. If the central bank uses inflation to obtain information about nominal spending, sticky prices impair the learning ability of the central bank and hence its ability to implement the right stabilization policy. Inflation targeting makes prices stickier, and worsens this learning problem. The key is a microfounded information-based model for price stickiness: taking into account how agents react to the adoption of inflation targeting makes explicit a basic conflict between inflation targeting and stabilization policy.

Keywords: Dual objective, conduct of monetary policy, anchoring of inflation expectations.

JEL codes: E31, E52, E61.

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1 Introduction

In actual economies the central bank does *not* observe the aggregate state of the economy directly but rather gathers statistical information to infer the aggregate state and decide on (monetary) policy. In this process, inflation is commonly used as a prime business cycle indicator to assess the size of output deviations from its frictionless or "potential" level (referred to as the "output gap", see Woodford 2010.) In this paper we are interested in this learning process. Our focus is on how a much central bank can learn from inflation, and how welfare is affected by the information available to the central bank. A key feature of our investigation is that we consider the possibility that the conduct of policy *itself* may affect this learning process by modifying the behavior of the private sector. To this end, we use a microfounded model for price stickiness (L'Huillier 2019). In this model the degree of price stickiness is derived endogenously, as a function of parameters of the economic environment, including monetary policy itself. In particular, there is an interaction between inflation targeting and the degree of price stickiness, and this interaction has implications for the learning problem of the central bank and for the compatibility of stabilization and inflation targeting as policy goals.

We start by laying down a simple and stylized model in which a central bank (CB) has the objective of stabilizing the economy from nominal disturbances. The key aspect of our model is that the CB does not observe these disturbances directly, but infers them by looking at aggregate inflation. Because the CB does not observe disturbances directly, the information conveyed by inflation is a crucial determinant of whether the CB can effectively stabilize the economy. The information conveyed by inflation is determined by the degree of price stickiness: when prices are very sticky, inflation conveys little information. As we show, price stickiness impairs the learning ability of the CB to the point of making it impossible for the CB to implement stabilization policy.

We then expand the model to encompass a dual objective for the CB: a short-run objective (to stabilize the economy from nominal disturbances) and a long-run objective (to achieve a long-run inflation target). We show that when prices are endogenously sticky, these objectives interfere in a non-trivial way. The long-run objective of the CB has a direct impact on consumers' expectations about prices they will face in the long run. Indeed, if the CB has credibly promised to achieve a long-run inflation target, consumers' expect the same long-run inflation rate no matter what the current state is. These consumer expectations generate greater price stickiness. This holds even when firms are perfectly informed about the current state, and they

do not face any menu costs for price adjustment. Thus, inflation targeting generates more *short-run* price stickiness. As in the basic model, price stickiness reduces the amount of information conveyed by inflation and makes it more difficult for the CB to implement stabilization policy.

To sum up, we obtain the following three main results. First, we show that if the extent of price stickiness is sufficiently great, the CB is not able to implement the optimal stabilization policy (Theorem 1). The reason is that in this case prices do not reveal the aggregate state to the CB. This leads to a loss of welfare in comparison to the benchmark economy in which the CB is directly informed (and does not have to draw inferences from prices). Second, we show that inflation targeting leads to a flattening of the Phillips curve (Theorem 2). This flattening is obtained in the following way: The (unconditional) slope of the Phillips curve depends on parameters of the model, as for instance the information available to consumers or firms' marginal costs. Under inflation targeting, the parameter region where the Phillips curve is flat is strictly bigger than when the CB has no long-run inflation considerations. The reason for this flattening is that the region in which firms choose sticky prices is larger when there is inflation targeting than when there is not. Third, inflation targeting interferes with the implementation of the optimal stabilization policy (Theorem 3). Again, the region in which prices are sticky – and hence when the CB is unable to learn the state of nominal disturbances and hence implement the optimal stabilization policy – is larger when there is inflation targeting than when there is not.

Our conclusion that inflation targeting can diminish the CB's ability to stabilize the economy stands in stark contrast to the conclusion obtained in much of mainstream monetary policy analyses. The key difference between our analysis and much of mainstream policy analyses is that we posit an endogenous reason for price stickiness. In a model with Calvo price stickiness, an analysis similar to ours would lead to the conclusion that the CB is always able to learn the aggregate state and improve welfare, even under inflation targeting (which does not modify the degree of stickiness.) The endogenous, microfounded mechanism for price stickiness is central to our argument.

Our exercise is motivated by U.S. monetary policy in the early to mid-2000s. In light of the 2008 financial crisis, and the large accumulation of household debt from 1999 to 2006, one may entertain the hypothesis that a monetary tightening could have been beneficial. However, a puzzling observation is that inflation rates during those years remained quite stable (see Figure 1), and thus did not signal any particular need to increase policy rates to moderate the economic boom. Our model

provides a resolution to this puzzle. Indeed, under the (plausible) assumption that inflation expectations were firmly anchored during this period, our model predicts that demand booms should not be very inflationary, precisely as observed. Notice also that our model says that this is *not* a reason to infer that the output gap was zero, because in these circumstances inflation is fairly uninformative about movements of the output gap.

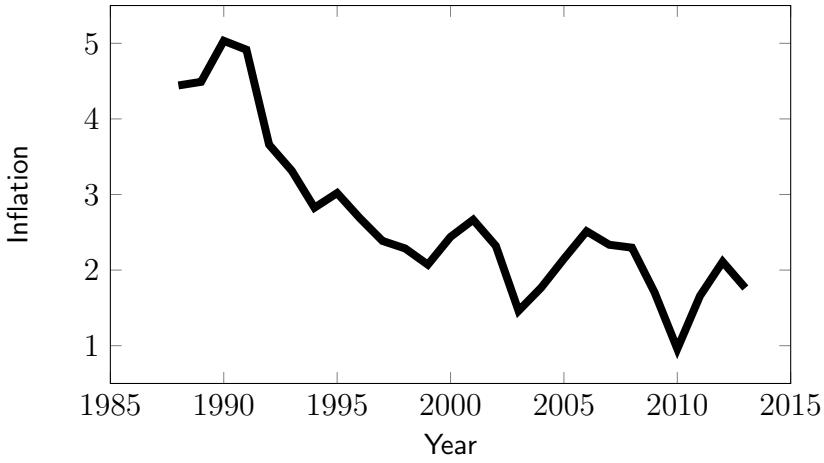

Figure 1: Core Annual Inflation, U.S. 1988 to 2013

Notes: Consumer Price Index for all urban consumers, all items less food and energy, percentage change from year ago. Source: U.S. Department of Labor: Bureau of Labor Statistics

In addition to setting out a formal model and deriving formal results, our paper makes two methodological contributions that are worth highlighting. First, it lays down a model in which three types of agents, firms, consumers, and the CB, are heterogeneously informed. Both consumers and the CB observe the actions of the firms, but the interaction between firms and consumers is strategic. The CB begins with less information than the firms, but is CB is able to improve consumer welfare if it gathers enough information. Handling the informational and game theoretic aspects required to answer our original question is a non-trivial task. Second, the paper derives results for optimal monetary policy in the framework set out by L'Huillier and Zame (2017) and L'Huillier (2019). The contribution here is to show that in this environment the CB acts as an insurer of consumers. Under price stickiness, consumers are subject to fluctuations in their consumption across states. Under certainty equivalence this insight leads to a sharp welfare characterization, which relates our results to the economics of insurance.

We have chosen to formalize our ideas in a highly stylized framework; this makes the ideas more transparent and avoids some technical difficulties. To make this policy

point, we have laid down what seems like the simplest model that encompasses the main ideas. For this reason, the reader should keep in mind that our proposed model is by no means to be interpreted as being realistic. We comment more on these modeling choices in the body and in the conclusion.

A vast literature studies optimal monetary policy. The usual assumption is that the monetary authority is perfectly informed about the state of the economy (Woodford 2003; Chari and Kehoe 1999; Gali 2008), but there are a number of papers that introduce imperfect information. For instance Lorenzoni (2010) analyzes optimal monetary policy in an economy with dispersed information and shows that the CB is quite able to stabilize the economy by influencing agents' responsiveness to private information. The distinguishing feature of Lorenzoni's framework is that better information about fundamentals arrive in future periods. By making announcements about how it will react to future information, the CB is able to influence the contemporaneous responses of agents and improve welfare. In our framework the CB does not observe the state directly at any date, but only indirectly, through the actions of the private sector. Information is dispersed in the private sector, and thus depending on the amount information conveyed by agents' actions, the CB is (or is not) able to stabilize the economy.[1] In an important paper, Angeletos and La'O (2012) show that in a class of economies with monopolistically competitive goods markets and flexible prices first-best allocations are no longer attainable and that optimal policy leans against the wind. In Angeletos and La'O (2013), they focus on the implications of endogenous learning. Other important studies that explore similar information structures where complementarities play a central role include Hellwig (2005), Adam (2007), Vives (2013), Colombo, Femminis, and Pavan (2014). Angeletos and Pavan (2007; 2008) have studied a similar question in more abstract environments. Vives (1988) and Morris and Shin (2002) have provided foundations for this type of analysis.

Our paper is most closely related to Paciello and Wiederholt (2014), who analyze optimal monetary policy under sticky prices due to rational inattention. There, firms choose how much attention to allocate to aggregate conditions, and this modulates the stickiness of prices as a function of policy. The key connection to our work is the endogeneity of the stickiness of prices, and how it interacts with policy. In the model below the stickiness of prices is modified by policy as well. Indeed, whether the CB has adopted a long-run inflation target or not has implications for the degree of stickiness (and modifies the policy conclusions.)

[1]Papers that have followed up on Lorenzoni's work include Rousakis (2012) and Baeriswyla and Cornand (2010).

A central piece of our analysis if the existence of a nominal rigidity based on an information friction. Even though in our setting the friction is on the consumer side, this connects our paper to a growing literature that generates a slow adjustment of prices due to imperfectly informed firms. The seminal contribution here is by Lucas (1972). Mankiw and Reis (2002) is a landmark contribution that revived attention to this topic. See Ball, Mankiw, and Reis (2005) for a policy analysis on this framework. Reis (2006) and (Reis 2009) extend similar ideas in a microfounded framework. On a similar vein, Mackowiak and Wiederholt (2009) and **?**) explore models with rational inattention.

There is an extensive literature on how prices provide information in the economy. We briefly mention some recent contributions to this topic. Amador and Weill (2010) set up the learning problem in a general equilibrium economy and study the welfare implications of learning from prices. Albagli, Hellwig, and Tsyvinski (2013; 2014a; 2014b) constitute a series of contributions on learning in markets with heterogenous and noisy information. **?**) focus on the long-run properties of learning in decentralized markets.

A number of recent papers have tackled the flattening of the Phillips curve from different angles; we note only a few. Simon, Matheson, and Sandri (2013) run regressions of long-run inflation expectations on deviations of current inflation from the central bank's inflation target and estimate New Keynesian Phillips curves using cross-country macroeconomic data. They find clear evidence of both a firmer anchoring of inflation expectations over the past decade or so, and of a flattening of the Phillips curve. This suggests that inflation targeting may have improved the anchoring of inflation expectations and led to a flatter Phillips curve, which is consistent with our results. This work is part of a small literature that has considered these and other explanations for the flattening of the Phillips curve, as for instance data problems, the impact of globalization through complementarities of domestic prices with prices abroad, or a negative correlation of cost-push shocks with the labor share (Kuttner and Robinson 2008; Matheson and Stravrev 2013; Williams 2006; Kuester, Mueller, and Stoelting 2007).

Following this introduction, we present our model and our main results. Section 2 lays down the simple model without inflation targeting. Section 3 present the model with inflation targeting. The Conclusion discusses our findings. Proofs and some numerical simulations are in the Appendices.

2 A Simple Model

We now describe a simple model in which prices are endogenously sticky, and the CB looks at an aggregate price index to learn the state of the economy. The CB is mandated to stabilize the economy in the short run, i.e. to follow an (optimal) *stabilization policy*. This policy maximizes the relevant welfare function. As we will see, the degree of price stickiness, which depends on the informational parameters of the economy, may impair the learning capability of the CB and thereby hinder its ability to implement the optimal stabilization policy.

In order to make this point in the most transparent and readable way, we will use the simplest model available. This has two implications for what follows. First, the model features only three periods. Second, the model is in partial equilibrium, in order to avoid the full description of all markets in the economy. This second simplification greatly simplifies the exposition and allows us to keep the focus on the essential channels for our analysis. However, it is possible to write a general equilibrium version of the same model and obtain the same results. Readers interested in understanding how this approach can be set up should refer to L'Huillier (2019).[2]

2.1 Short and Informal Description

We will model an economy in which *nominal spending* is subject to random disturbances. This is the only random aspect of the economy, so we identify nominal spending as the state.

All agents have common priors about the state, but some receive some additional information. Firms, by assumption, will be fully informed about the realization of the state. Among consumers, a fraction of them will also be fully informed. The remaining fraction of consumers will be uninformed, and thus just hold prior beliefs.

The CB is also uninformed. We motivate this assumption in two ways. First, it allows us to analyze the problem of a central bank that needs to collect information from the private sector in order to figure out the state of the economy. In the model, the central bank will be able to observe firms' prices and thereby make inferences about the state. Second, it corresponds to a view about nominal spending as representing shocks to financial markets which are not the direct product of a targeted CB action, as for instance unpredictable changes in the velocity of money. But the disturbance can also be interpreted more broadly as any other force that randomly affects nominal spending, as changes in consumer confidence, nominal wealth shocks,

[2]The general equilibrium version of the model is in the online appendix (of L'Huillier 2019).

etc.

In the first period, firms will post prices and trade between firms and consumers will happen. Then, the CB will be able to observe firms' prices and draw inferences about the state. In the second period, the CB will implement a stabilization policy in order to maximize welfare of agents in the economy. Conditionally on being informed of the true state, optimal policy of the CB is to keep nominal spending constant. This means: decrease nominal spending when it is higher than average, and increase it in the opposite case. However, the CB may not be informed of the true state, and thus it may be unable to keep nominal spending constant.

In the third period, a competitive market opens in which the price of consumption is flexible. Thus, here the price of consumption will adjust and the state of nominal spending will have no effect. This third period is simply a technical device to close the model.

The main result will be that, when many consumers in the economy are uninformed, firms' prices will be unresponsive to the state of nominal spending, blurring the CB's inference. Thus, welfare will be negatively affected. The reason will be that the CB will not be able to learn the state in order to keep nominal spending constant.

2.2 Environment

2.2.1 Agents

The economy is populated by firms, consumers, and a central bank (CB).

2.2.2 Time

Time is discrete and indexed by t. There are three periods $t = 1, 2, 3$, that we also refer to as *stages*. For expositional purposes, we qualify the first two periods, $t = 1, 2$, as the "short-run", and the last period, $t = 3$, as the "long-run". Also, and again for expositional purposes, we refer to the first period $t = 1$ as the *CB learning stage*, to the second $t = 2$ as the *stabilization policy stage*, and to the last $t = 3$ period as the *price flexible stage*.

2.2.3 Consumers and Goods Markets

In the first two periods, $t = 1, 2$, consumers have access to *decentralized* goods markets where they meet a monopolistic firm. Instead, in period $t = 3$ consumers have access to a *centralized* goods market, where they meet a representative, competitive, firm.

Following Lagos and Wright (2005) in terms of notation, we denote decentralized markets' variables in lower case, and centralized market's variables in upper case.

We index consumers by $i \in \mathfrak{I}$. Each consumer solves

$$\max_{c_{1i},c_{2i},C_{3i}} E[u(c_{1i}) + u(c_{2i}) + C_{3i}] \tag{1}$$

where c_{ti}, $t = 1, 2$, is consumption in the decentralized market, C_{3i} is consumption in the centralized market, and $E[\,\cdot\,]$ is an expectation operator conditional on contemporaneous information available to the consumer, and beliefs about the actions of the CB. Consumers are identical in terms of endowments and preferences, but will differ in terms of what they believe, in ways that will be explained below. To simplify, there is no discounting across periods. The maximization is subject to the budget constraint

$$p_1 c_{1i} + p_2 c_{2i} + P_3 C_{3i} = Income \tag{2}$$

where p_t, $t = 1, 2$, and P_3 are goods' prices. Because of quasilinearity this is an intertemporal budget constraint and, for any *Income*, the FOCs will hold date-by-date. We assume that *Income* is large enough so that $C_{3i} \geq 0$, for all i.

For simplicity we assume throughout the paper that $u(c) = c - 1/2 \cdot c^2$. We will exploit certainty equivalence to obtain sharp results for the optimal stabilization policy. However, the price rigidity mechanism itself can be obtained with any smooth strictly concave utility function $u(\cdot)$. Similar to Lagos and Wright 2005, we exploit quasilinearity in order to handle agent heterogeneity.

Trading in decentralized markets happens as follows. There is a mass of firms producing goods c_1 and c_2, each firm being indexed by $j \in \mathfrak{J}$. Decentralized markets are geographically defined by a mass of islands. On each island there is one firm, who is a price-setting monopolist. At each $t = 1, 2$ each firm (island) is visited by a mass of consumers.

Firms in decentralized markets have a linear cost function $k \cdot c$ and are heterogenous: some of them have low unit costs equal to k_{lc}, and the complement have high unit costs equal to k_{hc}, with $k_{lc} < k_{hc}$. The proportion of high cost firms is $\kappa \in (0, 1/2)$. The role of firm heterogeneity is to generate different degrees of price adjustment in decentralized markets. We make the following parametric assumption which ensure that profits are positive for the prices considered below.

Assumption 1 $k_{hc} < 1$.

Denote the price set by firm j at $t = 1, 2$ by p_{tj}.

Trading in the centralized market happens as follows. There is a representative firm producing an aggregate good C_3. This firm has a cost function $K(C_3)$. Denote the price of this aggregate good P_3. This is the aggregate price level at $t = 3$.

2.2.4 Evolution of the State

The state of the world at t is given by nominal spending M_t. The log of M_t follows the random walk

$$\log M_t = \log M_{t-1} + \tau_t \tag{3}$$

with initial condition M_0. Thus, M_t follows a multiplicative random walk. The random nominal disturbance τ_t belongs to the set $\mathfrak{T} = \{\tau_L, 0, \tau_H\}$, with $\tau_L < 0 < \tau_H$. The disturbances τ_t will be drawn from a discrete probability distribution, with all realizations equally likely $Pr(\tau_t = \tau_L) = Pr(\tau_t = 0) = Pr(\tau_t = \tau_H) = 1/3$. We make the following assumption regarding τ_L and τ_H.

Assumption 2 (Harmonic Property) *The set $\mathfrak{T} = \{\tau_L, 0, \tau_H\}$ of random nominal disturbances is such that*

$$E\left[e^{-\tau_t}\right] = 1$$

Assumption (2) implies that, for given M_{t-1}, the harmonic average of M_t is equal to M_{t-1}:

$$E\left[\frac{1}{M_t}\middle| M_{t-1}\right] = \frac{1}{M_{t-1}}$$

For this reason, we label the process for the state (3) in the no-central-bank economy a "harmonic random walk". The purpose of this assumption is to ensure that the value of a 1$ bill, $1/M_t$, is a martingale.

2.2.5 Information Structure and Beliefs

Firms. We assume that all firms in the economy are informed about the realization of the state M_t at all t.

Consumers. We assume that consumers are heterogeneous in terms of their information about the realization of the state M_t. At the beginning of every period *and before market interactions*, a fraction α of consumers is informed and the complement $1 - \alpha$ is uninformed. This fraction α is an exogenous parameter. Informed consumers know everything, in particular they know the realization of the state M_t

at t. Uninformed consumers do not know this realization but have beliefs that are based on the distribution of states M_t and on information received through their market interactions.

Consumer Learning from Prices at $t = 1$ and $t = 2$. Specifically, consumer learning happens as follows. As described above, firms are visited by consumers in the decentralized market. Specifically, each firm (island) is visited by a representative, random, sample of consumers. Thus, firm j is visited by a mass with fractions α of informed consumers, and $1 - \alpha$ of uninformed consumers.

Consumers update their beliefs based on their prior and the observed price. We assume the firm is the only one to observe total demand, and therefore uninformed consumers *cannot* learn the state from other consumers. We denote beliefs held by consumer i after the observation of the price μ_t. Note that due to firm heterogeneity, it is possible that in a given period not all firms behave the same, and thus some consumers learn, while at the same time other consumers do not learn anything.

Because at every t the state changes, the fraction of informed consumers *at the beginning of each period*, α, is constant. (The identity i of those informed can be allowed to change and this does not affect our results because each firm is visited by a representative random sample of consumers.)

Central Bank. We assume that the CB is *uninformed* about the realization of the the state M_1, but can make inferences by looking at firms' prices.

Information Dynamics: Central Bank Learning at $t = 1$. We now specify how the CB learns from prices. We could postulate that the CB observes an aggregate statistic of firms' prices. However, in this simple information structure the same information is obtained by the CB when it looks at all (non-zero measure) firms' prices. Thus, we simply assume that, at the end of $t = 1$, the CB observes the complete distribution of prices p_{1j}.[3] Having observed prices, the CB makes an inference regarding the state M_1. CB learning at $t = 1$ is central to our analysis. Because we want to focus on CB learning from prices, we assume that the CB does not observe consumers' demand at $t = 1$.

Information Dynamics: Central Bank Learning at $t = 2$. As it will become clear, CB learning at $t = 2$ is irrelevant for our analysis.

[3]Note that this distribution does not change if a single firm or a set of measure zero firms changes its price. Thus, in equilibrium a deviation by a single firm cannot be observed.

Information Dynamics: Final Period $t = 3$. Both consumers and the CB learn the state at $t = 3$ by observing the price in the centralized market P_3.

2.2.6 Central-Bank Economy: Stabilization Policy

At the beginning of the stabilization policy stage $t = 2$, the CB can affect nominal spending by picking the value for the rate τ_2. We allow the rate chosen by the CB to belong to $\mathbb{R}$, and denote it by τ_2^*. The CB chooses τ_2^* to maximize consumer welfare at $t = 2$. The appropriate welfare function will be written down when describing the equilibrium of the economy in Section 2.3 below.

2.3 Equilibrium

First we formally describe the game played between firms and consumers and define its equilibrium. Then we define an equilibrium for the whole economy. In this description we drop consumer and firm indexes (i and j) unless required.

2.3.1 Game

Game Played Between Firms and Consumers. Firms and consumers meet in a decentralized market. The market is composed by islands. On each island the local monopolist and consumers play the following one-shot game. First, the monopolist observes the realization M_t. Then, the monopolist posts a price p_{tj}. Consumers observe p_{tj}, form beliefs μ_t about M_t, and decide how much to demand from the monopolist. (Informed consumers already know M_t, and therefore do not update their beliefs. Uninformed consumers update their prior beliefs using Bayes' rule.) This game is played on all islands j at $t = 1, 2$. For simplicity, we assume that the unit cost k on a given island (either high or low) is common knowledge.

Beliefs of Consumers. At $t = 1, 2$:

- Informed consumers believe that

$$E\left[\frac{1}{P_3}\right] = E\left[\frac{1}{M_t}\right] = \frac{1}{M_t}$$

  where the first equality follows from the harmonic Assumption 2 and the second from the fact that they are informed.

- Uninformed consumers believe that

$$E\left[\frac{1}{P_3}\right] = E\left[\frac{1}{M_t}\right]$$

where the equality follows from the harmonic Assumption 2. Notice, in equilibrium these consumers update their beliefs as a function of the price posted by the firm.

Consumers' Optimality Conditions. Marginal utility of c_t is equated to the expected relative price of goods c_t and C_3:[4]

$$u'(c_t) = E\left[\frac{p_t}{P_3}\right] \tag{4}$$

Given that $u(\cdot)$ is quadratic, this leads to a linear demand function:

$$\mathcal{D}(E[p_t/P_3]) = 1 - E[p_t/P_3]$$

Notice that because of the quasilinearity of preferences, the demand does not depend on income.

We denote by $E_I[\,\cdot\,]$ the expectation operator of the informed consumers. Tacitly, we focus on symmetric strategies for uninformed consumers. In equilibrium their actions are uniquely pinned down by (4), and their beliefs are pinned down by the distribution of the state and information they receive. Thus, symmetry is only a restriction in off equilibrium path beliefs. With this considerations in hand, we write total demand as

$$\begin{aligned} \mathcal{D}_t(p_t, M_t, \mu_t) &= \alpha\mathcal{D}(E_I[p_t/P_3]) + (1-\alpha)\mathcal{D}(E_\mu[p_t/P_3]) \\ &= \alpha\mathcal{D}(p_t/M_t) + (1-\alpha)\mathcal{D}(E_\mu[p_t/P_3]) \end{aligned} \tag{5}$$

$E_\mu[\,\cdot\,]$ is the expectation operator of uninformed consumers, which in equilibrium depend on the price p_t used to update their beliefs. It can be seen that total demand $\mathcal{D}_t(p_t, M_t, \mu_t)$ depends on three objects. First, it depends directly on the price p_t. Second, it depends on beliefs held by the uninformed through their expectation $E_\mu[p_t/P_3]$, which in turn depends on the monopolist's price p_t. (Notice that at $t = 2$ not all uninformed consumers need to have the same beliefs because they may have

[4]To get this expression, substitute Income from (2) into (1) and then take the first order condition with respect to c_t.

observed different prices at $t = 1$, but the linearity of demand allows us to aggregate beliefs to the expectation $E_\mu[p_t/P_3]$ and corresponding μ_t.) Third, it depends on beliefs of the informed $E_I[p_t/P_3]$, which do not depend on the price posted by the monopolist but directly on the state M_t.

Monopolist's Problem. The monopolist chooses p_t to maximize profits:

$$\max_{p_t} (p_t - kM_t)\mathcal{D}_t(p_t, M_t, \mu_t) \tag{6}$$

Perfect Information Benchmark. To develop intuition, it is useful to consider the perfect information benchmark. In this case, total demand is

$$\mathcal{D}_t(p_t, P_3) = \mathcal{D}\left(p_t/M_t\right) \tag{7}$$

Plugging (7) into (6), the monopolist's problem is

$$\max_{p_t} (p_t - kM_t)\mathcal{D}\left(p_t/M_t\right) \tag{8}$$

where we used Assumption 2. The following definition is necessary to understand the properties of this benchmark.

Definition 1 (Flexible and Sticky Prices) *The nominal price p_t is flexible if p_t/M_t is constant across different realizations of M_t, and it is sticky otherwise.*

The following lemma establishes price flexibility in this benchmark.

Lemma 1 *If $\alpha = 1$, then the price p_t is flexible, and demand $\mathcal{D}\left(p_t/M_t\right)$ is the same in all states of nature.*

Proof. Taking the first order condition for the problem (8) and rearranging, get

$$\mathcal{D}\left(p_t/M_t\right) + (p_t/M_t - k)\mathcal{D}'\left(p_t/M_t\right) = 0 \tag{9}$$

From condition (9) one can conclude that the monopolist's optimal price is proportional to M_t. Thus, p_t is flexible. Also, since total demand depends on the monopolist's price divided by M_t, it is the same in all states of nature.

■

Equilibrium Definition for the Game. Denote by $\mathcal{M}_t$ the set of states at t, and by $S = card(\mathcal{M}_t)$ the number of states at t. Also, index states $M_t \in \mathcal{M}_t$ by their rank, M_{t1} being the lowest, and M_{tS} being the highest.

We now define a Perfect Bayesian Equilibrium. We first describe the strategy of the monopolist. We focus on pure strategies. At $t = 1, 2$, a pure strategy for the monopolist p_t is a mapping

$$p_t : \mathcal{M}_t \longrightarrow \mathbb{R}_+$$

that assigns a price p_t to each state of nature $M_t \in \mathcal{M}_t$.

Next, we describe beliefs about M_t, denoted μ_t, of uninformed consumers. We focus on symmetric beliefs. Beliefs are a probability distribution over $\mathcal{M}_t$ defined by a mapping

$$\mu_t : \mathbb{R}_+ \longrightarrow \Delta(S) \tag{10}$$

that assigns to each price p_t a probability to each possible state of M_t. Mapping (10) is consistent with Bayes' rule on the path of equilibrium play. According to these beliefs, $E_\mu [p_t/P_3]$ is simply the expectation of the relative price using beliefs $\mu_t(\cdot\,|p_t)$, that is,

$$E_\mu \left[\frac{p_t}{P_3}\right] = \sum_s \mu_t(s|p_t)\frac{p_t}{M_{ts}}$$

where we used both the fact that the expectation is conditional on p_t, because this was posted by the monopolist, and the harmonic property given by Assumption 2.

Given these definitions, we can now define an equilibrium formally.

Definition 2 *A Perfect Bayesian Equilibrium (PBE) is a list (p_t,$\mu_t(\cdot\,|p_t)$,$c_i(\cdot)$), such that*

1. *There is no profitable deviation from posting p_t, given consumers' play,*
2. *On the equilibrium path, beliefs of the uninformed $\mu_t(\cdot\,|p_t)$ are derived using Bayes' rule,*
3. *Consumption decisions of all consumers $c_i(\cdot) = \mathcal{D}(\cdot)$.*

Equilibrium of the Game. As it is usually the case with this type games in which the informed party moves first, this game has many equilibria. We will proceed as follows. We will restrict attention to two benchmark equilibria, a benchmark separating and a benchmark pooling equilibrium, fully characterized in the appendix. If k_{hc} is small enough (the appendix quantifies how small it must be), the benchmark pooling equilibrium exists. We will propose an equilibrium selection criterion based on the

maximization of expected real profits of the firm. This will lead to our price rigidity argument. Indeed, for high fractions of informed consumers, the best equilibrium for the firm will be the separating, or price flexible, equilibrium. On the opposite range of parameter values, i.e. for low fractions of informed consumers, the best equilibrium for the firm will be the pooling, or sticky price, equilibrium. Throughout the paper we will refer to the benchmark separating (pooling) equilibrium as "the Separating (Pooling) Equilibrium".

A preliminary consideration is useful to provide intuition. Consider beliefs of uninformed consumers, given by the expectation $E_\mu[p_t/P_3]$. This expectation is conditional on p_t, since consumers observe this price posted by the firm. Thus, it can be taken out of the expectation operator, to obtain the demand function

$$\mathcal{D}(E_\mu[p_t/P_3]) = 1 - p_t E_\mu\left[\frac{1}{P_3}\right] \tag{11}$$

At this point, it is important to notice that this demand depends on the price chosen by the monopolist p_t times (the inverse of) a deflator. This deflator is equal to the inverse of the long-run price level $1/P_3$. An interesting feature of (11) is that is it decreasing in the expected deflator $E_\mu\left[1/P_3\right]$. This fact has a clear implication for the strategic motives of the monopolist: it prefers uninformed consumers to believe that the long-run price level is high, because in that case the deflator is low, which increase their demand. Thus, separation in all states but the lowest one will sometimes require a cost related to incentive compatibility, necessary to mitigate the desire of the firm to misrepresent the state. This makes information transmission possible. The cost will be higher the lower the fraction of either informed consumers. This cost can be avoided by playing a pooling or sticky-price equilibrium, leading to optimal price rigidity.

When the fraction of informed consumers is small, the firm-best equilibrium is the Pooling Equilibrium. Instead, when the fraction of informed consumers is large, the firm-best equilibrium is the Separating Equilibrium. This is established formally as follows.

Lemma 2 (Cutoff for Price Adjustment) *There is a cutoff $\alpha_k \in (0,1)$ such that*

- *if $\alpha \in (0, \alpha_k]$, the Pooling Equilibrium is firm-best,*
- *if $\alpha \in (\alpha_k, 1]$, the Separating Equilibrium is firm-best.*

The proof is in the Appendix (p. 33). Armed with this result, for each (k,α) we will assume that the best equilibrium for the firm, among the pooling and the

separating, is played. This assumption can be given a mechanism design foundation, which is fully developed in L'Huillier and Zame (2017). There, the formulation of the problem admits a more general contractual environment, together with the specification of a communication protocol with the mechanism. For readability and focus on our original question we do not fully develop that argument here.

We now turn to the description of the equilibrium for the economy.

2.3.2 Economy

Equilibrium Definition for the Economy. Consider the first period $t = 1$. According to maximization of firms profits, for given k, the cutoff α_k is such that a firm with costs k does not adjust its price (i.e. it plays the Pooling Equilibrium) for $\alpha \in (0, \alpha_k]$, and it adjusts its price for $\alpha \in (\alpha_k, 1]$ (it plays the Separating Equilibrium). We denote these $t = 1$ cutoffs $\alpha_{1,lc}$ and $\alpha_{1,hc}$ for low and high cost firms respectively, and we assume throughout the paper that $\alpha_{1,lc} > \alpha_{1,hc}$. (This assumption is made without loss of generality. A milder implication of firm heterogeneity, mainly $\alpha_{1,lc} \neq \alpha_{1,hc}$, also delivers the results below.)[5]

In the second period $t = 2$, if $\alpha \in (0, \alpha_{1,hc}]$ uninformed consumers have learnt nothing and thus the cutoffs of price adjustment are obtained as in $t = 1$. (The value of the cutoffs will be different at $t = 2$ because the state space is different.) However, if $\alpha \in (\alpha_{1,hc}, 1)$, *some* uninformed consumers have learnt the state at $t = 1$ and this modifies the derivation of cutoffs of price adjustment at $t = 2$. We denote the $t = 2$ cutoffs $\alpha_{2,hc}$ and $\alpha_{2,lc}$. (These cutoffs are derived in the Appendix, p. 36.) We also assume that $\alpha_{2,lc} > \alpha_{2,hc}$.[6]

The CB updates his information about the state at $t = 1$, and then sets τ_2 to maximize welfare at $t = 2$. We begin by writing the welfare function that the CB maximizes.

Definition 3 (Welfare Function) *The welfare function is*

$$W(\tau_2) = E_0 \left[\int_{firms} \left[\alpha u \left(\mathcal{D}(E_I[p_{tj}/P_3]) \right) + (1-\alpha) u \left(\mathcal{D}(E_\mu[p_{tj}/P_3]) \right) \right] dj \right]$$

[5] In the Appendix we present numerical results showing that indeed $\alpha_{1,lc} > \alpha_{1,hc}$, i.e., low cost firms have stickier prices. The comparative statics of these cutoffs is not tractable analytically. As it will become clear, the opposite assumption ($\alpha_{1,hc} > \alpha_{1,lc}$) would deliver the same main results (by just changing which firms have more flexible prices.) In fact all we need is $\alpha_{1,lc} \neq \alpha_{1,hc}$, which is why we introduce firm heterogeneity in the model.

[6] Here again, all we need is $\alpha_{2,lc} \neq \alpha_{2,hc}$.

where $E_0[\,\cdot\,]$ is the ex-ante expectation operator over $\mathcal{M}_2$, and p_{tj} the price of firm j.

According to the stabilization policy, the objective of the CB is:

$$\max_{\tau_2} \; W(\tau_2) \tag{12}$$

Consistent to our previous exposition, we denote the solution to (12) τ_2^*. Notice that (12) implies that the CB has no ex-post discretion.

For simplicity, we assume that at $t = 3$ the price of the representative firm is equal to nominal spending:

$$P_3 = M_3 \tag{13}$$

We denote by $\hat{j}(i)$ the firm visited by consumer i at a given period.

We can now define an equilibrium for the economy.

Definition 4 *An equilibrium of this economy is a given by allocations in decentralized markets $\{c_{1i}, c_{2i}\}$ for each consumer, beliefs $\{\mu_1(\,\cdot\,|p_{1\hat{j}}), \mu_2(\,\cdot\,|p_{2\hat{j}})\}$ of uninformed consumers, prices $\{p_{1j}, p_{2j}\}$ for each firm, cutoffs $\{\alpha_{1,hc}, \alpha_{1,lc}, \alpha_{2,hc}, \alpha_{2,lc}\}$, policy $\{\tau_2\}$, and an allocation in the centralized market C_{3i} for each consumer such that*

1. *Allocations $\{c_{1i}, c_{2i}\}$, beliefs $\{\mu_1(\,\cdot\,|p_{1\hat{j}}), \mu_2(\,\cdot\,|p_{2\hat{j}})\}$, and prices $\{p_{1j}, p_{2j}\}$ satisfy the definition of an equilibrium for the game between firms and consumers,*
2. *Cutoffs $\{\alpha_{1,hc}, \alpha_{1,lc}, \alpha_{2,hc}, \alpha_{2,lc}\}$ are used to determine which, among the Pooling and the Separating equilibrium, is played,*
3. *Policy τ_2 solves Problem (12),*
4. *Consumers form beliefs about policy τ_2 and these beliefs are consistent with policy τ_2,*
5. *Allocations C_{3i} are such that the budget constraint of each consumer holds.*

2.4 Stabilization Policy with Central Bank Learning

2.4.1 No Central Bank

In order to characterize the impact of stabilization policy, we first describe allocations and welfare in the absence of a central bank. Thus, in this case τ_2 is assumed to be freely determined by process (3).

The following defines a useful welfare benchmark.

Definition 5 (Perfect Information Allocations and Welfare) *Suppose $\alpha = 1$. In this case, prices are flexible, and we have that*

$$E_I\left[\frac{1}{P_3}\right] = E_I\left[\frac{1}{M_t}\right] = \frac{1}{M_t}$$

Thus, at $t = 0$ there is no uncertainty regarding equilibrium allocation c_{2i}. Define welfare under these conditions by $\overline{W}$, and the corresponding allocation by $\bar{c}_2$.

In order to understand the impact of price rigidity on welfare, let us characterize welfare in an economy with imperfectly informed consumers ($\alpha < 1$). The following lemma is useful to understand some properties of allocations in this case. First, it discusses allocations in the Separating Equilibrium and then it considers the Pooling Equilibrium. A small issue here is that when $\alpha \in (\alpha_k, 1]$ and thus a firm with marginal costs k plays the Separating Equilibrium, for values of α close to α_k a distortion is present in the Separating Equilibrium. This distortion is fully described in the appendix. To get a sharp welfare characterization, we will make the assumption that, in this case the government will tax this firm to avoid this distortion. The uses of taxes and subsidies to avoid distortions that are tangential to the main analysis and gain tractability is commonplace in monetary economics (see for instance Woodford 2010, and the explanation in footnote 63.) With this assumption we are able to obtain a simple and intuitive characterization of allocations.

Lemma 3 (Allocations in the Separating and in the Pooling Equilibrium) *The following characterizes allocations in the Separating and in the Pooling Equilibrium.*

- *In the Separating Equilibrium all consumers get the allocation $c_{2i} = \bar{c}_2$.*
- *In the Pooling Equilibrium uninformed consumers that have learnt nothing at $t = 1$ get the allocation $c_{2i} = \bar{c}_2$. All other consumers get an allocation different from $\bar{c}_2$: $c_{2i} \neq \bar{c}_2$.*

An immediate corollary is the following Proposition, which characterizes welfare under these circumstances.

Proposition 1 (Welfare in the Absence of Stabilization Policy) *The following characterizes the equilibrium level of welfare in the absence of stabilization policy.*

- *If $\alpha \in (0, \alpha_{2,lc}]$, welfare is strictly below $\overline{W}$.*
- *If $\alpha \in (\alpha_{2,lc}, 1]$, welfare is $\overline{W}$.*

The intuition for this result is straightforward. For $\alpha \in (0, \alpha_{2,lc}]$, all firms do not adjust prices at $t = 2$. In some states, informed consumers meeting firms get an allocation $c_{2i} \neq \bar{c}_2$, and therefore they cannot equate their marginal utility for c_{2i} across states of the world, suffering an expected welfare loss. In the opposite case, for high enough α, enough consumers are informed so that all firms adjust prices, and all consumers equate their marginal utility across states. In this case, there is not welfare loss away from the perfect information benchmark.

2.4.2 Benchmark: Informed Central Bank

We now characterize the optimal policy τ_2^* with an informed CB, which is a useful benchmark for the case in the which the CB needs to learn.

An informed CB can fully correct the distortion caused when α is low, as shown by the following proposition.

Proposition 2 (Optimal Stabilization Policy) *For all α, an informed CB can reach $\overline{W}$ if the following state-contingent policy is implemented:*

$$\tau_2^* = \log(M_0/M_1)$$

At $t = 1$ there are three possible states for M_1, ranked from $s = 1$ (lowest) to $s = 3$ (highest). Policy τ_2^* is intuitive and simply sets $M_2 = M_0$. This means:

- If $s = 1$ (lowest $M_1 < M_0$), $\tau_2^* > 0$,
- If $s = 2$ (intermediate $M_1 = M_0$), $\tau_2^* = 0$,
- If $s = 3$ (highest $M_1 > M_0$), $\tau_2^* < 0$.

The proof is immediate and therefore it is omitted. The reason this policy reaches the perfect information welfare is the following. By bringing M_t back to M_0, the CB achieves a situation in which all consumers have the same beliefs. By rational expectations, irrespectively of whether consumers are informed or uninformed, $E_I[1/M_2] = E[1/M_2] = 1/M_0$. Thus, play happens according to the Separating equilibrium, by Lemma 3 allocations $c_{2i} = \bar{c}_2$ for all consumers and all states at $t = 2$, and thus welfare is $\overline{W}$. As a consequence, using this optimal policy, the CB is able to provide insurance to the informed by ensuring $c_{2i} = \bar{c}_2$.

The way the information structure is set up, the policy that achieves $\overline{W}$ is not unique. This does not interfere with our coming results that learning can interfere with the capability of the CB to achieve $\overline{W}$. Nevertheless, it is a point that deserves discussion. Given that α is known to all agents in the economy, all consumers know

that for high enough α the CB *will* become informed and therefore will be able to achieve a given M_2 by setting τ_2. Thus, any target M_2 can coordinate consumers' beliefs, achieve play in the Separating equilibrium, and ensure $c_{2i} = \bar{c}_2$. However, notice two points. First, this requires the CB to become informed (which is why our main results are not affected by this feature of the model). Second, this property of the model is clearly not robust. Suppose that α is *not* known by consumers, and that at $t = 1$ every island receives a random sample α_j with the property that $\int \alpha_j dj = \alpha$. In this case, even if agents knew α_j locally, the optimal policy is uniquely pinned down at $\tau_2^* = \log(M_0/M_1)$ because this equalizes beliefs of the informed to uninformed prior beliefs $E[1/P_3] = E[1/M_2] = 1/M_0$.

2.4.3 Uninformed Central Bank

Now consider the case in which the CB needs to learn the state at $t = 1$ in order to implement the policy τ_2. Whether the CB can learn the state or not depends on the fraction of informed consumers, because this determines the fraction of firms adjusting prices. So long as a non-zero fraction of firms adjust its price, the CB will be able to fully learn the state by looking at $\overline{p}_1$. However, if no firms adjust its price, the CB will not be able to learn anything about the state because $\overline{p}_1$ will not contain any information. This is possible when the fraction of informed consumers α is low enough. The following result fully characterizes welfare under a learning CB.

Theorem 1 (CB Learning and Welfare) *The equilibrium level of welfare depends on whether the CB learns or not, as follows.*

- *If $\alpha \in (0, \alpha_{1,hc}]$, the CB cannot learn the state s at $t = 1$. In this case, welfare is strictly below $\overline{W}$.*
- *If $\alpha \in (\alpha_{1,hc}, 1]$, the CB learns the state s at $t = 1$. In this case, welfare is $\overline{W}$.*

The proof is immediate and is therefore omitted. Notice that this Proposition does not say that in this model stabilization policy is useless with a learning CB. This depends on parameter values. Indeed, if $\alpha_{1,hc} < \alpha_{2,lc}$ and $\alpha \in (\alpha_{1,hc}, \alpha_{2,lc})$, in this intermediate range the CB learns the state because some firms do not pool. Thus, it can improve welfare even if it needs to learn the state. But, if $\alpha \in (0, \alpha_{1,hc}]$ stabilization policy is indeed severely limited by the need to learn the state.

Thus, in the interval $(0, \alpha_{1,hc}]$ welfare is strictly below $\overline{W}$. In the next Section we will analyze how CB learning interacts with a main institutional property of central banking in modern economies: inflation targeting. We will show that this has implications for the size of this interval.

3 The Model with Inflation Targeting

In this section we incorporate inflation targeting into the analysis. There are two reasons to do this. First, this seems like a relevant and natural extension of the simple model above, given that most central banks in modern economies adopt some commitment to manage long-run inflation. Second, the welfare implications of inflation targeting in this environment are very different to those obtained in standard models of stabilization policy (Woodford 2003). Indeed, this section will show that inflation targeting a) changes the behavior of inflation in the short-run by generating supplementary price stickiness, b) hinders the learning capability of the CB, and thereby, c) can generate a welfare loss.

3.1 Short and Informal Description

We will amend the previous environment as follows. The model will still have 3 periods, but the price level in the third period will be controlled by the CB. The CB has a mandate to achieve an inflation target. However, for unspecified reasons in the model, the CB will not achieve this target with some probability. We think about this uncertainty as reflecting political (or other) reasons why the CB might not be able to always follow its mandate (last minute changes in policy decisions, idiosyncracy of CB governors, etc.)

Consumers in this economy will have expectations about the long-run price level that are consistent with the probability that the CB will achieve the specified target. The more likely is the CB to achieve the target, the more consumers align their long-run expectations with this target. Thus, we think about the degree to which consumers align their expectations, which is determined by the actual probability described previously, as "anchoring of inflation expectations".

In the previous section we showed that the number of uninformed consumers was a key determinant of the degree of price stickiness. The more consumers are uninformed, the stickier are prices. In the amended setup the degree of anchoring of inflation expectations will also have implications for the degree of stickiness. In fact, the more anchored are expectations, the stickier are prices. This result has implications for the slope of the Philips curve. For enough anchoring of expectations the Philips curve is flat: nominal demand can have an effect on amounts transacted in goods markets without any change in prices.

Because the anchoring of expectations generates stickiness, it also interferes with the learning ability of the CB. Indeed, if the CB looks at prices in order to learn

the state of nominal spending, enough nominal rigidity can completely blur the CB's inferences. A straightforward corollary for stabilization policy is the inability of the CB to keep nominal spending constant. In this situation there will be a welfare loss.

3.2 Environment

The environment is the same, except for the following amendments.

3.2.1 Inflation Targeting.

The CB implements inflation targeting in the long-run $t = 3$. The target is defined as a rate of inflation from an (exogenous) period zero price level P_0 to $t = 3$, i.e. as a targeted value for P_3/P_0. We assume that this target implies no inflation in the long run.[7] Thus,

$$\frac{P_3}{P_0} = 1 \tag{14}$$

So long (13) holds, this target can be expressed as a restriction on M_3, which for given M_2, pins down the rate τ_3 needed to reach (14). In order to make this feasible for the CB, we assume that M_2 is revealed to the CB at the end of period $t = 2$. Notice that we are interested in the learning problem for stabilization policy purposes, *not* for inflation targeting purposes. Thus this is a simplifying assumption that does not interfere with our question.

We suppose there is uncertainty about whether the CB will control τ_3. Specifically, the assumption is that with probability β the CB will *not* control τ_3, in which case this nominal disturbance will be given by (3). Otherwise, with probability $1 - \beta$, the CB will control τ_3 and will achieve (14). Below we offer an interpretation of this modeling device in terms of the degree of central bank credibility. (We think about β as small rather than large.)

Inflation targeting and the related uncertainty is common knowledge. We allow for the (consistent) belief that the CB could fail. This can be formalized in a number of ways: either a fraction of the population believes the CB will fail for sure, or the entire population believes the CB will fail will with some probability, or some mixture of these. For our purpose only the aggregate beliefs matter. It is convenient to adopt the first formalization, but we will show that the alternative would lead to the same conclusions.

[7] This is a simplification. We can easily extend our results to the case of positive but small long-run inflation.

3.3 Equilibrium

The equilibrium notions used here are similar to those used in Section 2, with a couple of amendments.

3.3.1 Game

The game played here is the same as in Section 2. Notice that the presence of inflation targeting will affect beliefs of consumers about $1/P_3$.

Total Demand and Beliefs of Consumers. Total demand is:

$$\mathcal{D}_t(p_t, P_3) = \beta\alpha\mathcal{D}(E_I[p_t/P_3]) + \beta(1-\alpha)\mathcal{D}(E_\mu[p_t/P_3]) + (1-\beta)\mathcal{D}(p_t/P_0) \qquad (15)$$

Thus, there are 3 types of consumers, according to their beliefs: consumers that believe that the CB will reach its target (of mass $1-\beta$), consumers that do not believe that the CB will reach its target but are informed of the state at t (of mass $\beta\alpha$), and consumers that do not believe that the CB will reach its target but are uninformed of the state at t (of mass $\beta(1-\alpha)$).[8] Specifically, at $t = 1, 2$:

- Consumers believing that the CB will reach its long-run target believe that

$$E\left[\frac{1}{P_3}\right] = \frac{1}{P_0}$$

  We name these consumers "believing consumers".

- Consumers believing that the CB will fail and are informed about the state at t believe that

$$E_I\left[\frac{1}{P_3}\right] = E_I\left[\frac{1}{M_t}\right] = \frac{1}{M_t}$$

  where the first equality follows from the harmonic Assumption 2 and the second from the fact that they are informed. We name these consumers "disbelieving-informed consumers".

- Consumers believing that the CB will fail and are uninformed believe that

$$E_\mu\left[\frac{1}{P_3}\right] = E_\mu\left[\frac{1}{M_t}\right]$$

[8] Because of the linearity of demand, the alternative formalization of beliefs in which the entire population believes that the CB will fail with some probability leads to the same total demand function and to equivalent results.

where the first equality follows from the harmonic Assumption 2. Notice, in equilibrium these consumers update their beliefs as a function of the price posted by the firm. We name these consumers "disbelieving-uninformed consumers".

Thus, the lower the probability the CB does not achieve its long-run target (low β), the higher the fraction of believing consumers with fixed expectations that $E[1/P_3] = 1/P_0$. As it will be clear, the presence of believing consumers will endogenously modulate the amount of price stickiness in the economy. When the fraction of believing consumers is large, there will be more price stickiness.

Before proceeding to define an equilibrium for the economy with inflation targeting, we proceed to discuss a small issue relative to the Separating Equilibrium. For convenience we shall focus in the rest of this section on the same Separating Equilibrium as in the previous section, in which firms separate at $\arg\max\ (p_t - kM_t)\mathcal{D}(p_t/M_t)$. This will simplify our calculations considerably and allow us to get sharp results in what follows. In the Appendix, we resort to numerical methods and extend the analysis to the consideration of separating prices at $\arg\max\ (1-\beta)(p_t - kM_t)\mathcal{D}(p_t/P_0) + \beta(p_t - kM_t)\mathcal{D}(p_t/M_t)$ and we obtain the same results.

The equilibrium definition for the game is the same as in Section 2 and is therefore omitted. As before, we assume that among the Pooling and the Separating Equilibrium the best one for the firm is played. This leads to cutoffs of price adjustment $\alpha^{\beta}_{1,hc}, \alpha^{\beta}_{1,lc}, \alpha^{\beta}_{2,hc}, \alpha^{\beta}_{2,lc}$.

We can now define an equilibrium for the economy.

Definition 6 *An equilibrium of this economy is a given by allocations in decentralized markets $\{c_{1i}, c_{2i}\}$ for each consumer, beliefs $\{\mu_{1i}(\ \cdot\ |p_{1\hat{i}}), \mu_{2i}(\ \cdot\ |p_{2\hat{i}})\}$ of uninformed consumers, prices $\{p_{1j}, p_{2j}\}$ for each firm, cutoffs $\{\alpha^{\beta}_{1,hc}, \alpha^{\beta}_{1,lc}, \alpha^{\beta}_{2,hc}, \alpha^{\beta}_{2,lc}\}$, policies $\{\tau_2, \tau_3\}$, and an allocation in the centralized market C_{3i} for each consumer such that*

1. *Allocations $\{c_{1i}, c_{2i}\}$, beliefs $\{\mu_{1i}(\ \cdot\ |p_{1\hat{i}}), \mu_{2i}(\ \cdot\ |p_{2\hat{i}})\}$, and prices $\{p_{1j}, p_{2j}\}$ satisfy the definition of an equilibrium for the game between firms and consumers,*
2. *Cutoffs $\{\alpha^{\beta}_{1,hc}, \alpha^{\beta}_{1,lc}, \alpha^{\beta}_{2,hc}, \alpha^{\beta}_{2,lc}\}$ are used to determine which, among the Pooling and the Separating equilibrium, is played,*
3. *Policy τ_2 solves Problem (12), and policy τ_3 ensures (14) holds,*
4. *Consumers form beliefs about policies $\{\tau_2, \tau_3\}$ and these beliefs are consistent with policies $\{\tau_2, \tau_3\}$,*
5. *Allocations C_{3i} are such that the budget constraint of each consumer holds.*

3.4 Stabilization Policy with Inflation Targeting

3.4.1 Degree of Stickiness

We now analyze the degree of price stickiness and the capability of stabilization policy to improve welfare under inflation targeting. Consider, for ease of exposition, $t = 1$.[9] The first result, Lemma 4, establishes that under inflation targeting prices become stickier.

Lemma 4 (Increased Nominal Rigidity) *There is $\overline{\beta} \in (0, 1]$ such that for $\beta \in (0, \overline{\beta}]$, the cutoff of price adjustment α_{1k}^{β}, is such that $\alpha_{1k}^{\beta} \in (\alpha_{1k}, 1)$.*

The proof is in the Appendix. The intuition for this result is that the Pooling Equilibrium becomes more attractive to firms the less likely the CB is to fail to achieve its inflation target (the lower β). The reason is that this pins down long-run expectations about the price level to $1/M_0$. Equation (15) shows that this makes the pooling price more attractive to the firm.

3.4.2 Phillips Curves

We consider the following notion of Phillips curve. Consider, for ease of exposition, $t = 1$.[10] Unconditionally sample equilibrium pairs of inflation and aggregate consumption over realizations of the state M_1, where

$$Inflation = \int p_{1j} dj \Big/ P_0$$

and

$$Agg.\ Consumption = \int c_{1i} di$$

Then, plot these ($Inflation$, $Agg.\ Consumption$) pairs in a graph, having inflation in the vertical axis and aggregate consumption in the horizontal axis. This Phillips curve will be *flat* if prices in the whole economy (and thus for both high and low cost firms) are sticky and thus do not change, but $\alpha > 0$ and therefore demand $\mathcal{D}_t$ moves away from the perfect information benchmark quantity $\overline{c}_2$ depending on the realized state.

We now present our second main result, Theorem 2. This Theorem is concerned with the shape of the Phillips curve under inflation targeting. In the model of Section 2 the Phillips curve is flat when $\alpha \in (0, \alpha_{1,hc}]$, because in this interval all firms play

[9] The same remarks can be restated for $t = 2$.
[10] The same remarks can be restated for $t = 2$.

Pooling but demand of the non-zero mass of informed consumers α varies across states M_1. The result is that under inflation targeting the Phillips curve "flattens".

Theorem 2 (Flattening of the Phillips Curve) *Suppose that $\beta \in (0, \overline{\beta}]$. The interval over which the Phillips curve is* flat *is strictly larger than when $\beta = 1$: $(0, \alpha^{\beta}_{1,hc}] \supsetneq (0, \alpha_{1,hc}]$.*

The proof is in the Appendix.

3.4.3 Central Bank Learning

Because no price adjusts when the Phillips curve is flat, the CB cannot learn the state at $t = 1$. The following proposition expresses this result formally.

Proposition 3 (Central Bank Learning under Inflation Targeting) *Suppose that $\beta \in (0, \overline{\beta}]$. Then, inflation targeting blurs CB learning when $\alpha_1 \in (\alpha_{1,hc}, \alpha^{\beta}_{1,hc}]$, that is, for $\beta = 1$, $\mu_1^{CB}(s)$ is either 0 or 1, $\forall s$, but for $\beta \in [0, \overline{\beta})$, $\mu_1^{CB}(s) = 1/S$, $\forall s$.*

The proof is immediate and is therefore omitted.

3.4.4 Policy

Our third main result, Theorem 3, is concerned with stabilization policy and implied welfare when the CB needs to learn but also has a long-run inflation target. It shows that the region over which stabilization policy is unable to improve welfare is bigger than when the CB has no long-run inflation concerns. This is a direct consequence of the inability of the CB to learn the state.

Theorem 3 (Welfare Impact of Inflation Targeting) *Suppose that $\beta \in (0, \overline{\beta}]$. Then, the interval over which welfare is strictly below $\overline{W}$ is larger than when $\beta = 1$: $(0, \alpha^{\beta}_{1,hc}] \supsetneq (0, \alpha_{1,hc}]$.*

The proof is in the Appendix.

An important qualification of Theorem 3 is that we have only characterized the region where welfare is below $\overline{W}$. This does not allow to make conclusions about the desirability of inflation targeting based on welfare considerations because it also has direct welfare effects. We discuss this issue more in detail in the Conclusion.

4 Conclusion and Final Remarks

Being a central friction in many macroeconomic models, we find a microfoundation for price rigidity useful to analyze how this friction is modified by the environment, and most importantly, by economic policy itself. In this paper, we illustrate our point by considering a central bank with a dual objective: stabilization of economic activity from nominal disturbances in the short run, and achievement of an inflation target in the long run. The main lesson of our analysis is that taking into account how price rigidity is modified by the adoption of a long-run inflation target shows that, for a range of parameters of the environment, both objectives are not compatible which each other. In particular, inflation targeting limits the ability of the CB to get information about nominal disturbances, and this channel makes it unable to stabilize the economy.

It seems appropriate to discuss our modeling choices. First, for expositional simplicity we have adopted a simple, partial equilibrium, framework. Our goal was to focus on the informational structure and learning dynamics. However, it is possible to embed our model into a fuller general equilibrium framework in which money has an explicit role, and other markets as for labor and financial assets are present. The reader interested in such a framework should refer to L'Huillier (2019). Second, in the derivation of the cutoffs of price adjustment we have focused on a benchmark separating and a benchmark pooling equilibrium. This procedure greatly simplifies the game theoretic analysis of our model. Considering other equilibria (for instance, semi-separating equilibria, or pooling at other prices) would require to also use a richer specification of learning among consumers (besides our simplified informed-uninformed approach) in order to deliver similar results and does not seem to add relevant insights for our original question. Third, we have considered a limited amount of firm heterogeneity. Enriching the firm distribution would, keeping the rest of the informational structure of the model fixed, make it easier for the CB to learn. However, similar insights would be obtained if at the same time the rest of the model was enriched.[11]

Our main point regarding the non-compatibility of inflation targeting and stabilization policy is closely related to an old idea within monetary policy discussions referred to as "Goodhart's law". According to Charles Goodhart (1975), "*When a measure becomes a target, it ceases to be a good measure.*" Our contribution is to provide a microfounded model expressing formally a version of this point. Our microfounded model clarifies the role of price stickiness in making inflation a "bad

[11] For instance, one could imagine that the CB observes firms' prices with noise, etc.

measure", and how exactly trade-offs in firms' pricing policies can imply a flattened Phillips curve and make inflation a muted signal of the aggregate state of the economy. We are unaware of other formalizations of Goodhart's original idea.[12]

We do not attempt to make policy recommendations regarding the soundness of inflation targeting as an element of central banking. The main reason is that our current analysis is missing important ingredients that could change the welfare arithmetic. Two channels deserve discussion. First, it is quite possible that even though the CB cannot learn from inflation, it may learn from other indicators as, for instance, preliminary output growth figures or the stock market index. In fact, our above results suggest that central banks should pay a lot of attention to that type of indicators if inflation expectations are firmly anchored. We see the analysis of a model with a richer information structure that allows to analyze these issues as a fruitful research avenue. However, the most useful policy answer would probably come out of a quantitative version of such a model. These considerations are clearly out of the scope of this paper. Second, even in our simple model inflation targeting has a direct positive effect on welfare. The reason is that the smaller β, the more irrelevant become nominal disturbances, and thus welfare losses coming from across-states fluctuations are muted. Again, this is an issue that for clarity we have decided to leave aside. Assessing the net welfare effect of inflation targeting is a fascinating question that we leave for future work.

[12]For a recent informal discussion of Goodhart' law, see Chrystal and Mizen (2001).

A Full Characterization of the Game

In this section we fully describe the game between firms and consumers. For ease of notation, we shall drop consumer and firm indexes (i and j).

A.1 Basic Properties

Formally, the game between firms and consumers defines a signaling game. The sender in the signaling game is the firm. The type of the sender is defined by referring to different possible information sets he can access.[13] Therefore, there is one possible type of monopolist for each possible realization of M. The message of the sender is the price p. The receiver is the set of uninformed consumers, whose action is given by a demand function $\mathcal{D}_t(\cdot)$. This action depends on beliefs μ.

This signaling game belongs to the class of monotonic signaling games. To show this, it is necessary to define the following well-known property for a function of two variables.

The following lemma shows that under some conditions this game is a standard monotonic signaling game. To simplify the notation we write $\pi(p, M, \mu) = (p - kM)\mathcal{D}_t(p, M, \mu)$.

Lemma 5 (Game Properties) *If $\alpha > 0$, this is a monotonic signaling game, i.e, it satisfies:*

1. *Monotonicity.*

   Let μ' and μ be two possible beliefs of the uninformed. Denote the respective expectations $E'[1/P_3]$ and $E[1/P_3]$. If $E'[1/P_3] < E[1/P_3]$, then, for all $p < kM$ and all M, $\pi(p, M, \mu') > \pi(p_t, M, \mu)$.

2. *Single-crossing.*

   Let $E'_I[1/P_3]$ and $E_I[1/P_3]$ be two possible expectations of the informed. Suppose that $E'_I[1/P_3] = 1/M_{t,s+1}$ and $E_I[1/P_3] = 1/M_{ts}$. Then, for any p', p s.t. $p' > p$ and $p' < kM_{t,s+1}$, for arbitrary beliefs of the uninformed μ, $\pi(p', M_{ts}, \mu) \geq \pi(p, M_{ts}, \mu) \implies \pi(p', M_{t,s+1}, \mu) > \pi(p, M_{t,s+1}, \mu)$.

Proof. We first prove monotonicity, and then single-crossing.

1. Monotonicity.

[13] This is the standard definition of "type" in game theory.

The proof follows from the fact that the demand of the uninformed $\mathcal{D}(E_\mu[p/P_3])$ is strictly decreasing in $E[1/P_3]$. One can easily verify that, for all $p < 1$ and any beliefs μ' and μ s.t. $E'[1/M] < E[1/M]$, $\pi(p, M, \mu') > \pi(p, M, \mu)$.

2. Single-crossing.

   Consider p', p, s.t. $p' > p$ and $p < 1$, and assume

$$\pi(p', M_{ts}, \mu) \geq \pi(p, M_{ts}, \mu)$$

   This is equivalent to

$$\pi(p', M_{ts}, \mu) - \pi(p, M_{ts}, \mu) \geq 0$$

   Notice that for linear demand

$$\pi(p', M_{t,s+1}, \mu) - \pi(p, M_{t,s+1}, \mu) > \pi(p', M_{ts}, \mu) - \pi(p, M_{ts}, \mu) \geq 0$$

   and therefore

$$\pi(p', M_{t,s+1}, \mu) > \pi(p, M_{t,s+1}, \mu)$$

■

As we will see now, these two properties make this game tractable.

A.2 Equilibrium Play

The following proposition characterizes the Separating Equilibrium. This is the best separating equilibrium for the firm, also called "Least Cost Separating Equilibrium" in the signaling games literature.

Proposition 4 (Separating Equilibrium) *The following is the Best Separating Equilibrium. Consider the monopoly price when the state is s and all consumers have fixed beliefs that the state is s:*

$$p_{ts} = \arg\max(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts})$$

Then, there is $\alpha_1 \in (0, 1)$ such that,

- *For $\alpha \geq \alpha_1$ the firm separates by posting p_{ts} in each state s,*
- *For $\alpha < \alpha_1$, in $s = 1$ the firm separates by posting p_1, and by posting either p_{ts} or a price $\bar{p}_{ts} > p_{ts}$ in the other states s, with $\bar{p}_{ts}$ in at least one s.*

Proof.

- In order to find α_1, consider all IC constraints ensuring that types do not want to imitate each other. Because of monotonicity, it is sufficient to check local IC constraints only, i.e. check that

$$\begin{aligned}(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}) \;\geq\; & \alpha(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{ts}) \\ & +(1-\alpha)(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{t,s+1}) \quad (16)\end{aligned}$$

  Notice that

$$\begin{aligned}(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{t,s+1}) &> (p_{t,s+1} - kM_{t,s+1})\mathcal{D}(p_{t,s+1}/M_{t,s+1}) \\ &= (p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}) \cdot M_{t,s+1}/M_{ts} \\ &> (p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts})\end{aligned}$$

  where the first inequality follows from the profit function being single-peaked, which is the case with linear demand. Also, notice that

$$(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{ts}) < (p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts})$$

  Thus, for high enough α constraints (16) are satisfied. Then, pick α_1 as the lowest α such that this happens.

  In order to prove that separating at p_{ts} is an equilibrium, we also need to consider off-equilibrium deviations for the firm. We construct the separating equilibrium by imposing that off-equilibrium path beliefs are $E[1/P_3] = 1/M_1$. We know that type zero does not want to imitate other higher types. A fortiori, given that for higher types beliefs of the informed $E_I[1/P_3]$ are strictly lower than for the zero type, there are no profitable deviations.

  Because p_{ts} are the optimal prices given consumer beliefs, there are no profitable deviations.

  Given that consumers are choosing consumption according to their FOC (4), then for $\alpha \in [\alpha_1, 1)$ this is indeed an equilibrium.

- Now consider the case when $\alpha < \alpha_1$. Again, we start by looking at the IC constraints of the firm (16). We know that at least one of them is not satisfied

at p_{ts}. For the sake of the argument, say that for p_{ts} and $p_{t,s+1}$

$$\begin{aligned}(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}) &< \alpha(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{ts}) \\ &\quad +(1-\alpha)(p_{t,s+1} - kM_{ts})\mathcal{D}(p_{t,s+1}/M_{t,s+1})\end{aligned}$$

By single-crossing, we know that there is $\bar{p}_{t,s+1} > p_{t,s+1}$ such that

$$\begin{aligned}(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}) &= \alpha(p_{t,s+1} - kM_{ts})\mathcal{D}(\bar{p}_{t,s+1}/M_{ts}) \\ &\quad +(1-\alpha)(\bar{p}_{t,s+1} - kM_{ts})\mathcal{D}(\bar{p}_{t,s+1}/M_{t,s+1}) \quad (17)\end{aligned}$$

Moreover, again by single-crossing we know that there is a price such that the $s+1-nth$ type does not want to imitate the $s+2-nth$ type, and so on.

The rest of the proof is similar.

■

The intuition for prices in the Separating Equilibrium is as follows. α is a measure of the cost of information transmission, in the following sense. If all consumers were informed the firm would be able to post the monopoly price. So, for α strictly lower than 1 the firm would like to inform the consumers. However, it can not do so credibly because it has an incentive to pretend that the state is the highest and extract more profits. Thus, in order to inform the consumers credibly, in equilibrium the firm needs to incur a cost. The higher α, the less bad is the incentive problem (given that informed consumers know the state), and the lower this information transmission cost. As a result, for high enough α, there is no cost in equilibrium and thus the firm can transmit all information and post the monopoly prices. Instead, for low α the firm needs to create a distortion away from monopoly pricing at the top. This distortion is similar to distortions in typical signaling games (as for instance the job market signaling model).

We next characterize the Pooling Equilibrium. This equilibrium is based on the idea of pooling at the monopoly price that would prevail at $t = 0$, that is, supposing that all consumers have fixed beliefs $E[1/P_3]$ that $1/M_t = 1/M_0$, and that marginal costs are also proportional to M_0.

Proposition 5 (Pooling Equilibrium) *Set*

$$p_0 = \arg\max(p - kM_0)\mathcal{D}(p/M_0)$$

If

$$(p_0 - kM_{tS})\mathcal{D}(E_\mu[p_0/M]) > \max\ (p - kM_{tS})\mathcal{D}(p/M_{t1}) \tag{18}$$

then there is an $\alpha_2 \in (0,1)$ *such that for* $\alpha \in (0, \alpha_2]$ *then it is a pooling equilibrium for the firm to set the price* p_0 *in each state.*

Proof. We construct an equilibrium by imposing that off-equilibrium path beliefs are $E[1/P_3] = 1/M_1$. Inequality (18) ensures that the IC constraint is satisfied (and thus this is an equilibrium) when $\alpha = 0$ and the state is the highest. Pick a k such that (18) is satisfied. Now we need to check the IC constraints of all types for $\alpha > 0$:

$$\begin{aligned} &\alpha(p_0 - kM_{ts})\mathcal{D}(p_0/M_{ts}) + (1-\alpha)(p_0 - kM_{ts})\mathcal{D}(E_\mu[p_0/M]) \\ &\geq \alpha(p - kM_{ts})\mathcal{D}(p/M_{ts}) + (1-\alpha)(p - kM_{ts})\mathcal{D}(p_0/M_{t1}), \quad \forall p \end{aligned} \tag{19}$$

If (18) holds we have that

$$(p_0 - kM_{ts})\mathcal{D}(E_\mu[p_0/M]) > (p - kM_{ts})\mathcal{D}(p/M_1), \quad \forall p$$

and these constraints are strictly satisfied for $\alpha = 0$. Pick α_2 as the highest α so that all constraints are satisfied.

Consumers choose consumption according to their FOC (4). Then for $\alpha \in (0, \alpha_2]$ this is an equilibrium.

■

The intuition for prices in the Pooling Equilibrium is as follows. If the fraction of informed consumers is small, the asymmetry of information is strong enough that the firm is able to hide its information and post a price that is not contingent on the state. This leads to the derivation of cutoff α_2. For a fraction of informed consumers above the cutoff, the firm prefers to post the state-optimal price in at least one of the states of the world, and thus this pooling equilibrium is not sustainable and does not exist. Condition (19) quantifies how small costs have to be in order for the firm to be willing to play in this pooling equilibrium. Nominal costs are proportional to the state, and thus for high k it may be that costs rise by so much that the firm is not willing to pool. Condition (19) takes care of this problem.

We now prove Lemma 2.

Proof of Lemma 2. Define

$$\Pi^* = \max(p-k)\mathcal{D}(p)$$

and

$$p^* = \arg\max(p-k)\mathcal{D}(p)$$

The cutoff α_k is obtained by comparing expected real profits in both equilibria. We first show that these are increasing in α in the Separating Equilibrium. We then show that these are decreasing in α in the Pooling Equilibrium.

We first derive the cutoff in the case uninformed consumers have learnt nothing and $E_\mu[1/M_t|Pooling] = 1/M_0$. (This is the case, for example, of the economy at $t = 1$.) We derive the cutoff in the other case in Lemma 6 following (p. 36).

In the Separating Equilibrium these profits are

$$\Pi(\alpha) = \begin{cases} \sum_s Pr(s)(1/M_{ts})(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}), & \alpha \in [\alpha_1, 1] \\ \sum_s Pr(s)(1/M_{ts})(\hat{p}_{ts} - kM_{ts})\mathcal{D}(\hat{p}_{ts}/M_{ts}), & \alpha \in (0, \alpha_1) \end{cases}$$

where $\hat{p}_{ts}$ is either equal to p_{ts} or $\bar{p}_{ts}$, with $\hat{p}_{ts} = \bar{p}_{ts}$ for at least one s. This function is constant for $\alpha \in [\alpha_1, 1]$. From (17) it follows that $\bar{p}_{ts}$ is strictly decreasing in α, and thus over this range $\Pi(\alpha)$ is strictly increasing. Since

$$\lim_{\alpha \longrightarrow \alpha_1} \bar{p}_{ts} = p_{ts}$$

the function $\Pi(\alpha)$ is continuous.

In the Pooling Equilibrium these profits are

$$\Pi_0(\alpha) = \sum_s Pr(s)(1/M_{ts}) \left[\alpha(p_0 - kM_{ts})\mathcal{D}(p_0/M_{ts}) + (1-\alpha)(p_0 - kM_{ts})\mathcal{D}(p_0/M_0)\right]$$

Because

$$\sum_s Pr(s)(1/M_{ts})(p_0 - kM_{ts})\mathcal{D}(p_0/M_0) > \sum_s Pr(s)(1/M_{ts})(p_0 - kM_{ts})\mathcal{D}(p_0/M_{ts})$$

this function is strictly decreasing in α. Notice also that

$$\sum_s Pr(s)(1/M_{ts})(p_0 - kM_{ts})\mathcal{D}(p_0/M_0) = \sum_s Pr(s)(1/M_{ts})(p_s - kM_{ts})\mathcal{D}(p_{ts}/M_{ts}) = \Pi^*$$

and so these functions cross. Thus, there is a unique α_k at which

$$\Pi_0(\alpha_k) = \Pi(\alpha_k)$$

and such that $\Pi_0(\alpha_k) \leq \Pi(\alpha_k)$ for $\alpha \in (0, \alpha_k]$, and $\Pi_0(\alpha_k) > \Pi(\alpha_k)$ for $\alpha \in (\alpha_k, 1]$ as claimed.

We denote the cutoff α_k in this case α_{1k}.

■

We attempt now to provide further intuition into Lemma 2. Recall the discussion of α being a determinant of the cost of information transmission above. In the Separating Equilibrium expected real profits are increasing in α because the cost of information transmission goes down with the fraction of informed consumers. Pooling Equilibrium expected real profits are decreasing in α because posting the ex-ante optimal price is increasingly suboptimal the higher the fraction of informed. Lemma 2 trades-off these considerations for the firm, leading for given k to a unique cutoff α_k. Figure 2 illustrates this firm trade-off in the case of 2 states. On the left panel equilibrium we show prices in both equilibria. In the Separating Equilibrium, when $\alpha < \alpha_1$ the price of the high type is strictly higher than p^h. The price of the low type is constant and equal to the perfect information price. The lower α, the bigger the distortion. In the Pooling Equilibrium the price is constant. On the right panel we plot real expected ex-ante profits in this equilibrium. The plot shows that ex-ante profits are increasing in α, and reach Π^* when $\alpha \geq \alpha_1$. On the contrary expected profits are decreasing in the Pooling Equilibrium and reach the perfect information profits for $\alpha = 0$. Thus, there is a unique α_k where these profit functions cross.

The Separating Equilibrium reaches the perfect information profits for high α, because in that case prices and allocations are the same as under perfect information. We say one more word regarding the fact that the Pooling Equilibrium *also* reaches the same level of profits for $\alpha = 0$. The intuition comes from the definition of p_0. Since this price maximizes profits when beliefs are $1/M_0$, it is optimal when $\alpha = 0$. Using risk neutrality and averaging across states delivers that ex-ante profits are also maximum. Notice, the result holds even if nominal costs kM_{ts} vary across states. The reason is Assumption 2, which implies that expected real costs are k, and thus p_0 remains ex-ante optimal. In simpler terms, the intuition is that profit maximization does not require state contingency as long as there is lack of knowledge about the state among consumers.

We now derive the cutoffs of price adjustment for $t = 2$. These cutoffs are derived similarly as at $t = 1$ if $\alpha \in (0, \alpha_{1,hc}]$ and no uninformed consumer has leant anything.

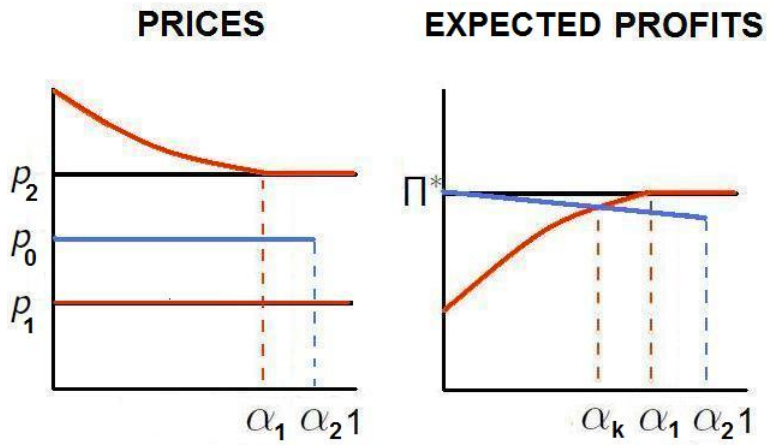

Figure 2: Equilibrium Prices (left) and Expected Profits (right) in the Separating and Pooling Equilibrium

However, when $\alpha \in (\alpha_{1,hc}, 1]$, κ high cost firms have adjusted prices at $t = 1$ and thus some uninformed consumers have learnt the state at $t = 1$.

Lemma 6 (Cutoffs when Uninformed Consumers Have Learnt) *Consider* $t = 2$. *For all* k, *there is a cutoff* α_{2k}, $\alpha_{2k} \in (0, 1)$ *such that*

- *if* $\alpha \in (0, \alpha_{2k}]$ *the Pooling Equilibrium is firm-best,*
- *if* $\alpha \in (\alpha_{2k}, 1]$, *the Separating Equilibrium is firm-best.*

Proof. In this case $E_\mu[1/M_t|Pooling] = \kappa(1/M_1) + (1-\kappa)(1/M_0)$.

Fix α and k. We will prove that so long as κ is small, the amount of learning by uninformed consumers at $t = 1$ is small enough so that the cutoff of price adjustment $\alpha_{2k} \in (0, 1)$.

Because when $\kappa = 0$, $\Pi_0(0) > \Pi(0)$, by continuity and monotonicity in κ there exists $\overline{\kappa} \in (0, 1)$ such that for $\kappa \in (0, \overline{\kappa}]$, $\Pi_0(0) > \Pi(0)$. Thus, again by continuity and monotonicity in α there is $\alpha_{2k} \in (0, 1)$ such that for $\alpha \in (0, \alpha_{2k}]$, $\Pi_0(\alpha) \geq \Pi(\alpha)$, as claimed.

■

Throughout the paper we assume that for $\alpha \in (\alpha_{1,hc}, \alpha_{1,lc}]$ and for low cost firms (and a fortiori for high cost firms) $\kappa \in (0, \overline{\kappa}]$. This ensures that prices are *still* sticky at $t = 2$ for $\alpha \in (0, \alpha_{2,lc}]$.

A.3 Numerical Exercises

In this section we present a couple of numerical results that complement our analysis.

First, we show that the cutoff of price adjustment in Section 2 α_k is a decreasing function of k. This exercise provides support to our assumption that $\alpha_{lc} > \alpha_{hc}$. To simplify our computations we have only performed these calculations in the case of 2 types. Figure 3 shows the results. (Besides plotting α_k as function of k, the figure also plots α_1 and α_2, showing that $\alpha_k \leq \alpha_1$ and $\alpha_k \leq \alpha_2$ and thus both the Separating Equilibrium and the Pooling Equilibrium exist at α_k.)

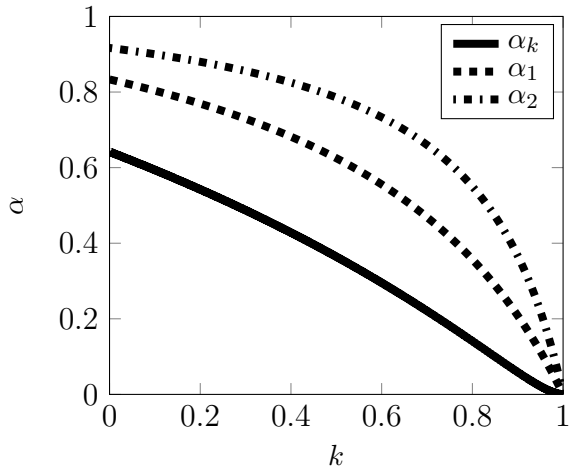

Figure 3: α_k, α_1, and α_2 as k varies ($M_1 = 1$, $M_2 = 1.2$)

Second, we show that the cutoff of price adjustment mentioned in Section 3 resulting from considering the Pooling Equilibrium and the separating equilibrium $\arg\max\ (1-\beta)(p_t - kM_t)\mathcal{D}(p_t/P_0) + \beta(p_t - kM_t)\mathcal{D}(p_t/M_t)$ is decreasing in β. This clarifies that an equivalent to Lemma 4 holds numerically under the consideration of this alternative separating equilibrium. To simplify our computations we have only performed these calculations in the case of 2 types. Figure 4 shows the results.

B Supplementary Proofs

B.1 Proof of Lemma 3

We first consider the Separating Equilibrium and then the Pooling Equilibrium.

- In the Separating Equilibrium, due to consumer learning, beliefs are $E_I[1/P_3] = E[1/M_t] = 1/M_t$.

  Consider the match between an informed consumer and a firm with marginal cost k. If $\alpha \in (\alpha_k, \alpha_1)$, the government imposes a tax on the firm to avoid

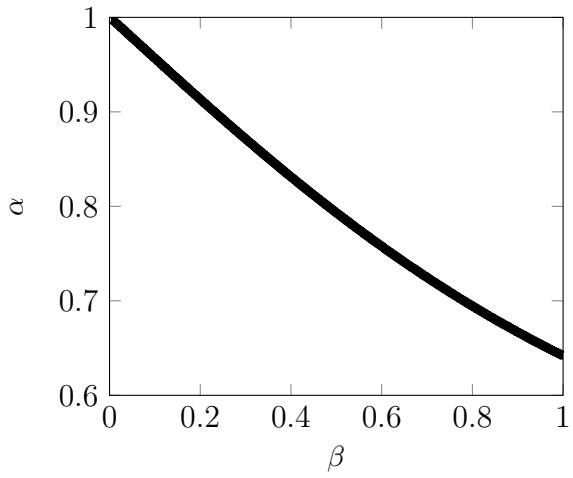

Figure 4: Cutoff for price adjustment in alternative separating equilibrium as β varies ($M_1 = 1$, $M_2 = 1.2$, $k = 0$)

distortions in the Separating Equilibrium. With marginal costs $k + tax$, $\alpha_1 \leq \alpha$, and the firm separates posting p_{ts}. There are no distortions and so $c_{2i} = \bar{c}_2$.

Otherwise, if $\alpha \in (\alpha_1, 1]$, $tax = 0$.

To conclude, all consumers get the allocation $c_{2i} = \bar{c}_2$ as claimed.

- In the Pooling Equilibrium, uninformed consumers that have learnt nothing (from firms' prices) at $t = 1$ have beliefs $E_\mu[1/M_2] = 1/M_0$. But $p_0/M_0 = p_{ts}/M_{ts}$, and therefore uninformed consumers get the allocation $c_{2i} = \bar{c}_2$.

  Uninformed consumers that learnt from prices have beliefs $E_\mu[1/M_2] = 1/M_1$. Thus, they will get an allocation $c_{2i} \neq \bar{c}_2$.

  Informed consumers have beliefs $E_I[1/M_2] = 1/M_2$. Thus, they will get an allocation $c_{2i} \neq \bar{c}_2$.

  All other consumers (the informed have beliefs $E_I[1/M_2] = 1/M_2$. Unless $M_2 = M_0$, informed consumers will get an allocation $c_{2i} \neq \bar{c}_2$, as claimed.

■

B.2 Proof of Proposition 1

If $\alpha \in (0, \alpha_{2,lc}]$, low cost (and high cost) firms do not adjust prices at $t = 2$. Informed consumers meeting low cost firms have welfare $E_0[u(\mathcal{D}(p_0/M_{2s}))] = \sum_s Pr(s)u(\mathcal{D}(p_0/M_{2s}))$. By Jensen's inequality we have

$$\sum_s Pr(s)u(\mathcal{D}(p_0/M_{2s})) < u\left(\sum_s Pr(s)\mathcal{D}(p_0/M_{2s})\right)$$

By the linearity of $\mathcal{D}(\cdot)$,

$$\begin{aligned} u\left(\sum_s Pr(s)\mathcal{D}(p_0/M_{2s})\right) &= u\left(\mathcal{D}\left(\sum_s Pr(s)(p_0/M_{2s})\right)\right) \\ &= u\left(\mathcal{D}\left(\sum_s Pr(s)(p_{ts}/M_{2s})\right)\right) = \overline{W} \end{aligned}$$

where the second equality follows from the definition of p_0. Thus, informed consumers meeting low cost firms suffer a welfare loss. Then, if $\alpha \in (0, \alpha_{2,lc}]$ welfare falls strictly below $\overline{W}$.

Also, if $\alpha \in (\alpha_{2,lc}, 1]$, by Lemma 3 all consumers get the allocation $\bar{c}_2$ and welfare is $\overline{W}$.

■

B.3 Proof of Lemma 4

Consider a firm with marginal costs kM. Consider expected profits in the Pooling Equilibrium

$$\begin{aligned} \Pi_0(\alpha, \beta) &= \sum_s Pr(s)(1/M_{ts}) \left[(1-\beta)(p_0 - kM_{ts})\mathcal{D}(p_0/M_0)\right. \\ &\quad \left. +\beta\left(\alpha(p_0 - kM_{ts})\mathcal{D}(p_0/M_{ts}) + (1-\alpha)(p_0 - kM_{ts})\mathcal{D}(p_0/M_0)\right)\right] \end{aligned}$$

If $\beta = 0$ and $\alpha = \alpha_k$, $\Pi_0(\alpha_k, 0) = \Pi^*$.

Now consider expected profits in the Separating Equilibrium

$$\Pi(\alpha, \beta) = \begin{cases} \sum_s Pr(s)(1/M_{ts}) \left[(1-\beta)\left((p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_0)\right)\right. \\ \qquad\qquad \left. +\quad \beta\left((p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_{ts})\right)\right], \qquad \alpha \in [\alpha_1, 1] \\ \sum_s Pr(s)(1/M_{ts}) \left[(1-\beta)\left((\hat{p}_{ts} - kM_{ts})\mathcal{D}(\hat{p}_{ts}/M_{ts})\right)\right. \\ \qquad\qquad \left. +\quad \beta\left((\hat{p}_{ts} - kM_{ts})\mathcal{D}(\hat{p}_{ts}/M_{ts})\right)\right], \qquad \alpha \in (0, \alpha_1) \end{cases}$$

If $\beta = 0$ and $\alpha = \alpha_1$, using the linearity of demand,

$$\begin{aligned}
\Pi(\alpha_1, 0) &= \sum_s Pr(s)(1/M_{ts})(p_{ts} - kM_{ts})\mathcal{D}(p_{ts}/M_0) \\
&= \sum_s Pr(s)(p^* - k)\mathcal{D}(p_{ts}/M_0) \\
&= (p^* - k)\mathcal{D}\left(\sum_s Pr(s)p_{ts}/M_0\right) \\
&= (p^* - k)\mathcal{D}\left(p^*\left(\sum_s Pr(s)M_{ts}/M_0\right)\right) < \Pi^*
\end{aligned}$$

where the last inequality follows from $\sum_s Pr(s)M_{ts} > [\sum_s Pr(s) \cdot 1/M_{ts}]^{-1} = M_0$.[14]

Thus, $\Pi(\alpha_1, 0) < \Pi_0(\alpha_k, 0)$, and since $\Pi(\alpha_k, 0) < \Pi(\alpha_1, 0)$, $\Pi_0(\alpha_k, 0) > \Pi(\alpha_k, 0)$. Thus, by continuity of these two ex-ante profit functions in β, there is $\overline{\beta} \in (0, 1)$ such that for $\beta \in (0, \overline{\beta})$, the cutoff of price adjustment has to satisfy $\alpha_k^\beta \in (\alpha_k, 1)$, for all k.

■

B.4 Proof of Theorem 2

By Lemma 4 we know that if $\beta \in (0, \overline{\beta}]$, $\alpha_{1k}^\beta \in (\alpha_{1k}, 1)$. Thus, $\alpha_{1,hc}^\beta \in (\alpha_{1,hc}, 1)$. As a result, when $\beta \in (0, \overline{\beta}]$, the interval over which the Phillips curve is flat is larger than when $\beta = 1$ (case of no inflation targeting), $(0, \alpha_{1,hc}^\beta] \supsetneq (0, \alpha_{1,hc}]$.

■

B.5 Proof of Theorem 3

For $\alpha \in (0, \alpha_{hc}^{\beta,1}]$ the CB does not learn the state and is unable to implement the optimal stabilization policy. Thus, welfare is strictly below $\overline{W}$. Because $\alpha_{1,hc}^\beta \in (\alpha_{1,hc}, 1]$, the set $(0, \alpha_{hc}^{\beta,1}]$ is a strict superset of $(0, \alpha_{1,hc}]$, i.e. $(0, \alpha_{1,hc}] \subset (0, \alpha_{hc}^{\beta,1}]$.

■

[14] It is a general property of averages that the harmonic average is strictly smaller than the arithmetic average.

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