Multistep prediction error decomposition in DSGE models: estimation and forecast performance

George Kapetanios, Simon Price and Konstantinos Theodoridis

Motivation

- Policy relevant forecasts commonly required at quarterly horizons from 1 - 12
- Policy *analysis* requires structural models - often DSGE models
- DSGE Models estimated by ML or Bayesian methods - in both cases using one-step prediction errors to build likelihood
- Might $h > 1$-step forecast errors
  - produce sensible estimates?
  - improve forecast performance?

Method

- State space representation of the linearised DSGE model

$$y_t = H\xi_t, \quad t = 1, .., T$$
$$\xi_t = C\xi_{t-1} + v_t.$$

- y_t observed variables, x_t unobserved vector of states that may be estimated using the Kalman filter to provide $\hat{\xi}_{t|t-1} = E(\xi_t|Y_{1,t-1})$ and $\hat{\xi}_{t|t} = E(\xi_t|Y_{1,t})$ where $Y_{s,t} = (y_s, ..., y_t)'$
- As parameter matrices H and C are unknown, can be conveniently estimated using ML based on the prediction error decomposition
- Normally uses a 1-step error to do so, but can equally use h-step where $h > 1$, thus using a vector of errors
- Might be interpreted as a MOM approach akin to cross-validation
- Precursor: Frank Schorfheide JoE 2005 *VAR forecasting under misspecification*, who specifies a loss function in terms of prediction errors

Model

- Use Smets and Wouters 2007
- Seven macro series - output, consumption and investment growth, inflation, wage growth, hours, interest rate
- Estimate by maximising likelihood (as in eg Ireland JEEA 2013)
- S&W 1966Q1-2004Q4: we use 1954Q3-1997Q4 to use earlier data while still retaining observations for forecast evaluation
- Despite shorter and overlapping sample, results similar to S&W

Global optimum?

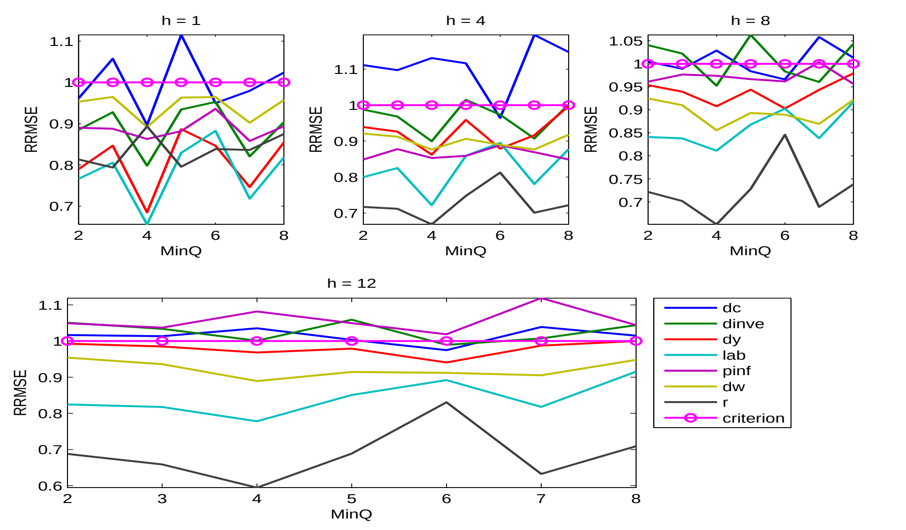
- Core results outcome of maxima from 100 starting points
- To check robustness, compared to results from only 10 starting points
- Most estimates similar
- Where there appear to be large differences, in fact not economically significant (as in insensitive parts of the parameter space)

Forecasts

- Sample 1954Q3-1997Q4 evaluated 1998Q1-2007Q4 to exclude crisis
- Sample 1954Q3-200Q4Q4 evaluated 2001Q1-2010Q2 to include crisis
- Recursive estimates starting 1954Q3-1997Q4 recursively evaluated 1998Q1-2010Q2

RMSFE 1998Q1-2007Q4

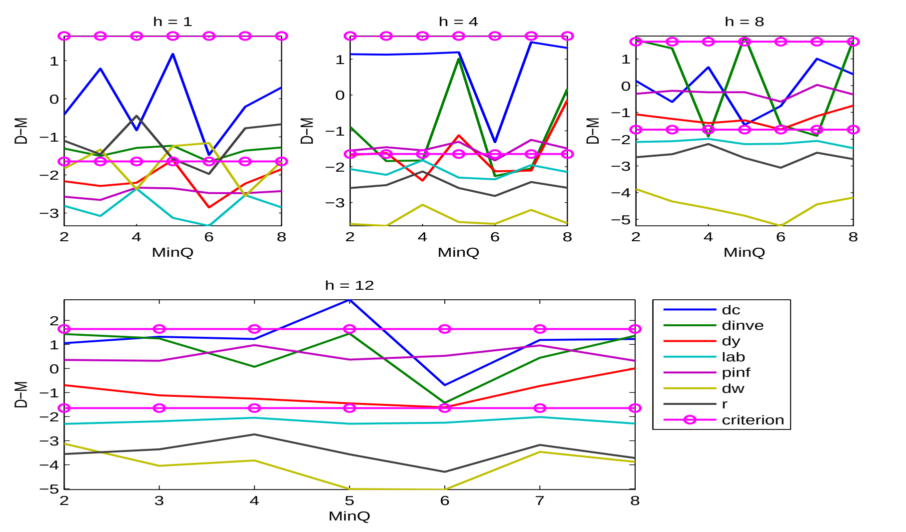
- Multi-step outperforms benchmark in most cases, some cases by large margins



- Might hypothesise that forecast performance at h optimised using h-step errors; but not so

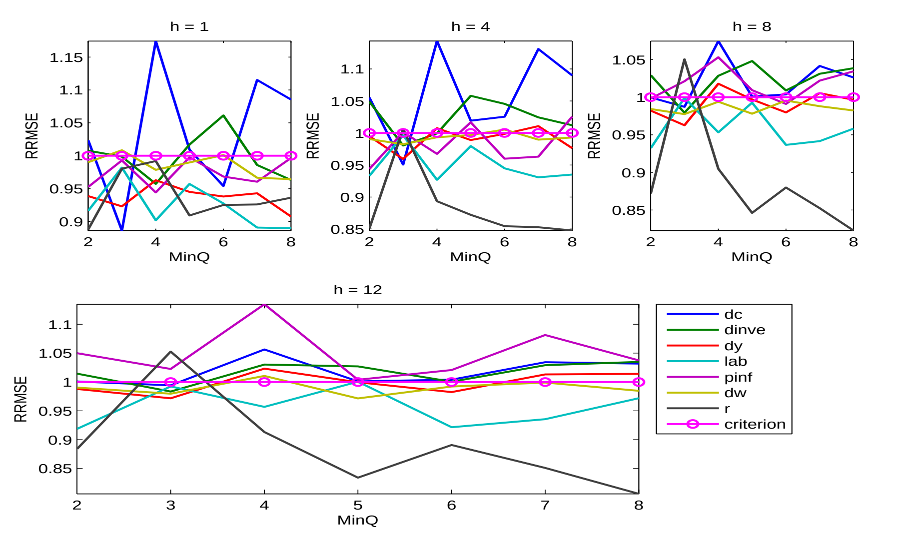
DM stats 1998Q1-2007Q4

- Also *significantly* better in many cases

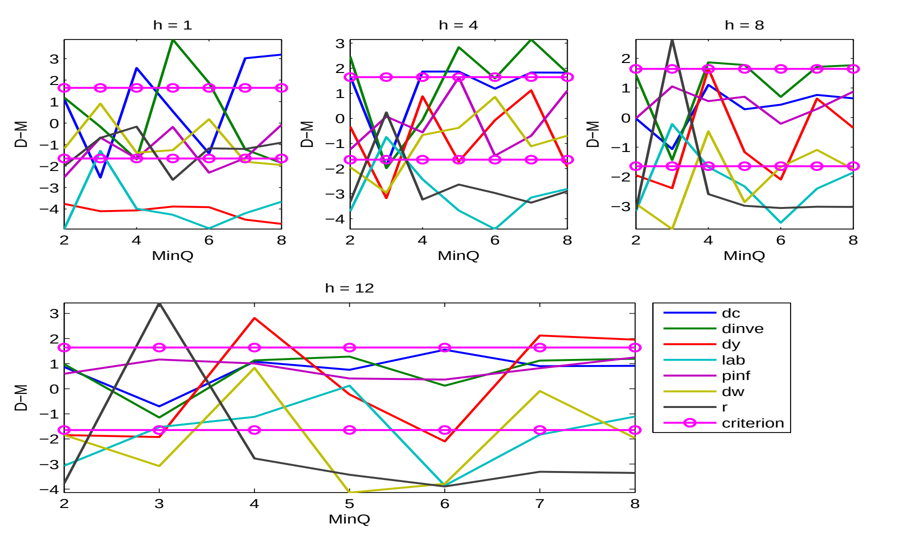


RMSFE 2001Q1-2010Q2

- Less good post-crisis but multi-step still better

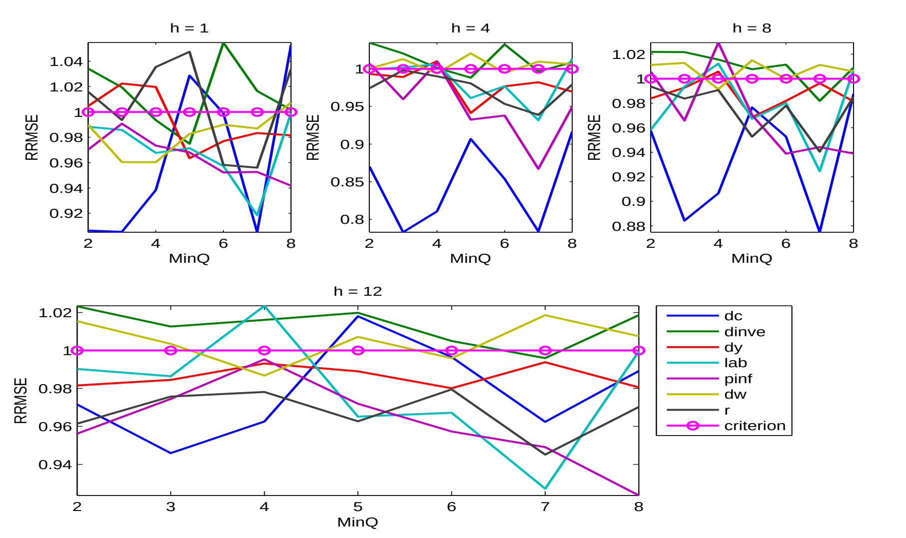


DM stats 2001Q1-2010Q2



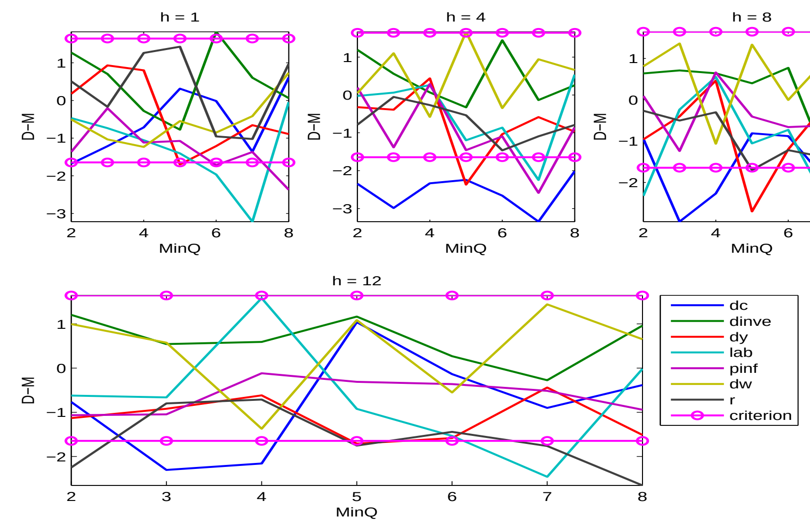
RMSFE rolling 1998Q1-2010Q2

- Again multi-step best performer



DM stats rolling 1998Q1-2010Q2

- Rather less significant cases



Why does this work?

- In general, introducing additional moment conditions
- As parameter estimates largely unchanged, we hypothesise improvement due partly to improved estimate of the state

Monte Carlo

- 200 data points are simulated from true DGP
- Add noise to the observed variables (noise to signal ratios of 0.5, 1.0 and 2.0)
- Estimate the state vector for $h = 1$, 4 and 8
- Calculate MSFE (between the actual state observations and the smoothed estimates) for last 50 periods and repeat 1000 times

Noise to Signal Ratio	Relative Mean Square Forecast Error	
	$\frac{MSFR(h=4)}{MSFR(h=1)}$	$\frac{MSFR(h=8)}{MSFR(h=1)}$
0.5	0.991	0.975
1.0	0.987	0.968
2.0	0.932	0.894

- As h rises estimates improve due to cross-equation restrictions
- As the noise to signal ratio increases these restrictions become more important
- Expanding evaluation period from 50 to 200 performance increases dramatically - evaluation over entire sample even more so
- The additional cross-equation restrictions particularly useful for estimation of initial values of state vector

Paper and code

- Published in *Economic Letters* as *A New Approach to Multi-Step Forecasting using Dynamic Stochastic General Equilibrium Models* - longer version in BoE WP series
- Code available from Kostas' site:

https://sites.google.com/site/konstantinostheodoridis/publications

Conclusions

- If concerned with forecasts as well as structure, may help to optimise over predictive power
- Turns out to improve forecasts with minimal changes to structure
- In most cases RRMSFE improved, in many cases significantly