Fiscal Unions*

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We study cross-country insurance for members of a currency union using an open economy model with nominal price and wage rigidities. We provide two results that build the case for the creation of a fiscal union within a currency union. First, we show that, if financial markets are incomplete, the value of gaining access to any given level of insurance is greater for countries that are members of a currency union. Second, we show that, even if financial markets are complete, private insurance is inefficiently low. A role emerges for government intervention in macro insurance to both guarantee its existence and to influence its operation. The efficient insurance arrangement can be implemented by contingent transfers within a fiscal union. The benefits of such a fiscal union are larger, the bigger the asymmetric shocks affecting the members of the currency union, the more persistent these shocks, and the less open the member economies.

1 Introduction

The ongoing crisis in the eurozone patently exposes the weakness of a currency union that is not also a fiscal union. Macroeconomic imbalances and the ineffectiveness of a single monetary policy to cope with asymmetric shocks across its members are coupled with liquidity and solvency problems in banks and sovereigns. Opinions oscillate between predicting the inevitable breakup of the monetary union, or suggesting its salvation by furthering the ties using some combination of a fiscal, banking and political union.

This paper tackles the design of a fiscal union within a currency union using a simple model. We show that a currency union creates macroeconomic externalities that call for

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insurance arrangements among members to provide transfers to countries in the most dire circumstances. Transfers help smooth consumption, the usual direct role of insurance, but this is not our focus. Instead, our first key observation is that under a fixed exchange rate, transfers also have an indirect effect: they promote spending and this helps mitigate recessions (or, in the other direction, curb booms). The social benefits from insurance are greater than what is appreciated by private economic agents, since they do not internalize these indirect macro stabilizing effects and only value the direct private consumption smoothing role. Indeed, even under ideal, complete-market conditions the equilibrium without intervention underinsures relative to the Pareto efficient level of insurance. A fiscal union can arrange for transfers among members to implement efficient insurance, going beyond the complete market solution to provide greater insurance.

Fiscal unions may mean different things to different people, so let us clarify the scope of our analysis at the outset. One perspective is that a fiscal union is needed to set rules for the division of seignorage (e.g. Casella and Feinstein, 1988; Aizenman, 1992) or, relatedly, that due to its budgetary effects, monetary and fiscal policy are inseparable (Sibert, 1992; Sims, 1999; Bottazzi and Manasse, 2002). Another perspective focuses on the role that the union's central bank may play as the lender of last resort to both sovereigns (e.g. De Grauwe, 2011) and banks (e.g. Goodhart, ed, 2000); the latter is sometimes referred to as a banking union. We believe that all these perspectives are important. Our contribution is to offer a different, more macroeconomic, perspective.

We focus on the fact that a common currency effectively fixes exchange rates and this constrains monetary policy, limiting its potential to stabilize asymmetric shocks across members. To single this effect out, we abstract from banks and sovereign financing issues. Despite this, we find an important role for fiscal arrangements within a monetary union. In this way our perspective is also closer to ideas from the original Optimal Currency Area literature (for the pioneering articles, see Mundell, 1961; McKinnon, 1963; Kenen, 1969) and lends support to the view originally formulated by Kenen (1969) that fiscal integration is an important condition for successful currency unions,

> "It is a chief function of fiscal policy, using both sides of the budget, to offset or compensate for regional differences, whether in earned income or in unemployment rates. The large-scale transfer payments built into fiscal systems are interregional, not just interpersonal [...]" (pg. 47)

Countries such as the United States, which can be thought as a currency and fiscal union of regions, share federal revenue and transfers—through the unemployment insurance program, federal income and social security taxes and, in extreme cases, direct federal assistance—in a manner that provides automatic stabilizers across regions. Our results also qualify a view often presented in the Optimal Currency Area literature that transfers and risk sharing through private financial markets are substitutes—both providing adequate buffers against asymmetric macroeconomic shocks in a currency union. For example, Mundell (1973) argues that a common currency could help improve risk sharing, by increasing cross holdings of assets or deepening financial markets.[1] While our model is silent on whether a currency union may facilitate the development of private insurance, it shows that the benefits of insurance are larger in a currency union and that government intervention is needed to reap the full benefits. Indeed, we establish that private risk sharing is not Pareto efficient in a currency union, so that financial integration alone is not sufficient.

These ideas come across robustly in a variety of standard open economy models. We begin our analysis with the simplest possible model: a static setting with a traded good, a non-traded good and labor as in Obstfeld and Rogoff (1995). We then extend the analysis to a standard dynamic model featuring non-trivial intra-temporal trade and price dynamics that builds on Gali and Monacelli (2005). The key friction in both settings is price or wage stickiness. Countries form a currency union that constrains their monetary policy response to asymmetric shocks. In this context, we set up the planning problem for efficient insurance transfers among members.

Our main result provides a case for government intervention in macro insurance markets even in the ideal situation where individuals can insure in frictionless complete asset markets. The argument is based on the observation that in the presence of nominal price or wage rigidities, financial wealth transfers affect spending, which in turn affects output and hence income or wealth—a mechanism first discussed in the famous Transfer Problem debate involving Keynes (1929) and Ohlin (1929). We show that under laissez-faire private agents do not purchase the efficient amount of insurance because they do not internalize these macroeconomic stabilization effects. Market insurance, or its lack thereof, is inefficient and tends to exacerbate booms and busts.

Importantly, we do not reach the same conclusion for countries outside a currency union, with flexible exchange rates. As long as they exercise their independent monetary policy optimally, no intervention is required in the financial market. Our argument for government involvement in macro insurance relies on membership in a currency union precisely because this constrains monetary policy and prevents stabilization of asymmetric shocks. Fiscal and monetary unions go hand in hand.

Our result rests on a key formula relating the social marginal utility of transfers to the

[1] For a recent textbook treatment and discussion of many of these ideas see De Grauwe (2012).

private marginal utility and the state of the economy,

$$\text{Social Marginal Utility} \;=\; \text{Private Marginal Utility} \;\times \left(1 + \frac{\text{Labor Wedge}}{\text{Openness}}\right).$$

Openness is measured precisely by the relative share of spending on foreign goods. The labor wedge represents a distortion in labor markets; it is positive when the economy is depressed. According to our formula, social and private marginal utilities diverge unless monetary policy is capable of perfect stabilization, defined precisely here as achieving the efficient level of labor in all states of the world. The discrepancy factor $\frac{\text{Labor Wedge}}{\text{Openness}}$ captures the macroeconomic externality calculation as follows. A positive transfer increases spending on domestically produced goods in proportion to the reciprocal of the degree of openness; this additional spending expands domestic output and labor, which enhances welfare if and only if the labor wedge is positive, since this signals that employment is too low and that expansions improve efficiency.

Currency unions buffeted by asymmetric shocks experience booms and busts that imply nonzero labor wedges, ensuring a discrepancy between social and private marginal utilities. However, our formula and conclusions are more general and apply to situations outside currency unions. Any constraint on monetary policy that induces labor wedges will create a discrepancy between social and private marginal utilities, creating a role for government intervention in ex ante financial decisions.

How can well-functioning financial markets lead to bad outcomes? Some economists have proposed models where "pecuniary externalities" are to blame (e.g. Geanakoplos and Polemarchakis, 1985; Caballero and Krishnamurthy, 2001; Bianchi and Mendoza, 2010; Jeanne and Korinek, 2010; Korinek, 2011; Bianchi, 2011). When markets are incomplete or when prices affect borrowing constraints, price-taking individuals will not internalize the effect that their collective financial decisions have on current and future prices, which, in turn, affect the financial possibilities of other individuals. Thus, in these models inefficiencies arise from price fluctuations and their interaction with borrowing constraints or incomplete markets. Note that the root of the inefficiency can be traced to the financial market itself and that the argument has nothing to do with currency unions. We propose a completely different mechanism, with inefficiencies arising from price *inflexibility*, instead of price variability. Moreover, the root of our inefficiency lies outside the financial market. Indeed, our results hold even if we assume that financial markets are complete and that borrowing constraints do not bind. The problem lies elsewhere, in the market for goods or labor, which suffers from price or wage stickiness.

The inefficiency of market insurance can be addressed by government intervention.

Indeed efficient outcomes can be implemented in a number of ways. If individuals do have access to private asset markets that are complete, then efficiency can be ensured by providing tax incentives that distort their individual portfolios choices. We provide a simple formula for the required tax system: the subsidy on the portfolio return in a particular state of the world equals the product of the labor wedge (a measure of the state of the business cycle) and the relative expenditure share of non-traded goods. A second possibility is for the government to take over macro insurance by assuming the necessary insurance positions in financial markets itself. Equivalently, instead of using financial markets, it can arrange ex ante for state contingent transfers or "bailouts" with other union members. In either case, it must then also take steps to ensure that the private sector does not undo these arrangements, by setting up the aforementioned tax incentive system or employing more extreme measures, such as banning private macro insurance.

We view the complete financial markets paradigm as a useful assumption to highlight that the inefficiency of private insurance that we derive does not arise from inefficiencies in financial markets. However, our preferred interpretation is that financial markets are incomplete, so that macro insurance markets are imperfect or nonexistent. This only strengthens the argument for building a fiscal union that creates insurance arrangements across members within a currency union.[2] Indeed, the efficient insurance arrangement can then be implemented through ex-post transfers or "bailouts" that are contingent on the shocks experienced by each country. Since agents have no access to macro insurance, no taxes or bans on private insurance are needed. Under this interpretation, our paper can be seen as offering a precise characterization of these ex-post transfers and clarifying that for members of a currency union: (i) the value of gaining access to insurance, for any given level of insurance, is greater; and (ii) transfers go beyond emulating the outcome that private risk sharing would reach if only asset markets were complete. These two points are distinct but complement each other to motivate the formation of fiscal unions within currency unions.

In particular, we emphasize three key determinants of the effectiveness of transfers as a stabilization tool in a currency union, which have received considerable attention in the Optimal Currency Area Literature: the asymmetry of the shocks hitting the members of the currency union, the persistence of these shocks (in the dynamic version of the model) and the openness of the member economies. Indeed, symmetric shocks can be accommodated with union wide monetary policy so that transfers should be used only in the face

[2] Atkeson and Bayoumi (1993) examine cross-regional insurance in the United States and conclude that "integrated capital markets are [...] unlikely to provide a substantial degree of insurance against regional economic fluctuations [...] This task will continue to be primarily the business of government."

asymmetric shocks. Efficient transfers are increasing in the persistence of the shocks, but hump-shaped as a function of openness. However a given transfer is more effective at stabilizing the economy when the economy is more closed. Hence more stabilization is achieved at the optimum both when the economy is more closed, and when shocks are more persistent. Indeed, we show that full stabilization is achieved in the limit as shocks become permanent and the economy becomes closed. This contrasts with the ideas in McKinnon (1963), who discusses reasons why openness may mitigate the costs of currency unions. However, our results are fully compatible with the notion that openness is beneficial in a currency union lacking a fiscal union because our results only apply when an optimal fiscal union is in place.

Interestingly, although there is a role for government at the national level, we find no need for coordination at the supranational level as long as countries cannot influence prices. The efficient risk sharing arrangement is obtained when each country manages its own insurance in a competitive international financial market—provided such markets are available, of course, otherwise, there is an obvious need to convene to create these markets or recreate them by arranging for transfers between members. Nevertheless, all these transactions or arrangements are mutually beneficial and no concerted effort is required to control individual members' insurance goals. Overall, macro insurance is not a policy tool that requires international coordination to internalize spillovers across countries. When countries are large, (or even when they are small, if they have monopoly power over the production of some traded goods), then this result is overturned and coordination is needed to prevent countries from engaging in terms of trade manipulation.

The rest of the paper is organized as follows. The static model is covered in Sections 2 and 3. The dynamic model is contained in Sections 4 and 5. Section 6 contains our conclusions.

2 A Static Model of a Currency Union

We start with a simple static model that illustrates our main idea most transparently. Later we show that the same effects are present in standard dynamic open economy models. The model's environment builds on the model with traded and non-traded goods presented in Obstfeld and Rogoff (1995). It features a traded good, a non-traded good and labor. The traded good is supplied inelastically and traded competitively. The non-traded good is supplied from labor by monopolistic firms. The prices set by these monopolistic firms are sticky.

We offer two market settings and associated policy interventions for the same model environment. The first assumes complete markets and features portfolio taxes as the policy instrument to influence equilibrium risk sharing across countries. The second assumes incomplete markets, so that private agents have no opportunities to share risk. In this case we focus on government arranged fiscal transfers across countries to provide international risk sharing. Importantly, we show that both settings lead to the same set of implementable allocations. This allows us to characterize efficient allocations using the same Ramsey planning problems for both settings in Section 3.

In our view, the first setting, while less realistic offers several conceptual advantages. First, it allows us to make the point that efficient allocations require government intervention *even if* financial markets are complete. By implication, if markets are incomplete, government intervention should not simply mimic the complete-market outcome. Second, we can provide simple formulas for the intervention in the form of portfolio taxes. The incomplete markets setting, on the other hand, seems more realistic and the implementation of efficient allocations involves cross country insurance through fiscal transfers, providing a foundation for fiscal unions. In any case, although we favor the incomplete-market setting and its implementation in practical terms, the characterization using complete markets sheds light on both.

2.1 Households

There is a single period and a continuum of countries indexed by $i \in [0,1]$. We start by assuming that all countries belong to a currency union, but will relax this later. Uncertainty affects preferences and technology: the state of the world $s \in S$ has density $\pi(s)$ and determines preferences and technology, possibly asymmetrically, in all countries.

In each country $i \in I$, there is a representative agent with preferences over non-traded goods, traded goods and labor given by the expected utility

$$\int U^i(C^i_{NT}(s), C^i_T(s), N^i(s); s)\pi(s)ds.$$

Below we make some further assumptions on preferences.

In the complete-market setting, agents can trade in a complete set of financial markets before the realization of the state of the world $s \in S$ (we discuss the incomplete market setting in subsection 2.5). Households are subject to the following budget constraints

$$\int D^i(s)Q(s)\pi(s)ds \leq 0, \tag{1}$$

$$P^i_{NT}C^i_{NT}(s) + P_T(s)C^i_T(s)$$
$$\leq W^i(s)N^i(s) + P_T(s)E^i_T(s) + \Pi^i(s) + T^i(s) + (1+\tau^i_D(s))D^i(s), \quad (2)$$

where P^i_{NT} is the price of non-traded goods which as we will see shortly, does not depend on s due to the assumed price stickiness; $P_T(s)$ is the price of traded goods in state s; $W^i(s)$ is the nominal wage in state s; $E^i_T(s)$ is country i's endowment of traded goods in state s; $\Pi^i(s)$ represents aggregate profits in state s; $T^i(s)$ is a lump sum rebate; $D^i(s)$ is the nominal payoff of the household portfolio in state s; $Q(s)$ is the price of one unit of currency in state s in world markets, normalized by the probability of state s; and $\tau^i_D(s)$ is a state contingent portfolio return subsidy.[3] The lump sum rebate $T^i(s)$ is used to rebate the proceeds from the tax on financial transactions to households. We sometimes also consider lump-sum transfers over and above such rebates to redistribute wealth across countries. Note that the nominal price of traded goods is assumed to be the same across countries, reflecting the law of one price and the fact that all countries in the union share the same currency.

The households' first order conditions can be written as

$$\frac{U^i_{C_T}(s)(1+\tau^i_D(s))}{Q(s)P_T(s)} = \frac{U^i_{C_T}(s')(1+\tau^i_D(s'))}{Q(s')P_T(s')}, \quad (3)$$

$$\frac{U^i_{C_T}(s)}{P_T(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}, \quad (4)$$

$$-\frac{U^i_N(s)}{W^i(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}. \quad (5)$$

2.2 Firms

We assume that the traded good is in inelastic supply: each country is endowed with a quantity $E^i_T(s)$ of traded goods. These goods are traded competitively in international markets.

Non-traded goods are produced in each country by competitive firms that combine a continuum of non-traded varieties indexed by $j \in [0,1]$ using the constant returns to scale CES technology

$$Y^i_{NT}(s) = \left(\int_0^1 Y^{i,j}_{NT}(s)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{1}{1-\frac{1}{\epsilon}}},$$

[3] Above we assumed that the returns from firms are not subsidized. Another possibility is to subsidize profits $\Pi^i(s)$ at the same rate $\tau^i_D(s)$ as financial returns. None of our analysis or conclusions are affected by this modeling choice.

with elasticity $\varepsilon > 1$.

Each variety is produced by a monopolist using a linear technology:

$$Y^{i,j}_{NT}(s) = A^i(s) N^{i,j}(s).$$

Each monopolist hires labor in a competitive market with wage $W^i(s)$, but pays $W^i(s)(1 + \tau^i_L)$ net of a country specific tax on labor. Monopolists must set prices in advance, at the beginning of the period, before the realization of uncertainty. The demand for each variety is given by $C^i_{NT}(s)(P^{i,j}_{NT}/P^i_{NT})^{-\varepsilon}$ where $P^i_{NT} = (\int (P^{i,j}_{NT})^{1-\varepsilon} dj)^{1/(1-\varepsilon)}$ is the price of non traded goods.

With complete markets (we discuss the incomplete markets case further below) they solve

$$\max_{P^{i,j}_{NT}} \int \frac{Q(s)}{1 + \tau^i_D(s)} \Pi^{i,j}(s) \pi(s) ds,$$

where

$$\Pi^{i,j}(s) = \left(P^{i,j}_{NT} - \frac{1 + \tau^i_L}{A^i(s)} W^i(s) \right) C^i_{NT}(s) \left(\frac{P^{i,j}_{NT}}{P^i_{NT}} \right)^{-\varepsilon}.$$

Aggregate profits are given by $\Pi^i(s) = \int \Pi^{i,j}(s) dj$. In a symmetric equilibrium, all monopolists in country i set the same profit maximizing price. Rearranging the first-order condition yields the familiar expression for the price as a markup over a weighted average across states of the marginal cost

$$P^i_{NT} = (1 + \tau^i_L) \frac{\varepsilon}{\varepsilon - 1} \frac{\int \frac{Q(s)}{1+\tau^i_D(s)} \frac{W^i(s)}{A^i(s)} C^i_{NT}(s) \pi(s) ds}{\int \frac{Q(s)}{1+\tau^i_D(s)} C^i_{NT}(s) \pi(s) ds}. \tag{6}$$

2.3 Government

The government is subject to the budget constraint

$$T^i(s) = \tau^i_L W^i(s) N^i(s) - \tau^i_D(s) D^i(s) + \hat{T}^i(s). \tag{7}$$

Here $\hat{T}^i(s)$ are net international fiscal transfers, satisfying

$$\int \hat{T}^i(s) di = 0, \tag{8}$$

for all $s \in S$, that redistributes resources across countries via the governments' budgets.

2.4 Equilibrium with Complete Markets

An equilibrium specifies quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$, bonds $\{D^i(s)\}$, prices and wages $\{P_T(s), P^i_{NT}, w^i(s), Q(s)\}$, taxes $\{\tau^i_L, \tau^i_D(s), T^i(s)\}$ and international fiscal transfers $\{\hat{T}^i(s)\}$ such that households and firms maximize, the government's budget constraint is satisfied, and markets clear[4]:

$$C^i_{NT}(s) = A^i(s)N^i(s), \tag{9}$$

$$\int C^i_T(s)di = \int E^i_T(s)di. \tag{10}$$

These conditions imply that the bond market is cleared, i.e. $\int D^i(s)di = 0$ for all $s \in S$.

The conditions for an equilibrium (1)–(10) act as constraints on the planning problem we study next in Section 3.[5] However, in a spirit similar to Lucas and Stokey (1983), we seek to drop variables and constraints as follows. Given quantities, equations (3), (5) and (6) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (1), (2), (3), (5), (6), (7), and (8). We summarize these arguments in the following proposition.

Proposition 1 (Implementability, Complete Markets). *An allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ form part of an equilibrium with complete markets if and only if equations (4) and (9) hold for all $i \in I, s \in S$ and equation (10) holds for all $s \in S$.*

Proof. We have already proved that the conditions in the proposition are necessary for an allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ to form part of an equilibrium with complete markets. We now need to establish these conditions are sufficient. The proof is constructive. Start with an allocation together with prices that satisfy these conditions. We choose wages $W^i(s)$ to satisfy the labor-leisure condition (5) for each $i \in I$ and $s \in S$. Given some set of state prices $Q(s)$, we pick portfolio taxes $\tau^i_D(s)$ to satisfy the risk sharing condition (3) for each $i \in I$ and $s \in S$. Note a first dimension of indeterminacy here: we can always multiply state prices $Q(s)$ and portfolio taxes $1 + \tau^i_D(s)$ by some arbitrary common function $\lambda(s)$ of s. We then pick labor taxes τ^i_L to satisfy the price setting equation (6). Finally, for a given set of ex-post fiscal transfers $\hat{T}^i(s)$ that satisfy the country budget constraint $\int Q(s)\hat{T}^i(s)\pi(s)ds = \int Q(s)\left[P_T(s)(C^i_T(s) - E^i_T(s))\right]\pi(s)$ and

[4]Our notation already takes into account the symmetry of prices, output and labor across varieties j within each country i.

[5]In addition, the budget constraints (1) and (2) must hold as an equality.

the condition that aggregate net international transfers are zero in every state (8), we compute transfers to households $T^i(s)$ using the government budget constraint (7). We can then compute the required portfolio positions $D^i(s)$ using the ex-post household budget constraint (2). These choices guarantee that the ex-ante household budget constraint (1) is verified. Note a second dimension of indeterminacy, as we have some degree of freedom in choosing ex-post fiscal transfers $\hat{T}^i(s)$. □

Importantly, we cannot dispense with equation (4). This equation summarizes the restriction imposed by a currency union, that the price of traded goods cannot vary across countries, and price stickiness, that the price of non-traded goods cannot vary across states of the world. Consider attempting to use equation (4) as a residual to back out prices that support an allocation, as we did with equations (3), (5) and (6). Equation (4) requires that the relative price of traded to non-traded goods equal $U^i_{C_T}(s)/U^i_{C_{NT}}(s)$. For any arbitrary allocation, this required relative price can be computed, but the problem is that it may not be possible to express it as a ratio of a price that is independent of i and a price that is independent of s, i.e. as a ratio $P_T(s)/P^i_{NT}$. This is why we must keep equation (4) as a constraint.

Our constructive proof shows that an allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ that satisfy the conditions in the propositions are actually part of several equilibria. We have emphasized two dimensions of indeterminacy. First, we can choose any set of state prices $Q(s)$. Second, we can choose different ex-post fiscal transfers $\hat{T}^i(s)$. These two dimensions are actually related in the sense that different state prices require different ex-post fiscal transfers.

The first dimension of indeterminacy can be intuitively understood as follows. The relevant state prices for households are adjusted for portfolio taxes $\frac{Q(s)}{1+\tau^i_D(s)}$. Scaling up state prices $Q(s)$ and the corresponding portfolio taxes $1+\tau^i_D(s)$ by a function $\lambda(s)$ leaves these tax-adjusted state prices unchanged. However this change indirectly transferring resources across countries and states. These indirect transfers need to be compensated by adjusting ex-post fiscal transfers $\hat{T}^i(s)$.

The second dimension of indeterminacy can be intuitively understood as follows. How much transfers across countries actually operate through financial markets $D^i(s)$ or ex-post fiscal transfers $\hat{T}^i(s)$ is not pinned down. For example, one possibility is to constrain ex-post fiscal transfers to be non-state contingent $\hat{T}^i(s) = \hat{T}^i$. All the insurance is then being delivered through financial markets, and portfolio taxes are required to make sure that private agents secure the right amount of insurance $D^i(s)$. Another possibility is to set $\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i_T(s))$. In that case, all the insurance is being delivered through ex-post fiscal transfers. Portfolio taxes are then required to ensure that private agents do not "undo" these transfers and indeed choose $D^i(s) = 0$.

2.5 Equilibrium with Incomplete Markets

We also consider an alternative setup where markets are incomplete, in the sense that there are no financial markets before the realization of the state of the world $s \in S$. We split the representative agent in country i into a continuum of households $j \in [0,1]$. Household j is assumed to own the firm of variety j. Households j maximizes utility

$$\int U^i(C^i_{NT}(s), C^i_T(s), N^i(s); s)\pi(s)ds,$$

by choosing $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ and the prices set by its own firm $P^{i,j}_{NT}$, taking aggregate prices and wages $\{P_T(s), P^i_{NT}, W^i(s)\}$ and aggregate demand $\{\bar{C}^i_{NT}(s)\}$ as given, subject to

$$P^i_{NT}C^i_{NT}(s) + P_T(s)C^i_T(s) \leq W^i(s)N^i(s) + P_T(s)E^i_T(s) + \Pi^{i,j}(s) + T^i(s), \tag{11}$$

where

$$\Pi^{i,j}(s) = \left(P^{i,j}_{NT} - \frac{1+\tau^i_L}{A^i(s)}W^i(s)\right)\bar{C}^i_{NT}(s)\left(\frac{P^{i,j}_{NT}}{P^i_{NT}}\right)^{-\varepsilon}.$$

The corresponding first-order conditions are symmetric across j and given by (4) and (5) and the price setting condition

$$P^i_{NT} = (1+\tau^i_L)\frac{\varepsilon}{\varepsilon-1}\frac{\int \frac{U^i_{C_T}(s)}{P_T(s)}\frac{W^i(s)}{A^i(s)}\bar{C}^i_{NT}(s)\pi(s)ds}{\int \frac{U^i_{C_T}(s)}{P_T(s)}\bar{C}^i_{NT}(s)\pi(s)ds}. \tag{12}$$

Of course, in equilibrium we impose the consistency condition that $\bar{C}^i_{NT}(s) = C^i_{NT}(s)$ for all i and s.

The government budget constraint simplifies to

$$T^i(s) = \tau^i_L W^i(s)N^i(s) + \hat{T}^i(s). \tag{13}$$

We can now define an equilibrium with incomplete markets. An equilibrium specifies quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$, prices and wages $\{P_T(s), P^i_{NT}, w^i(s)\}$, taxes $\{\tau^i_L, T^i(s)\}$ and international fiscal transfers $\{\hat{T}^i(s)\}$ such that households and firms maximize, the government's budget constraint is satisfied, and markets clear. More formally, the conditions for an equilibrium are given by (4), (5), (8), (11) holding with equality, (12) with $\bar{C}^i(s) = C^i(s)$, and (13).

As in the complete markets implementation, we can drop variables and constraints as follows. Given quantities, equations (5) and (12) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (5), (8), (11), (12), and (13). We summarize these arguments in the following proposition.

Proposition 2 (Implementability, Incomplete Markets)**.** *An allocation* $\{C_T^i(s), C_{NT}^i(s), N^i(s)\}$ *together with prices* $\{P_T(s), P_{NT}^i\}$ *form part of an equilibrium with incomplete markets if and only if equations* (4) *and* (9) *hold for all* $i \in I, s \in S$ *and equation* (10) *holds for all* $s \in S$*.*

Proof. We have already proved that the conditions in the proposition are necessary for an allocation $\{C_T^i(s), C_{NT}^i(s), N^i(s)\}$ together with prices $\{P_T(s), P_{NT}^i\}$ to form part of an equilibrium with complete markets. We now need to establish these conditions are sufficient. The proof is constructive. Start with an allocation together with prices that satisfy these conditions. We choose wages $W^i(s)$ to satisfy the labor-leisure condition (5) for each $i \in I$ and $s \in S$. We then pick labor taxes τ_L^i to satisfy the prices setting equation (12). We choose transfers $T^i(s)$ to satisfy the household budget constraint (11). We then choose ex-post fiscal transfers $\hat{T}^i(s)$ to satisfy the government budget constraint (13). We can verify that these choices satisfy (8). □

Propositions 1 and 2 reach the same implementability conditions for the complete- and incomplete-market settings. Of course, although the set of implementable quantities $\{C_T^i(s), C_{NT}^i(s), N^i(s)\}$ and prices $\{P_T(s), P_{NT}^i\}$ is the same, the required policy instruments are different.

Under complete markets, portfolio taxes $\{\tau_D^i(s)\}$ are needed, and international transfers $\{\hat{T}^i(s)\}$ are largely indeterminate. This can easily be seen by starting with the household's budget constraint, holding with equality, and substituting out profits $\Pi^i(s)$ and transfers $T^i(s)$ to arrive at the following country budget constraint

$$\int Q(s)\left[P_T(s)(C_T^i(s) - E_T^i(s))\right]\pi(s) = \int Q(s)\hat{T}^i(s)\pi(s)ds,$$

which states that the value of the trade balance must be covered by the value of international fiscal transfers. Indeed, this is the only constraint on fiscal transfers, any $\{\hat{T}^i(s)\}$ satisfying this equation helps implements an equilibrium. One simple case is to assume that transfers that are not state contingent, making $\hat{T}^i(s)$ independent of s for all i.

In contrast, in the incomplete market setting no restriction on private portfolios are introduced since no assets are available to private agents. In this case, the international transfers $\{\hat{T}^i(s)\}$ are uniquely determined and are typically state contingent.

2.6 Homothetic Preferences

Next, we characterize this key condition (4) further by making some weak assumptions on preferences. We make two assumptions on preferences: (i) preferences over consumption goods are weakly separable from labor; and (ii) the preference over consumption goods are homothetic. Denoting by $p^i(s) = \frac{P_T(s)}{P^i_{NT}}$ the relative price of traded goods in state s in country i, these assumptions imply that

$$C^i_{NT}(s) = \alpha^i(p^i(s); s) C^i_T(s),$$

for some function $\alpha^i(p; s)$ that is increasing and differentiable in its first argument. This conveniently encapsulates the restriction implied by the first order condition (4). This condition is crucial because the stickiness of non-traded prices, together with the lack of monetary independence, places restrictions on the possible variability across $i \in I$, for any state of the world s, in the relative price $p^i(s)$.

3 Efficient Macro Insurance in the Static Model

Define the indirect utility function

$$V^i(C_T, p; s) \equiv U^i\left(\alpha^i(p; s)C_T, C_T, \frac{\alpha^i(p; s)}{A^i(s)}C_T; s\right).$$

In an equilibrium with $C^i_T(s)$ and $p^i(s)$, ex post welfare in state s in country i is then given by

$$V^i(C^i_T(s), p^i(s); s).$$

The derivatives of the indirect utility function will prove useful for our analysis. To describe these derivatives, it is useful to first introduce the labor wedge[6]

$$\tau^i(s) \equiv 1 + \frac{1}{A^i(s)} \frac{U^i_N(s)}{U^i_{C_{NT}}(s)}.$$

[6] In this and other expressions and functions we streamline the notation by leaving the dependence on some of the arguments implicit.

The labor wedge is zero at a first-best allocation.

Proposition 3. *The derivatives of the value function are*

$$V^i_p(C^i_T(s), p^i(s); s) = \frac{\alpha^i_p(s)}{p^i(s)} C^i_T(s) U^i_{C_T}(s)\, \tau^i(s),$$

$$V^i_{C_T}(C^i_T(s), p^i(s); s) = U^i_{C_T}(s) \left(1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s)\right).$$

These observations about the derivatives and their connection to the labor wedge will be key to our results. A private agent values a transfer in traded goods according to its marginal utility $U^i_{C_T}(s)$, but the actual marginal value in equilibrium is $V^i_{C_T}(s)$. The wedge between the two equals $\frac{\alpha^i(s)}{p^i(s)}\tau^i(s) = \frac{P^i_{NT} C_{NT}(s)}{P_T(s) C_T(s)}\tau^i(s)$, the labor wedge weighted by the relative expenditure share of non-traded goods relative to traded goods. We will sometimes refer to it as the *weighted labor wedge* for short.

In particular, a private agent undervalues transfers $V^i_{C_T}(s) > U^i_{C_T}(s)$ whenever the economy is experiencing a recession, in the sense of having a positive labor wedge $\tau^i(s) > 0$. Conversely, private agents overvalue the costs of making transfers $V^i_{C_T}(s) < U^i_{C_T}(s)$ whenever the economy is booming, in the sense of having a negative labor wedge $\tau^i(s) < 0$. These effects are magnified when the economy is relatively closed, so that the relative expenditure share of non-traded goods is large.

When country i receives a transfer, its consumers feel richer and increase their spending on both traded and non-traded goods in equal proportions. Since prices are fixed, the resulting increased demand for non-traded goods translates one-for-one into an increase in output. This in turn generates more income, further raising spending etc. This mechanism is at the core of the famous Transfer Problem controversy between Keynes (1929) and Ohlin (1929). These equilibrium effects, which are not internalized by private agents, open up a wedge between the social and private marginal values of transfers.

Since the increase in demand for both goods is proportional, the "dollar-for-dollar" output multiplier of transfers is precisely given by the relative expenditure share of non-traded to traded goods $\frac{P^i_{NT} C^i_{NT}(s)}{P_T(s) C^i_T(s)}$. The labor wedge $\tau^i(s)$ summarizes the net calculation for utility of the increase in non-traded consumption and the increase in labor that accompany the increase in output. This explains why the wedge between the social and private marginal valuations is precisely $\frac{P^i_{NT} C^i_{NT}(s)}{P_T(s) C^i_T(s)}\tau^i(s)$.

It is theoretically possible for the marginal value of a transfer to be negative $V^i_{C_T}(s) < 0$ if the labor wedge is sufficiently negative, especially if the share of non traded goods, relative to traded goods, is large enough. In this extreme case a country can improve welfare by making gift transfers, without any counterpart transfer in the opposite direction.

Corollary 1. *If τ^i is sufficiently negative then unilateral gift transfers to other countries are welfare enhancing for country i.*

This extreme case will not be our focus and is not employed in any of our results below. However, it is a stark example of just how divergent public and private valuations of transfers can become.

3.1 Ramsey Planning Problem

We consider a planning problem that allows us to characterize constrained Pareto efficient allocations.

Constrained Pareto efficient allocations. The planning problem is indexed by a set of nonnegative Pareto weights λ^i. By varying these Pareto weights, we can trace out the entire constrained Pareto frontier. The planning problem is

$$\max_{P_T(s), P^i_{NT}, C^i_T(s)} \int \int V^i \left(C^i_T(s), \frac{P_T(s)}{P^i_{NT}}; s \right) \lambda^i \pi(s) \, di \, ds \tag{14}$$

subject to

$$\int C^i_T(s) di = \int E^i_T(s) di.$$

Let $\mu(s)\pi(s)$ be the multiplier on the resource constraint in state $s \in S$. The first order conditions for $C^i_T(s)$, $P_T(s)$ and P^i_{NT} are, respectively,

$$V^i_{C_T}(s)\lambda^i = \mu(s),$$

$$\int V^i_p(s) \frac{1}{P^i_{NT}} \lambda^i di = 0,$$

$$\int V^i_p(s) p^i(s) \pi(s) ds = 0.$$

These first-order conditions tightly characterize the solution. The first order condition for P^i_{NT} implies our first proposition.

Proposition 4 (Optimal Price Setting). *At a constrained Pareto efficient equilibrium, for every country i, a weighted average of labor wedges across states is zero:*

$$\int \alpha^i_p(s) \, C^i_T(s) \, U^i_{C_T}(s) \, \tau^i(s) \, \pi(s) \, ds = 0.$$

In the absence of uncertainty this proposition implies a zero labor wedge $\tau^i(s) = 0$, obtained by setting the labor tax to cancel the monopolistic markup: $\tau_L^i = -1/\varepsilon$. With uncertainty, in general $\tau_L^i \neq -1/\varepsilon$ and the labor wedge takes on both signs with a weighted average of zero.[7]

The first-order condition for $P_T(s)$ implies the following proposition.

Proposition 5 (Optimal Monetary Policy)**.** *At a constrained Pareto efficient equilibrium, in every state s, a weighted average of labor wedges across countries is zero:*

$$\int \alpha_p^i(s) C_T^i(s) U_{C_T}^i(s)\, \tau^i(s) \lambda^i di = 0.$$

This proposition establishes that optimal monetary policy targets a weighted average across countries for the labor wedge. It sets this target to zero in each state of the world. The intuition for the result is that monetary policy can be chosen at the union level, and can adapt across states to the average condition. If all countries are identical and the shock is symmetric, then we obtain perfect stabilization in each country: $\tau^i(s) = 0$ for all $i \in I, s \in S$. By contrast, when shocks across countries are not symmetric then perfect stabilization is impossible. However, at the union level the economy is stabilized in the sense that the weighted average for the labor wedge across countries is set to zero for all states of the world $s \in S$.[8]

Finally, the first order condition for $C_T(s)$ says that the marginal utility of transfers in traded goods adjusted for the Pareto weight $\lambda^i V_{C_T}^i(s)$ should be equalized across countries for every state s. It is more revealing to rewrite this condition using our expressions for the derivative of $V_{C_T}^i(s)$.

Proposition 6 (Optimal Risk Sharing)**.** *For every pair of states (s, s'), and pair of countries (i, i'), optimal risk sharing takes the following form:*

$$\frac{U_{C_T}^i(s)\left(1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)\right)}{U_{C_T}^i(s')\left(1 + \frac{\alpha^i(s')}{p^i(s')}\tau^i(s')\right)} = \frac{U_{C_T}^{i'}(s)\left(1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)}\tau^{i'}(s)\right)}{U_{C_T}^{i'}(s')\left(1 + \frac{\alpha^{i'}(s')}{p^{i'}(s')}\tau^{i'}(s')\right)}. \tag{15}$$

If portfolio taxes are not employed, then the risk sharing condition (3) imposes the

[7] When the sub-utility function between C_{NT} and C_T is a CES so that $\alpha(\cdot; s)$ has constant elasticity, independent of s, then $\tau_L^i = -1/\varepsilon$ is optimal even with uncertainty. The proof is contained in the appendix.

[8] The result is related to the result in Benigno (2004) and Gali and Monacelli (2008) that optimal monetary policy in a currency union ensures that the union average output gap, in a linearized version of the model, is zero in every period. Here the result is obtained without linearizing the model and it is expressed in terms of the labor wedge, instead of the output gap.

additional constraint that for every pair of states (s, s'), and pair of countries (i, i'),

$$\frac{U^i_{C_T}(s)}{U^i_{C_T}(s')} = \frac{U^{i'}_{C_T}(s)}{U^{i'}_{C_T}(s')}. \tag{16}$$

Comparing these conditions, one may expect the private risk sharing condition (16) to be incompatible with the efficiency condition (15) except in special cases. Indeed, we next show that because labor wedges must average to zero across states and countries according to Propositions 4 and 5, they are indeed incompatible unless the first best is attainable. This implies that equilibria with privately optimal risk sharing (without portfolio taxes) are constrained Pareto inefficient.

Proposition 7 (Inefficiency of Private Risk Sharing)**.** *An equilibrium with complete markets and no portfolio taxes ($\tau^i_D(s) = 0$ for all $i \in I, s \in S$) is constrained Pareto inefficient unless $\tau^i(s) = 0$ for all $i \in I, s \in S$, in which case it is first best.*

Proof. Consider an equilibrium such that $\tau^i(s) \neq 0$ for some $i \in I, s \in S$. Assume, towards a contradiction, that the allocation is constrained Pareto efficient.

We consider two cases in turn. First, suppose that $V^i_{C_T}(s) = U^i_{C_T}(s)(1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)) < 0$ for some set $\Omega \subset I \times S$ of positive measure of countries and states. Define the sections $\Omega(s) = \{i : (i, s) \in \Omega\}$. Then there exists a perturbation that for each $s \in S$: (a) lowers $C^i_T(s)$ for $i \in \Omega(s)$ and improves welfare $V^i(s)$; (b) increases $C^i_T(s)$ for $i \notin \Omega(s)$ and improves welfare $V^i(s)$; and (c) satisfies the resource constraint $\int C^i_T(s)di = \int E^i_T(s)di$. This perturbation is feasible and creates a Pareto improvement, a contradiction.

Next, consider the case where $1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s) \geq 0$ for all $i \in I, s \in S$. For each state s consider ranking countries by their weighted labor wedge $\frac{\alpha^i(s)}{p^i(s)}\tau^i(s)$. By Proposition 6 it must be that

$$\frac{1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)}{1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)}\tau^{i'}(s)} = \frac{1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s')}{1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)}\tau^{i'}(s')}$$

for all i, i', s and s'. This implies that the ranking must be the same in all states s. It follows that there is a country i^* that is at top of the ranking for all states s, i.e. $i^* \in \cap_{s \in S} \arg\max_{i \in I} \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)$. Proposition 5 then implies that this country has a positive labor wedge: $\tau^{i^*}(s) \geq 0$ for all s. Proposition 4 then implies that $\tau^{i^*}(s) = 0$ for all s. Therefore we have that $\tau^i(s) \leq 0$ for all $i \in I, s \in S$. Proposition 5 then implies that actually $\tau^i(s) = 0$ for all $i \in I, s \in S$. □

Under laissez-faire, private agents do not purchase the optimal amount of macro-insurance. They do not fully internalize the macroeconomic stability consequences of their portfolio decisions, opening a role for government intervention in macro-insurance markets.[9] Government intervention secures additional transfers from low weighted labor wedge countries ("boom" countries) to high weighted labor wedge countries ("bust" countries). This reduces the demand for non-traded goods in the boom countries and increases it in the bust countries, stabilizing output and income. These stabilization benefits are not internalized by private agents, hence the need for government intervention.

3.2 Implementation

We now turn to the implementation of constrained Pareto efficient allocations. With complete markets, constrained Pareto efficient equilibria can be decentralized with appropriate labor taxes τ_L^i and corrective portfolio taxes $\tau_D^i(s)$. Proposition 6 leads to a neat characterization of the required taxes.

Proposition 8 (Complete Markets and Portfolio Taxes)**.** *If private asset markets are complete, constrained Pareto efficient allocations can be implemented by subsidized private insurance with the portfolio return subsidy rates given by the formula*

$$\tau_D^i(s) = \frac{\alpha^i(s)}{p^i(s)}\tau^i(s).$$

Insurance for bad states of the world, where the weighted labor wedge is high, should be relatively subsidized. It is interesting to note that the taxes do not depend directly on the Pareto weights $\{\lambda^i\}$, but only indirectly through the relative expenditure share of non-traded goods and the labor wedge. This underscores the fact that they are imposed to correct a macroeconomic externality and not to redistribute. As we move along the constrained Pareto efficient frontier by varying Pareto weights $\{\lambda^i\}$, the net present value of transfers to each country varies according to

$$\int U^i_{C_T}(s)(1+\tau_D^i(s))\frac{\hat{T}^i(s)}{P_T(s)}\pi(s)ds = \int U^i_{C_T}(s)(1+\tau_D^i(s))(C_T^i(s)-E_T^i(s))\pi(s)ds.$$

When markets are complete, how much transfers across countries actually operate through financial markets or ex-post fiscal transfers is indeterminate. For example, one possibility is to constrain ex-post fiscal transfers to be non-state contingent $\hat{T}^i(s) = \hat{T}^i$

[9] We should also point out that the Propositions 5 and 6 go through if non-traded goods prices are entirely predetermined (i.e. are exogenously fixed).

where $\hat{T}^i = \frac{\int U^i_{C_T}(s)(1+\tau^i_D(s))(C^i_T(s)-E^i_T(s))\pi(s)ds}{\int U^i_{C_T}(s)(1+\tau^i_D(s))\frac{1}{P_T(s)}\pi(s)ds}$. In this case all the insurance is being delivered through financial markets, and portfolio taxes are required to make sure that private agents secure the right amount of insurance. Another possibility is to set $\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i_T(s))$. In this case, all the insurance is being delivered through ex-post fiscal transfers, and portfolio taxes are required to ensure that agents do not "undo" this insurance.

The implementation of the socially optimum with corrective portfolio taxes is only one interesting possibility. Another equally interesting interpretation of our results assumes that private asset markets are nonexistent, so that private opportunities for risk sharing are unavailable. The optimum can then be implemented through ex-post transfers contingent on the shocks experienced by each country.

Proposition 9 (Incomplete Markets and Ex-Post Transfers)**.** *If private asset markets are incomplete so that state contingent-assets are unavailable, constrained Pareto efficient allocations can also be implemented through ex-post transfers contingent on the shock experienced by each country*

$$\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i(s)).$$

Under this alternative implementation, no restriction on private portfolios are needed since no assets are available to private agents. Our results can then be seen as offering a precise characterization of the required ex-post transfers. A key conclusion of our analysis is that these transfers would go beyond replicating the outcome that private risk sharing decisions would achieve if markets were complete.

It is also possible to imagine implementations that are in between the two polar cases of corrective portfolio taxes with complete markets and ex-post transfers with incomplete markets. In general, government positions in asset markets, or ex-post transfers contingent on the shocks experienced by each country, combined with some restrictions or tax incentives on agents private portfolios are required.

3.3 Countries outside the currency union

Up to this point we have assumed that all countries belong to the currency union. Now, imagine that only a subset of countries $I \subseteq [0,1]$ are members. The rest manage monetary policy independently as follows. Country $i \notin I$ sets its own local nominal price for the traded good $P^i_T(s) = E^i(s)P_T(s)$ in its home currency by manipulating the level of its exchange rate $E^i(s)$ against the union's currency.[10] The planning problem becomes

$$\max \int_{i\in I} V^i \left(C_T^i(s), \frac{P_T(s)}{P_{NT}^i}; s \right) \lambda^i di + \int_{i\notin I} V^i \left(C_T^i(s), \frac{P_T^i(s)}{P_{NT}^i}; s \right) \lambda^i di \tag{17}$$

subject to

$$\int C_T^i(s) di = \int E_T^i(s) di.$$

For a country $i \notin I$ outside the union, the first order condition for $P_T^i(s)$ is

$$V_p^i(C_T^i(s), p^i(s); s) = \frac{\alpha_p^i(s)}{p^i(s)} C_T^i(s) U_{C_T}^i(s)\, \tau^i(s) = 0.$$

By implication

$$\tau^i(s) = 0 \qquad \text{for all } s \in S,\ i \notin I.$$

A flexible exchange rate leads to perfect stabilization, in the sense that the labor wedge is set to zero for all states of the world. This result is reminiscent of the arguments set forth by Friedman (1953) and Mundell (1961) in favor of flexible exchange rates. For countries in the currency union optimal monetary policy is still imperfect and characterized by the average condition for the labor wedge in Proposition 5.

The optimal risk sharing condition in Proposition 6 still applies to all countries, inside or outside the currency union. However, since $\tau^i(s) = 0$ for $s \in S$, $i \notin I$, this condition coincides with the privately optimal risk sharing condition for countries outside the currency union. As a result, there is no need to upset private risk sharing.

Proposition 10 (Countries Outside the Currency Union)**.** *None of the results are affected by considering countries outside the union. Countries that have independent monetary policy manage to obtain a zero labor wedge $\tau^i(s) = 0$. If markets are incomplete, they should not subsidize macro insurance $\tau_D^i(s) = 0$. If markets are incomplete, they should seek to secure ex-post transfers $\hat{T}^i(s)$ that replicate private risk sharing outcomes.*

If markets are incomplete, then ex-post fiscal transfers might be required even outside a currency union. Interestingly, we will show in the dynamic version of the model with only traded goods and home bias in preferences, there are cases (the Cole-Obstfeld case) where ex-post fiscal transfers are not be required for countries outside a currency union, whereas they are required for countries inside a currency union. Crucially, our

[10]Since the price of traded goods is modeled as flexible here, we do not require assumptions about producer currency pricing (PCP) versus local currency pricing (LCP); these are alternative assumptions regarding the form price stickiness takes.

results establish that that inside a currency union, ex-post fiscal transfers should go beyond replicating the outcome that would arise if markets were complete. In this sense, our results yield two important insights. First currency unions and fiscal unions go hand in hand. Second, fiscal integration and financial integration are not perfect substitutes.

How are attitudes towards risk affected by membership in a union? We show that members are more risk averse in the following sense. Suppose country i belongs to the currency union with equilibrium relative price $p^i(s)$. The advantage of leaving the union is that the relative price p^i is not constrained and welfare attains the first best level conditional on C_T^i. It follows that

$$v^i(C_T^i; s) \equiv V^i(C_T^i, p^i(s); s) \leq \max_p V^i(C_T^i, p; s) \equiv V^{i*}(C_T^i; s), \tag{18}$$

with equality if and only if $p^i(s) \in \arg\max_p V^i(C_T^i, p; s)$, in which case the labor wedge is zero, $\tau(s) = 0$. Thus, for every state s, the function V^{i*} is the upper envelope over v^i and is tangent to it precisely at a level of C_T^i that implies $\tau(s) = 0$. In this sense, v^i is more concave than V^{i*} and member countries are more risk averse. We shall put this inequality to use in the next section.

3.4 Value of Insurance

Our simple model allows for three random disturbances: (i) shocks to productivity of labor in the production of non-traded goods; (ii) shocks to preferences (demand); and (iii) shocks to the endowment of traded goods. Proposition 7 shows that if the equilibrium without portfolio taxes does not attain the first best, then it is constrained inefficient. As we show next, this is true except in a knife-edge cases. Examining these knife-edge cases turns out to be interesting, because even when the equilibria coincides with the first best we find that the planner values the availability of insurance strictly more than private agents do. Macro insurance is of greater public value than the aggregate private valuation. Extrapolating beyond our model, this could help explain why macro insurance markets may be missing, even if their social value is significant.

To concoct an example where the first best is attainable it is useful to specialized our model to the utility function

$$U^i(C_T, C_{NT}, N; s) = \log(C_T) + \alpha^i(s) \log(C_{NT}) - \frac{1}{1+\phi} N^{1+\phi}, \tag{19}$$

with $\phi \geq 0$.

Proposition 11. *Suppose the utility function is given by (19), then the equilibrium without portfolio taxes is constrained efficient if and only if productivity shocks and preference shocks are such for all pairs of countries* (i, i'),

$$\frac{A^i(s)}{A^{i'}(s)}\left(\frac{\alpha^i(s)}{\alpha^{i'}(s)}\right)^{\frac{-\phi}{1+\phi}}$$

is constant for all $s \in S$*; the shocks to the endowment of traded goods* $E^i(s)$ *can be arbitrary.*

This proposition defines a precise notion of symmetric shocks to productivity and preferences for which the first best allocation is attainable without portfolio taxes. For example, if the only shocks are to productivity, then this condition requires that productivity vary proportionally across countries. A currency union can handle such a shock using union-wide monetary policy. A similar point applies to taste shocks. More generally, the key constraint imposed by nominal rigidities and a single monetary policy is condition (4), rewritten here for convenience as

$$\frac{U^i_{C_{NT}}(s)}{U^i_{C_T}(s)} = \frac{P^i_{NT}}{P_T(s)}$$

where $P_T(s)$ is only allowed to vary with s not i, while P^i_{NT} is allowed to vary with i but not s. In other words, one can handle fixed differences across countries and union-wide shocks to this marginal rate of substitution, but not individual variations. This refines the notion of symmetric shocks that is required for the first best. Monetary policy in a currency union is constrained, affecting the adjustment in prices, but in some special circumstances no adjustment is needed.

This discussion highlights just how special these circumstances are. Note, however, that the proposition implies that endowment shocks can be properly insured without portfolio taxes. To understand this result, suppose we only have shocks to endowments. Then the first best features perfect risk sharing in the consumption of traded goods: only aggregate fluctuations in traded goods affect the consumption of traded goods. Due to separability of preferences, the first best allocation for non traded goods and labor is not affected by these shocks. It follows that the marginal rate of substitution only varies with union-wide shocks and the first best is implementable as an equilibrium. The marginal rate of substitution only varies with union-wide shocks—and does so symmetrically—implying that the first best is implementable as an equilibrium.[11]

[11] In more detail, suppose $A^i(s) = A^i$ and $\alpha^i(s) = \alpha^i$. The first best allocation features $C^i_T(s) = \frac{1}{\lambda^i}\int_0^1 E^i(s)di$, $N^i(s) = (\alpha^i)^{\frac{1}{1+\phi}}$, and $C^i_{NT}(s) = A^i (\alpha^i)^{\frac{1}{1+\phi}}$. This allocation is supported as an equilibrium

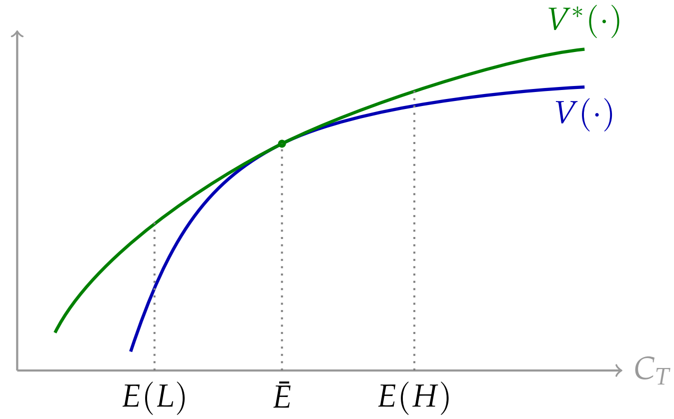

Figure 1: Welfare as perceived by individual agents (upper green curve) and country as a whole (lower blue curve).

Of course, the case of endowment shocks is somewhat artificial, relying on the modeling asymmetry that non traded goods are produced but traded goods are not. If instead traded goods were produced from labor and another fixed input (capital or land) subject to (industry specific) productivity shocks, then these shocks would also have to satisfy the restriction of being symmetric to attain the first best—just as in the case of productivity shocks in the non traded goods.

It is useful to have a case, however artificial, where private insurance is efficient so that we can isolate a separate result. We show that members of a currency union value this insurance more than non members. Moreover, this is is not the true of the value placed on insurance by private individuals. This highlights the role of the macroeconomic externality from insurance, which is not internalized by private agents.

Proposition 12. *Suppose there are only endowment shocks and that all risk is idiosyncratic, so that the aggregate endowment is constant across states:*

i. If we exclude a country from insurance markets, then its utility loss is greater if it belongs to a currency union.

ii. If we excluded a single individual within a country from insurance markets, then his utility loss is the same whether or not his country belongs to a currency union.

Figure 1 illustrates the basic logic behind the first part this proposition for an example with two the equiprobable endowment values. Since the aggregate endowment is

without portfolio taxes by $P^i_{NT} = (\alpha^i)^{\frac{\phi}{1+\phi}} / (\lambda^i A^i)$, $P_T(s) = (\int_0^1 E^i(s) di)^{-1}$, $W^i(s) = (\alpha^i)^{\frac{\phi}{1+\phi}} / \lambda^i$, $Q(s) = 1$ and $1 + \tau^i_L = \frac{\varepsilon - 1}{\varepsilon}$.

constant, the price of traded goods is constant and perfect financial markets offer fair insurance. The resulting equilibrium features constant consumption of the traded good at the average value of the endowment and constant prices and wages. This is true for both members and non members. When the country is excluded from insurance its consumption of the traded good must now fluctuate with its endowment, creating a mean-preserving spread in consumption of traded goods and a loss in expected utility. The crucial point is that the loss is greater for union members because they are more risk averse, according to inequality (18). Indeed, given that prices are constant and the utility function is independent of the state s this inequality simplifies to

$$V^i(C_T^i, \bar{p}) \leq \max_p V^i(C_T^i, p) \equiv V^{i*}(C_T^i).$$

These two value functions are depicted in the figure. They are tangent at the average value of the endowment $\bar{E}$ because this represents the equilibrium consumption level with insurance.

As to the second part of the proposition, it follows easily from the observation that the equilibrium with insurance is the same whether or not the country belongs to the currency union. In both cases the first best allocation is attained. Therefore, if an individual is excluded from insurance markets he faces the same prices whether the country is a member or not. Thus, the drop in utility is the same.

3.5 Coordination

Our next results establishes that we can let governments pick the tax rates on their households' portfolios (with complete private markets) or the state-contingent fiscal transfers (with incomplete private markets) in isolation, with no need for coordination at the supranational level. It highlights that there is no conflict of interest in the degree of insurance that each country should seek, given the terms $Q(s)$ offered to it.

With complete markets, and for fixed ex-post fiscal transfers $\hat{T}^i(s)$, the corrective portfolio taxes allow each country's government to control the country's portfolio $D^i(s)$, subject to the budget constraint $\int D^i(s)Q(s)\pi(s)ds \leq 0$, where the government takes the price $Q(s)$ of insurance in state s as given.

With incomplete markets, then some concerted effort is required to recreate optimal insurance arrangements. Country members of a currency union may jointly design a fiscal union involving state-contingent transfer payments amongst them. But we can let each government simply choose state-contingent government transfers $\hat{T}^i(s)$ subject to the requirement that the net present value of transfers $\int \hat{T}^i(s)Q(s)\pi(s)ds$ be the same as under the allocation to be implemented, with the same price $Q(s)$ as above.[12]

Proposition 13 (No Need for Coordination)**.** *Constrained Pareto efficient allocations can be achieved by each country's government arranging insurance payments acting as a price taker in a competitive international insurance market. No coordination is required.*

It is key for this result that countries are small. With large countries, Proposition 13 fails, and there are benefits from coordination. The reason large countries would seek to manipulate the state prices $Q(s)$ to their advantage by lowering the transfers that they seek to achieve in states of the world where they receive comparatively larger transfers. The force behind this results is similar to that behind the optimal tariff argument in trade theory, except that here countries manipulate the terms of trade across states rather than the terms of trade across goods in a given state.[13] In both cases, as long as countries have some monopoly power (which is the case if they are large), then it is optimal from their individual private perspective to exercise it. It is also socially suboptimal, and so coordination is needed to prevent countries from doing so.

It is important to realize that these observations would also apply if prices were flexible or if countries were not part of a currency union. In other words, the case for coordination in macro insurance among large countries is there whether or not countries are in a currency union and there are nominal rigidities.

3.6 Sticky Wages

We now show that all our results go through, with minimal changes, if wages are nominally rigid instead of prices. It should be clear that we could also manage a situation that combines wage and price rigidity.

In order to have a well defined wage setting problem we assume that labor services are produced by combining a variety of differentiated labor inputs according to the constant returns CES technology

$$N^i(s) = \left(\int_0^1 N^{i,h}(s)^{1-\frac{1}{\varepsilon_w}} dh \right)^{\frac{1}{1-\frac{1}{\varepsilon_w}}}.$$

The rest of the technology is as before. We assume that in each country there is a contin-

[12]If the allocation to be implemented is $\{C_T^i(s), C_{NT}^i(s), N^i(s)\}$ together with prices $\{P_T(s), P_{NT}^i\}$, then this value is simply given by $\int P_T(s)(C_T^i(s) - E_T^i(s))Q(s)\pi(s)ds$.

[13]There are some similarities with(Costinot et al., 2011) who show how capital controls can be used to manipulate terms of trade over time rather than across states.

uum of workers $h \in [0,1]$, each supplying a particular variety $h \in [0,1]$ with preferences

$$\int U^i(C^{i,h}_{NT}(s), C^{i,h}_T(s), N^{i,h}(s); s)\pi(s)ds.$$

The budget constraints are the same as before

$$\int D^{i,h}(s)Q(s)\pi(s)ds \leq 0,$$

$$P^i_{NT}(s)C^{i,h}_{NT}(s) + P_T(s)C^{i,h}_T(s) \leq (1-\tau^i_L)W^{i,h}N^{i,h}(s) \\ + P_T(s)E^i_T(s) + \Pi^i(s) + T^i(s) + (1+\tau^i_D(s))D^{i,h}(s),$$

except that the wage $W^{i,h}$ is now specific to each worker h but independent of s because wages are set in advance of the realization of the state s. Note that prices of non-traded goods are now state contingent. For convenience, we now assume that the worker pays for the labor tax; firms are untaxed.

Workers set their own wages $W^{i,h}$ taking into account that in each state of the world s labor demand is given by $N^i(s)(W^{i,h}/W^i)^{-\varepsilon_w}$ where $W^i = (\int (W^{i,h})^{1-\varepsilon_w} dh)^{1/(1-\varepsilon_w)}$ is the wage index for labor services. In a symmetric equilibrium, all workers set the same wage $W^{i,h} = W^i$, and consume and work the same so that $C^{i,h}_{NT}(s) = C^i_{NT}(s)$, $C^{i,h}_T(s) = C^i_T(s)$ and $N^{i,h}(s) = N^i(s)$. The wage W^i is given by

$$W^i = \frac{1}{1-\tau^i_L}\frac{\epsilon_w}{\epsilon_w - 1}\frac{\int -N^i(s)U^i_N(s)\pi(s)ds}{\int \frac{U^i_{C_{NT}}(s)}{P^i_{NT}(s)}N^i(s)\pi(s)ds}.$$

All varieties sell at the same price so that $P^{i,j}_{NT}(s) = P^i_{NT}(s)$. This price is given by

$$P^i_{NT}(s) = \frac{\epsilon}{\epsilon - 1}\frac{W^i}{A^i(s)}.$$

All the results that we derived in the version of the model with sticky prices carry through with no modification to this specification with sticky wages. In particular, Propositions 1–13 are still valid.

4 A Dynamic Model

The static model reveals some key results in a simple and transparent manner. However, it is perhaps too simple to explore the issues in greater depth. We now build a richer, dynamic model similar to Farhi and Werning (2012) which in turn builds on Gali and Monacelli (2005, 2008). We present the model with incomplete markets where agents can only trade short-term risk free bonds as in Farhi and Werning (2012), although we will also compare it to the complete financial market case when we turn to the log-linearized version of the model in Section 5.

In Farhi and Werning (2012), we focused on capital controls. Here instead we do no consider capital controls. Instead, our focus, just as in the static model, is on the design of ex-post transfers between countries that are contingent on the shocks experienced by all countries.

We focus on one-time shocks, starting in a symmetric steady state. At $t = 0$, the path for productivity in each country is realized. There is no further uncertainty. In the log-linearized version of the model, which we focus our analysis on, it is well known that a certainty equivalence principle holds so that this assumption is irrelevant. In other words, our analysis can simply be understood as an impulse response characterization in a setup where shocks might keep occurring in every period.

4.1 Households

There is a continuum measure one of countries $i \in [0,1]$. We focus attention on a single country, which we call Home, and can be thought of as a particular value $H \in [0,1]$. In every country, there is a representative household with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{20}$$

where N_t is labor, and C_t is a consumption index defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}$ is an index of consumption of domestic goods given by

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $j \in [0,1]$ denotes an individual good variety. Similarly, $C_{F,t}$ is a consumption index of imported goods given by

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

where $C_{i,t}$ is, in turn, an index of the consumption of varieties of goods imported from country i, given by

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Thus, ϵ is the elasticity between varieties produced within a given country, η the elasticity between domestic and foreign goods, and γ the elasticity between goods produced in different foreign countries. An important special case obtains when $\sigma = \eta = \gamma = 1$. We call this the Cole-Obstfeld case, in reference to Cole and Obstfeld (1991). This case is more tractable and has some special implications that are worth highlighting. Thus, we devote special attention to it, although we will also derive results away from it.

The parameter α indexes the degree of home bias, and can be interpreted as a measure of openness. Consider both extremes: as $\alpha \to 0$ the share of foreign goods vanishes; as $\alpha \to 1$ the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias; the former a closed economy barely trading with the outside world.

Households seek to maximize their utility subject to the sequence of budget constraints

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + D_{t+1} + \int_0^1 E_{i,t} D_{t+1}^i di$$
$$\leq W_t N_t + \Pi_t + T_t + (1 + i_{t-1}) D_t + \int_0^1 E_{i,t} (1 + i_{t-1}^i) D_t^i di$$

for $t = 0, 1, 2, \ldots$ In this inequality, $P_{H,t}(j)$ is the price of domestic variety j, $P_{i,t}$ is the price of variety j imported from country i, W_t is the nominal wage, Π_t represents nominal profits and T_t is a nominal lump sum transfer. All these variables are expressed in domestic currency. The portfolio of home agents is composed of home and foreign bond holding: D_t is home bond holdings of home agents, D_t^i is bond holdings of country i of home agents. The returns on these bonds are determined by the nominal interest rate in the home country i_t, the nominal interest rate i_t^i in country i, and the evolution of the nominal exchange rate $E_{i,t}$ between home and country i.

The nominal lump sum transfer is the focus of our analysis. More precisely, we allow for ex-post transfers across countries, contingent on the shocks experienced by these countries. We will provide a sharp characterization of these optimal transfers in the log-linearized version of the model. We will also compare these transfers to the implicit transfers that would occur through financial markets if asset markets were complete and private agents freely chose their portfolios.

4.2 Firms

Technology. A typical firm in the home economy produces a differentiated good with a linear technology given by

$$Y_t(j) = A_{H,t} N_t(j) \tag{21}$$

where $A_{H,t}$ is productivity in the home country. We denote productivity in country i by $A_{i,t}$.

We allow for a constant employment tax $1+\tau^L$, so that real marginal cost deflated by Home PPI is given by

$$MC_t = \frac{1+\tau^L}{A_{H,t}} \frac{W_t}{P_{H,t}}.$$

We take this employment tax to be constant in our model. We pin this tax rate down by assuming that it is optimally set cooperatively at a symmetric steady state with flexible prices. The tax rate is simply set to offset the monopoly distortion so that $\tau^L = -\frac{1}{\varepsilon}$.

Price-setting assumptions. As in Gali and Monacelli (2005), we maintain the assumption that the Law of One Price (LOP) holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency. This assumption is known as Producer Currency Pricing (PCP) and is sometimes contrasted with the assumption of Local Currency Pricing (LCP), where each variety's price is set separately for each country and quoted (and potentially sticky) in that country's local currency. Thus, LOP does not necessarily hold. It has been shown by Devereux and Engel (2003) that LCP and PCP may have different implications for monetary policy. However, for our purposes, these two polar cases are equivalent since, for the most part, we will study the model assuming fixed exchange rates.

We consider Calvo price setting, where in every period, a randomly selected fraction $1-\delta$ of firms can reset their prices. Those firms that get to reset their price choose a reset price P_t^r to solve

$$\max_{P_t^r} \sum_{k=0}^{\infty} \delta^k \left(\prod_{h=1}^{k} \frac{1}{1+i_{t+h}} \right) (P_t^r Y_{t+k|t} - P_{H,t} MC_t Y_{t+k|t})$$

where $Y_{t+k|t} = \left(\frac{P_t^r}{P_{H,t+k}}\right)^{-\epsilon} C_{t+k}$, taking the sequences for MC_t, Y_t and $P_{H,t}$ as given.

4.3 Terms of Trade, Exchange Rates and UIP

It is useful to define the following price indices: home's Consumer Price Index (CPI) $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$, home's Producer Price Index (PPI) $P_{H,t} = [\int_0^1 P_{H,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$, and the index for imported goods $P_{F,t} = [\int_0^1 P_{i,t}^{1-\gamma} di]^{\frac{1}{1-\gamma}}$, where $P_{i,t} = [\int_0^1 P_{i,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$ is country i's PPI.

Let $E_{i,t}$ be nominal exchange rate between home and i (an increase in $E_{i,t}$ is a depreciation of the home currency). Because the Law of One Price holds, we can write $P_{i,t}(j) = E_{i,t} P_{i,t}^i(j)$ where $P_{i,t}^i(j)$ is country i's price of variety j expressed in its own currency. Similarly, $P_{i,t} = E_{i,t} P_{i,t}^i$ where $P_{i,t}^i = [\int_0^1 P_{i,t}^i(j)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$ is country i's domestic PPI in terms of country i's own currency. We therefore have

$$P_{F,t} = E_t P_t^*$$

where $P_t^* = [\int_0^1 P_{i,t}^{i1-\gamma} di]^{\frac{1}{1-\gamma}}$ is the world price index and E_t is the effective nominal exchange rate.[14]

The effective terms of trade are defined by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

where $S_{i,t} = P_{i,t}/P_{H,t}$ is the terms of trade of home versus i. The terms of trade can be used to rewrite the home CPI as

$$P_t = P_{H,t}[1 - \alpha + \alpha S_t^{1-\eta}]^{\frac{1}{1-\eta}}.$$

Finally we can define the real exchange rate between home and i as $\mathcal{Q}_{i,t} = E_{i,t} P_t^i / P_t$.

[14] The effective nominal exchange rate is defined as $E_t = [\int_0^1 E_{i,t}^{1-\gamma} P_{i,t}^{i1-\gamma} di]^{\frac{1}{1-\gamma}} / [\int_0^1 P_{i,t}^{i1-\gamma} di]^{\frac{1}{1-\gamma}}$.

We define the effective real exchange rate be

$$\mathcal{Q}_t = \frac{E_t P_t^*}{P_t}.$$

4.4 Equilibrium Conditions

We now summarize the equilibrium conditions. Equilibrium in the home country can be described by the following equations. We find it convenient to group these equations into two blocks, which we refer to as the demand block and the supply block.

The demand block is independent of the nature of price setting. It is composed of the Backus-Smith condition

$$C_t = \Theta^i C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}, \tag{22}$$

where Θ^i is a relative Pareto weight which depends on the realization of the shocks, the goods market clearing condition

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha \int_0^1 C_t^i (S_t^i S_{i,t})^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta} di\right], \tag{23}$$

were S_t^i is denotes the effective terms of trade of country i, the labor market clearing condition

$$N_t = \frac{Y_t}{A_{H,t}} \Delta_t \tag{24}$$

where Δ_t is an index of price dispersion $\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}$, the Euler equation

$$1 + i_t = \beta^{-1} \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} \Pi_{t+1}$$

where $\Pi_t = \frac{P_{t+1}}{P_t}$ is CPI inflation, the arbitrage condition between home and foreign bonds

$$1 + i_t = (1 + i_t^i) \frac{E_{i,t+1}}{E_{i,t}}, \tag{25}$$

for all $i \in [0,1]$, and the country budget constraint

$$NFA_t = -\left(P_{H,t} Y_t - P_t C_t\right) + \frac{1}{1+i_t} NFA_{t+1} \tag{26}$$

where NFA_t is the country's net foreign assets at t, which for convenience, we measure in home numeraire. We also impose a No-Ponzi condition so that we can write the budget constraint in present-value form

$$NFA_0 = -\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1+i_s} \right) (P_{H,t}Y_t - P_tC_t) . \tag{27}$$

The value of NFA_0, which depends on the realization of shocks, is a measure of the (net present value) transfer to the home country. Characterizing the optimal value of NFA_0 depending on the shocks is of the main focuses of our analysis below. Absent ex-post transfers across countries, we would have $NFA_0 = 0$ since countries are ex-ante identical and only risk-free bonds can be traded. We will also compare the optimal value of NFA_0 to the value that would obtain if private agents could engage in risk-sharing through a complete set of financial markets. One of our main results will establish that these values differ, and to characterize how they differ.

Finally with Calvo price setting, the supply block is composed of the equations summarizing the first-order condition for optimal price setting. These conditions are provided in Appendix A.3. We will only analyze a log-linearized version of the model with Calvo price setting (see Section 5).

For most of the paper, we will be concerned with fixed exchange rate regimes (either pegs or currency unions) in which case we have the additional restriction that $E_t = E_0$ for all $t \geq 0$ where E_0 is predetermined.

5 Efficient Transfers in the Dynamic Model

As is standard in the literature, we work with a log-linearized approximation of the model. As before, at $t = 0$, the economy is hit with an unanticipated shock. It is convenient to work with a continuous time version of the model. This does not affect our results, but it is useful because it implies that no price index can jump at $t = 0$ and this simplifies the derivation of initial conditions characterizing the equilibrium. We denote the instantaneous discount rate by ρ, and the instantaneous arrival rate for price changes by ρ_δ.

From now on we focus on the Cole-Obstfeld case $\sigma = \eta = \gamma = 1$. This case is attractive for two reasons. First, with flexible prices, it is not optimal to use insurance or transfers since perfect risk sharing is achieved through movements in the real exchange rate and trade remains balanced. Second, even when prices are sticky, the laissez-faire equilibrium with incomplete markets coincides with its complete markets counterpart. Once again, risk sharing is delivered with balanced trade. This means that we can interpret any deviation from balanced trade at the optimum with transfers as an indication that private risk sharing through complete financial markets (if those were available) would be suboptimal. Third, it is possible to derive a simple second-order approximation of the welfare function around the symmetric deterministic steady state. Away from the Cole-Obstfeld case the welfare function is more involved.

We start by considering the case where all countries are members of the same currency union. Later, we consider the case where some countries are in a currency union, while others retain outside, with a flexible exchange rate and independent monetary policy. We show transfers are nonzero only for countries within a currency union.

The natural allocation. We define a reference allocation which corresponds to the flexible price allocation, with no transfers across countries over and above the privately optimal transfers (the complete markets solution). Note that we impose flexible prices in every country. We describe this allocation in log deviations from the symmetric steady state with a lower case, and a double bar. We denote with a star the union average of a given variable. For example, $\bar{\bar{y}}_t^* = \int_0^1 \bar{\bar{y}}_t^i di$ and $\bar{\bar{c}}_t^* = \int_0^1 \bar{\bar{c}}_t^i di$. At the natural allocation, output in country i is given by

$$\bar{\bar{y}}_t^i = a_{i,t},$$

consumption is given by

$$\bar{\bar{c}}_t^i = \alpha \int_0^1 a_{i,t} di + (1-\alpha) a_{i,t},$$

labor is given by

$$\bar{\bar{n}}_t^i = 0,$$

and the terms of trade are given by

$$\bar{\bar{s}}_t{}^i = a_{i,t} - \int_0^1 a_{i,t} di.$$

In addition, trade is balanced.

Finally, aggregate output is equal to aggregate consumption and is given by

$$\bar{\bar{y}}_t^* = \bar{\bar{c}}_t^* = \int_0^1 a_{i,t} di.$$

Note that by construction $\int_0^1 \bar{\bar{s}}_t^i di = 0$.[15]

[15]Although we do not need it for our analysis, note that the natural interest rate is given by $\bar{\bar{r}}_t^i = \dot{a}_{i,t}$.

Summarizing the system in gaps. We denote by $\hat{y}_t^i$ and $\hat{\hat{\theta}}^i$ the deviations of y_t^i and θ^i from their flexible price counterparts. We denote by $\tilde{y}_t^i = \hat{y}_t^i - \hat{y}_t^*$ and $\tilde{\theta}^i = \hat{\hat{\theta}}^i - \hat{\hat{\theta}}^*$ where $\hat{y}_t^* = \int_0^1 \hat{y}_t^i di$ and $\hat{\hat{\theta}}^* = \int_0^1 \hat{\hat{\theta}}^i di = 0$ the deviations of these variables from their corresponding aggregates; also let $\tilde{\pi}_{H,t}^i = \pi_{H,t}^i - \pi_t^*$ where $\pi_t^* = \int_0^1 \pi_{H,t}^i di$. Note that $\hat{\hat{\theta}}^i$ is already a normalized variable so that $\hat{\hat{\theta}}^i = \tilde{\theta}^i$.

The trade balance is constant and equals $-\alpha\tilde{\theta}^i$. The net foreign asset position must pay for the present value of the trade deficits, so that starting from a position of zero net foreign assets, transfers must bring the net foreign asset position to

$$N\tilde{F}A_0^i = \frac{\alpha}{\rho}\tilde{\theta}^i.$$

The disaggregated variables solve the ordinary differential equations, corresponding to the Phillips curve and the Euler equation,

$$\dot{\tilde{\pi}}_{H,t}^i = \rho\tilde{\pi}_{H,t}^i - \hat{\kappa}\tilde{y}_t^i - \lambda\alpha\tilde{\theta}^i,$$
$$\dot{\tilde{y}}_t^i = -\tilde{\pi}_{H,t}^i - \tilde{\bar{s}}_t^i,$$

with initial condition

$$\tilde{y}_0^i = (1-\alpha)\tilde{\theta}^i - \tilde{\bar{s}}_0^i,$$

where $\lambda = \rho_\delta(\rho + \rho_\delta)$ and $\hat{\kappa} = \lambda(1+\phi)$ index price flexibility.

Since $\int_0^1 \tilde{\bar{s}}_t^i di = 0$, as long as $\int_0^1 \tilde{\theta}^i di = 0$ the following aggregation constraints are verified for any bounded solution of the system above:

$$\int_0^1 \tilde{y}_t^i di = 0,$$
$$\int_0^1 \tilde{\pi}_{H,t}^i di = 0.$$

We will assume that the zero lower bound on the nominal interest is not binding. Then the only constraint on the aggregates is that they must satisfy the aggregate New Keynesian Philips Curve

$$\dot{\pi}_t^* = \rho\pi_t^* - \hat{\kappa}\hat{y}_t^*.$$

Thus, there are many possible paths for the aggregate variables, depending on the stance of monetary policy at the union level.

From these equations we can infer aggregate consumption

$$\hat{\hat{c}}_t^* = \hat{\hat{y}}_t^*.$$

We can also infer the disaggregated variables for country i as follows. The terms of trade gap $\tilde{s}_t^i$ can be backed out from

$$\tilde{y}_t^i = (1-\alpha)\tilde{\theta}^i + \tilde{s}_t^i.$$

which combines the market clearing condition with the Backus-Smith condition. Similarly, we can back out the employment gap $\tilde{n}_t^i$ and the consumption gap $\tilde{c}_t^i$ from technology and market clearing

$$\tilde{y}_t^i = \tilde{n}_t^i,$$
$$\tilde{y}_t^i = \tilde{c}_t^i + \alpha\tilde{s}_t^i - \alpha\tilde{\theta}^i.$$

Loss function. We are interested in the symmetric constrained Pareto efficient allocation that provides optimal ex-ante insurance behind the veil of ignorance, before shocks are realized. To solve for this we maximize an unweighted Utilitarian welfare function. A simple representation of the loss function associated with this welfare criterion is as follows (see Farhi and Werning, 2012):

$$\frac{1}{2}\int_0^\infty \int_0^1 e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}_{H,t}^i + \pi_t^*)^2 + (\tilde{y}_t^i + \hat{y}_t^*)^2 + \alpha_\theta(\tilde{\theta}^i)^2\right] di\, dt,$$

where $\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}$ and $\alpha_\theta = \frac{\alpha(2-\alpha)}{1+\phi}$.[16] The first two terms in the loss function are familiar in New-Keynesian models and are identical to those obtained by Gali and Monacelli (2005, 2008). The third term captures the direct welfare effects of transfers—it penalizes deviations from efficient private risk sharing. In the closed economy limit, as $\alpha \to 0$, this term goes to zero since $\alpha_\theta \to 0$.

Note that from the perspective of an individual country i, transfers also have a first order effect on welfare—the loss function of an individual country inherits a term

$$-\frac{1}{2}\int_0^\infty e^{-\rho t}\frac{2\alpha(2-\alpha)}{1+\phi}\tilde{\theta}^i dt,$$

This term represents the pure distributional aspect of transfers. These distributional concerns are zero sum and wash out in the aggregate since $\int_0^1 \tilde{\theta}^i = 0$.

[16]This welfare function assumes that labor taxes are set to maximize total welfare at the symmetric deterministic steady state.

5.1 Laissez-Faire with Incomplete Markets in a Currency Union

Before analyzing the optimal solution, we first analyze the laissez faire solution. Using the fact that $\int_0^1 \tilde{y}_t^i di = \int_0^1 \tilde{\pi}_{H,t}^i di = 0$, we are led to the following planning problem:

$$\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[\alpha_\pi (\tilde{\pi}_{H,t}^i)^2 + (\tilde{y}_t^i)^2 + \alpha_\pi (\pi_t^*)^2 + (\hat{y}_t^*)^2 \right] di\, dt$$

subject to

$$\dot{\tilde{\pi}}_{H,t}^i = \rho \tilde{\pi}_{H,t}^i - \hat{\kappa} \tilde{y}_t^i,$$
$$\dot{\tilde{y}}_t^i = -\tilde{\pi}_{H,t}^i - \dot{\bar{\tilde{s}}}_t^i,$$

$$\tilde{y}_0^i = -\bar{\tilde{s}}_0^i,$$

$$\dot{\pi}_t^* = \rho \pi_t^* - \hat{\kappa} \hat{y}_t^*,$$

where the minimization is over the variables $\tilde{\pi}_{H,t}^i$, π_t^*, $\tilde{y}_t^i$, $\hat{y}_t^*$. Note that since $\tilde{\theta}^i = 0$, the two aggregation constraints $\int_0^1 \tilde{y}_t^i di = 0$ and $\int_0^1 \tilde{\pi}_{H,t}^i di = 0$ are automatically verified.

The solution of the planning problem is then simply $\hat{y}_t^* = \pi_t^* = 0$ for the aggregates. This result is a restatement of the result in Benigno (2004) and Gali and Monacelli (2008) that optimal monetary policy in a currency union ensures that the union average output gap and inflation are zero in every period.[17] Monetary policy can be chosen at the union level so that monetary conditions are adapted to the average country. The disaggregated variables $\tilde{\pi}_{H,t}^i$ and $\tilde{y}_t^i$ solve the following system of differential equations,

$$\dot{\tilde{\pi}}_{H,t}^i = \rho \tilde{\pi}_{H,t}^i - \hat{\kappa} \tilde{y}_t^i,$$
$$\dot{\tilde{y}}_t^i = -\tilde{\pi}_{H,t}^i - \dot{\bar{\tilde{s}}}_t^i,$$

with initial condition

$$\tilde{y}_0^i = -\bar{\tilde{s}}_0^i.$$

Proposition 14 (Laissez-Faire)**.** *The laissez-faire solution with incomplete markets ($\tilde{NFA}_0^i = \tilde{\theta}^i = 0$) coincides with its complete markets counterpart. Moreover, union-wide aggregates are*

[17] Gali and Monacelli (2008) established this result under laissez-faire with complete markets in the Cole-Obstfeld case. As is well known, complete and incomplete markets coincide in this case. Hence their results can be seen as characterizing the laissez-faire solution that we analyze here. Benigno (2004) allows for more general preferences and establishes the result under incomplete markets and laissez-faire. Moreover, he allows for heterogeneity in nominal rigidities across regions and shows that the weighted average of inflation that should be targeted places more weight on countries with more price rigidity.

zero

$$\hat{\hat{y}}_t^* = \pi_t^* = 0.$$

A property of the Cole-Obstfeld case is that the laissez-faire solution with complete markets coincides with the incomplete markets solution where no-state contingent assets are available. Indeed, it coincides with complete financial autarky. The lack of complete markets is not a constraint on private risk sharing.

5.2 Transfer Multipliers in a Currency Union

Before solving the normative problem it is useful to review the positive effects of transfers. The next proposition characterizes the response of the economy to a marginal increase in transfers.

Proposition 15 (Transfer Multipliers)**.** *Let $\nu = \frac{\rho - \sqrt{\rho^2 + 4\hat{\kappa}}}{2}$. Transfer multipliers are given by*

$$\frac{\partial \tilde{y}_t^i}{\partial N\tilde{F}A_0^i} = e^{\nu t}\rho\frac{1-\alpha}{\alpha} - (1 - e^{\nu t})\rho\frac{1}{1+\phi},$$

$$\frac{\partial \tilde{\pi}_{H,t}^i}{\partial N\tilde{F}A_0^i} = -\nu e^{\nu t}\left[\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{1+\phi}\right],$$

$$\frac{\partial \tilde{s}_t^i}{\partial N\tilde{F}A_0^i} = -[1 - e^{\nu t}]\left[\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{1+\phi}\right].$$

The presence of the discount factor ρ in all these expressions is natural because what matters is the annuity value $\rho N\tilde{F}A_0^i$ of the transfer. Note that the terms of trade gap equals accumulated inflation: $\tilde{s}_t = -\int_0^t \tilde{\pi}_{H,s}^i ds$.

Transfers have opposite effects on output in the short and long run. In the short run, when prices are rigid, there is a Keynesian effect due to the fact that transfers stimulate the demand for home goods: $\frac{\partial \tilde{y}_0^i}{\partial N\tilde{F}A_0^i} = \rho\frac{1-\alpha}{\alpha}$. In the long run, when prices adjust, the neoclassical wealth effect on labor supply lowers output: $\lim_{t\to\infty} \frac{\partial \tilde{y}_t^i}{\partial N\tilde{F}A_0^i} = -\rho\frac{1}{1+\phi}$. In the medium run, the speed of adjustment, from the Keynesian short-run response to the neoclassical long-run response, is controlled by the degree of price flexibility $\hat{\kappa}$, which affects ν.[18]

Note that the determinants of the Keynesian and neoclassical wealth effects are very different. The strength of the Keynesian effect hinges on the relative expenditure share

[18] Note that ν is decreasing in $\hat{\kappa}$, with $\nu = 0$ when prices are rigid ($\hat{\kappa} = 0$), and $\nu = -\infty$ when prices are flexible ($\hat{\kappa} = \infty$).

of home goods $\frac{1-\alpha}{\alpha}$: the more closed the economy, the larger the Keynesian effect. The strength of the neoclassical wealth effect depends on the elasticity of labor supply ϕ: the more elastic labor supply, the larger the neoclassical wealth effect.

Positive transfers also increase home inflation. The long-run cumulated response in the price of home produced goods equals $\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{1+\phi}$. The first term $\rho\frac{1-\alpha}{\alpha}$ comes from the fact that transfers increase the demand for home goods, due to home bias. The second term $\rho\frac{1}{1+\phi}$ is due to a neoclassical wealth effect that reduces labor supply, raising the wage. How fast this increase in the price of home goods occurs depends positively on the flexibility of prices through its effect on ν.[19]

The effects echo the celebrated Transfer Problem controversy of Keynes (1929) and Ohlin (1929). With home bias, a transfer generates a boom when prices are sticky, and a real appreciation of the terms of trade when prices are flexible. The neoclassical wealth effect associated with a transfer comes into play when prices are flexible, and generates an output contraction and a further real appreciation.

5.3 Optimal Transfers in a Currency Union

Having solved for the positive effects of transfers, we now explore the associated normative question: what is the optimal use of transfers in a currency union? Using the fact that $\int_0^1 \tilde{y}_t^i di = \int_0^1 \tilde{\pi}_{H,t}^i di = 0$, we are led to the following coordinated planning problem:

$$\min \frac{1}{2}\int_0^\infty \int_0^1 e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}_{H,t}^i)^2 + (\tilde{y}_t^i)^2 + \alpha_\theta(\tilde{\theta}^i)^2 + \alpha_\pi(\pi_t^*)^2 + (\hat{y}_t^*)^2\right] di\, dt \tag{28}$$

subject to

$$\dot{\tilde{\pi}}_{H,t}^i = \rho\tilde{\pi}_{H,t}^i - \hat{\kappa}\tilde{y}_t^i - \lambda\alpha\tilde{\theta}^i, \tag{29}$$

$$\dot{\tilde{y}}_t^i = -\tilde{\pi}_{H,t}^i - \dot{\tilde{\bar{s}}}_t^i, \tag{30}$$

$$\tilde{y}_0^i = (1-\alpha)\tilde{\theta}^i - \tilde{\bar{s}}_0^i, \tag{31}$$

$$\int_0^1 \tilde{\theta}^i di = 0, \tag{32}$$

$$\dot{\pi}_t^* = \rho\pi_t^* - \hat{\kappa}\hat{y}_t^*, \tag{33}$$

where the minimization is over the variables $\tilde{\pi}_{H,t}^i$, π_t^*, $\tilde{y}_t^i$, $\hat{y}_t^*$, $\tilde{\theta}^i$.

We can break down the planning problem into two parts. First, there is an aggregate

[19] Recall that ν is decreasing in the degree of price flexibility $\hat{\kappa}$.

planning problem determining the average output gap and inflation $\hat{y}^*_t$ and π^*_t

$$\min \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi(\pi^*_t)^2+(\hat{y}^*_t)^2\right]dt \tag{34}$$

subject to (33).

Second, there is a disaggregated planning problem determining deviations from the aggregates for output gap, home inflation and consumption smoothing, $\tilde{y}^i_t$, $\tilde{\pi}^i_{H,t}$ and $\tilde{\theta}^i_t$

$$\min \frac{1}{2}\int_0^\infty \int_0^1 e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}^i_{H,t})^2+(\tilde{y}^i_t)^2+\alpha_\theta(\tilde{\theta}^i)^2\right]didt \tag{35}$$

subject to (29), (30), (31), (32). Note that because the forcing variables in this linear quadratic problem satisfy $\int_0^1 \bar{\bar{s}}^i_t di = 0$, the aggregation constraint (32) is not binding. We can therefore drop it from the planning problem. The resulting relaxed planning problem can be broken down into separate component planning problems for each country $i \in [0,1]$

$$\min \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}^i_{H,t})^2+(\tilde{y}^i_t)^2+\alpha_\theta(\tilde{\theta}^i)^2\right]dt \tag{36}$$

subject to (29), (30) and (31).

Aggregates. It turns out that the optimal level of aggregates is the same whether we allow for transfers or not: aggregates are perfectly stabilized in both cases.

Proposition 16 (Aggregates)**.** *At the optimum, union-wide aggregates are zero $\hat{y}^*_t = \pi^*_t = 0$ (exactly as under laissez-faire).*

This proposition, which echoes Proposition 5 from the static model, shows that the result in Benigno (2004) and Gali and Monacelli (2008), that optimal monetary policy at the union level targets the average output gap and inflation to zero, also holds when transfers are allowed.

Optimum with rigid prices. We first treat the case of rigid prices. In this case, $\hat{\kappa} = 0$ and the constraint set boils down to

$$\dot{\tilde{y}}^i_t = -\dot{\bar{\bar{s}}}^i_t,$$

$$\tilde{y}^i_0 = (1-\alpha)\tilde{\theta}^i - \bar{\bar{s}}^i_0,$$

which can be re-expressed as

$$\tilde{y}^i_t = (1-\alpha)\tilde{\theta}^i - \bar{\bar{s}}^i_t.$$

We are therefore left with the following component planning problem

$$\min \frac{1}{2}\int_0^\infty e^{-\rho t}\left[((1-\alpha)\tilde{\theta}^i - \bar{\bar{s}}_t^i)^2 + \alpha_\theta(\tilde{\theta}^i)^2\right]dt.$$

The solution implies the following result.

Proposition 17 (Rigid Prices)**.** *Suppose prices are rigid, then the optimum has*

$$N\tilde{F}A_0^i = \frac{\alpha(1-\alpha)}{(1-\alpha)^2 + \alpha_\theta}\int_0^\infty e^{-\rho t}\bar{\bar{s}}_t^i dt,$$
$$\tilde{\theta}^i = \frac{\rho(1-\alpha)}{(1-\alpha)^2 + \alpha_\theta}\int_0^\infty e^{-\rho t}\bar{\bar{s}}_t^i dt.$$

Importantly, we find that $N\tilde{F}A_0^i \neq 0$ and $\tilde{\theta}^i \neq 0$, so that the optimal solution *does not* coincide with the laissez-faire solution with complete markets. Government insurance, either through ex-post transfers or through assets markets, is a necessary feature of the optimum.

Countries experiencing shocks that depreciate their natural terms of trade $\bar{\bar{s}}_t^i$ should receive positive transfers. The optimal transfers are increasing in the size and persistence of shocks. This helps alleviate the recession resulting from the inability of the terms of trade to adjust to that level in the short-run. With positive home bias ($\alpha < 1$), transfer increases the demand for home goods and reduces that for foreign goods—once again, a manifestation of the Transfer Problem.

Optimal transfers are increasing the persistence of the shocks. This is intuitive. Transfers affect the economy permanently and are therefore better suited to deal with persistent shocks.

Optimal transfers $N\tilde{F}A_0^i$ depend crucially on the the openness of the economy, as captured by the degree of home bias α. They are non-monotonic in the degree of openness. Indeed, $N\tilde{F}A_0^i$ is zero for both $\alpha = 0$ (closed economy) and $\alpha = 1$ (fully open economy). In contrast, the coefficient $\tilde{\theta}^i$ equals ρ for $\alpha = 0$ and zero for $\alpha = 1$.

This shows that the reason for zero transfers for $\alpha = 0$ and for $\alpha = 1$ are very different. Basically for α close to 0 (extreme home bias), small transfers have large expenditure switching effect across different goods. Small transfers therefore have large effects on output. For α close to 1, transfers have no expenditure switching effects, and therefore have no effects on output. So for α close to 0, we get small transfers because small transfers are very effective (they have very large effects on output). By contrast, for α close to 1, we get small transfers because transfers are very ineffective (they have small effects on output).

The effectiveness of small transfers when α is small can be further illustrated in the case $\alpha \to 0$ and permanent shocks $\bar{\bar{s}}^i_t = \bar{\bar{s}}^i$ in which case we get perfect stabilization $\tilde{y}^i_t = 0$ at the optimum (we achieve the natural allocation). We show this conclusion holds more generally, even when prices are not perfectly rigid, in Corollary 2.

Optimum with sticky prices in the closed economy limit $\alpha \to 0$. We now return to the case where prices are not entirely rigid, $\hat{\kappa} > 0$, so that the costs of inflation must also be weighed against the stabilization of output gaps. Things simplify in the closed economy limit $\alpha \to 0$.

Proposition 18 (Closed Economy Limit)**.** *In the closed economy limit, when $\bar{\bar{s}}^i_t = \bar{\bar{s}}^i_0 e^{-\psi t}$, we have*

$$N\tilde{F}A^i_0 = 0,$$

$$\tilde{\theta}^i = \bar{\bar{s}}^i_0 \left[1 - \frac{\psi^2}{(\psi+\nu)(\psi+\rho-\nu)} + \frac{\psi(\nu\alpha_\pi\hat{\kappa}+\psi)}{(\psi+\nu)(\psi+\rho-\nu)^2}\frac{\rho-2\nu}{\alpha_\pi\nu^2+1}\right].$$

For α close to 0 (extreme home bias), small transfers have large expenditure switching effect across different goods. Small transfers therefore have large effects on output. Indeed, in the limit, we get $\tilde{\theta}^i \neq 0$ despite the fact that $N\tilde{F}A^i_0 = 0$. Transfers are particularly useful in the case where shocks are permanent: if $\psi = 0$ then $\tilde{\theta}^i = \bar{\bar{s}}^i_0$ and we get perfect stabilization of output and inflation.

Corollary 2 (Closed Economy Limit, Permanent Shocks)**.** *In the closed economy limit, in response to a permanent shock $\bar{\bar{s}}^i_t = \bar{\bar{s}}^i_0$*

$$N\tilde{F}A^i_0 = 0,$$
$$\tilde{\theta}^i = \bar{\bar{s}}^i_0,$$

and perfect stabilization is achieved: $\tilde{y}^i_t = \tilde{\pi}^i_{H,t} = 0$.

This result is striking. For rather closed economies in a currency union, modest transfers achieve large stabilization benefits. This result is interesting as a contrast to the arguments presented by McKinnon (1963) that common currencies are more costly for economies that are more closed. McKinnon did not consider transfers, however. Our result shows that this matters: closed economies make transfers more potent.

Numerical Exploration. We show in the appendix that $\tilde{\theta}^i$ solves a simple static quadratic minimization problem that is very tractable.

For our simulations, we follow Gali and Monacelli (2005) and set the benchmark parameters at: $\phi = 3$, $\rho = 0.04$, $\epsilon = 6$ and $\rho_\delta = -\log(0.75^4)$. We explore different values of the remaining parameters.

Figure 2 displays the behavior of the economy with optimal transfers and with no transfers in response to a permanent shock with $\bar{\bar{s}}^i_t = 0.05$. The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. In this figure, time is measured in years and inflation is annualized. The allocation without transfers features deflation and a recession (in gaps) in the short run which vanishes in the long run as prices adjust: the output gap increases from -5% to 0 and the inflation rate from -3% to 0. The allocation with transfers features less deflation and smaller recession in the short run, but lower output in the long run (in gaps). For example, with $\alpha = 0.1$, the output gap at impact is only -1.2% and the inflation rate -0.8%. The allocation without transfer is independent of openness α. By contrast, the solution with optimal transfers is more stable, the more closed the economy (the lower α). Optimal transfers stabilize the economy more effectively when the economy is more closed.

Figure 3 displays a measure of stabilization due to transfers. We compare the impact on the output gap of a shock with and without optimal transfers and report the mitigation factor—the difference between the two as a fraction of the latter. We feed in exponentially decaying shocks $\bar{\bar{s}}^i_t = e^{-\psi t}\bar{\bar{s}}^i_0$ and normalize the initial shock $\bar{\bar{s}}^i_0$ to 0.01. We then plot our stabilization measure as a function of openness α and the persistence of the shock as measured by its half life $(-\log(0.5)/\psi)$. Using the same shock, Figure 4 displays transfers $N\tilde{F}A_0$ as a function of the same two parameters; these numbers can be interpreted as transfers as a fraction of GDP.

Stabilization is increasing in the persistence of the shock and decreasing in openness. The optimal transfer is increasing in the persistence of the shock starting at zero for fully transitory shocks, but hump-shaped as a function of openness, starting at zero at $\alpha = 0$. Significant stabilization is achieved with relatively modest transfers when the economy is relatively closed and shocks are relatively permanent.

5.4 The Role of Fixed Exchange Rates: Countries Outside a Currency Union

In this section, we seek to clarify the role of fixed exchange rates. We now assume that only a subset of countries $I \subseteq [0, 1]$ are in the currency union. These countries have flexible exchange rates. We can write down the corresponding planning problem as follows:

$$\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[\alpha_\pi (\tilde{\pi}^i_{H,t})^2 + (\tilde{y}^i_t)^2 + \alpha_\theta (\tilde{\theta}^i)^2 + \alpha_\pi (\pi^*_t)^2 + (\hat{y}^*_t)^2 \right] di\, dt$$

subject to

$$\dot{\pi}^*_t = \rho \pi^*_t - \hat{\kappa} \hat{y}^*_t,$$

$$\int_0^1 \tilde{\theta}^i di = 0,$$

for $i \in I$,

$$\dot{\tilde{\pi}}^i_{H,t} = \rho \tilde{\pi}^i_{H,t} - \hat{\kappa} \tilde{y}^i_t - \lambda \alpha \tilde{\theta}^i,$$

$$\dot{\tilde{y}}^i_t = -\tilde{\pi}^i_{H,t} - \dot{\bar{\bar{s}}}^i_t,$$

$$\tilde{y}^i_0 = (1 - \alpha) \tilde{\theta}^i - \bar{\bar{s}}^i_0,$$

and for $i \notin I$,

$$\dot{\tilde{\pi}}^i_{H,t} = \rho \tilde{\pi}^i_{H,t} - \hat{\kappa} \tilde{y}^i_t - \lambda \alpha \tilde{\theta}^i.$$

For countries outside the currency union the only constraint is the Phillips curve. The Euler equation and the initial condition do not appear as constraints because with a flexible exchange rate $\tilde{e}^i_t$ these become

$$\dot{\tilde{y}}^i_t = \dot{\tilde{e}}^i_t - \tilde{\pi}^i_{H,t} - \dot{\bar{\bar{s}}}^i_t,$$

$$\tilde{y}^i_0 = \tilde{e}^i_t + (1 - \alpha) \tilde{\theta}^i - \bar{\bar{s}}^i_0.$$

Thus, these equations simply define the required value for the exchange rate $\tilde{e}^i_t$. As a result, the solution entails $\tilde{\pi}^i_{H,t} = \tilde{y}^i_t = \tilde{\theta}^i = 0$ for $i \notin I$. These countries do not send or receive transfers. The laissez-faire solution is optimal for them.

Proposition 19 (Countries Outside the Currency Union)**.** *Laissez-faire is optimal for countries outside the currency union and they do not make or receive any transfers to other countries* $\tilde{\theta}^i = 0$. *They achieve perfect stabilization* $\tilde{\pi}^i_{H,t} = \tilde{y}^i_t = 0$.

It follows that any role for transfers can be solely attributed to the fixed exchange rates prevailing in a currency union. This result echoes Proposition 10, but solves for transfers instead of the portfolio return taxes.

5.5 Coordination

We now consider what happens when countries do not coordinate on macro insurance.[20] To do so, we now assume that countries can access complete asset markets to purchase insurance. In the log-linearized model this amounts to having country i choose $\tilde{\theta}^i$ contingent on the shock realization, subject to a budget constraint, which turns out to be simply $\mathbb{E}[\tilde{\theta}^i] = 0$, taking the evolution of aggregates as given. Specifically, for small α, country i solves

$$\min \frac{1}{2}\mathbb{E}\int_0^\infty e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}^i_{H,t})^2 + 2\alpha_\pi \pi^*_t \tilde{\pi}^i_{H,t} + (\tilde{y}^i_t)^2 + 2\hat{y}^*_t \tilde{y}^i_t + \frac{2\alpha}{1+\phi}\tilde{y}^i_t + \alpha_\theta(\tilde{\theta}^i)^2\right] dt$$

subject to (29), (30), (31) and $\mathbb{E}[\tilde{\theta}^i] = 0$, where the minimization is over the (random) variables $\tilde{\pi}^i_{H,t}$, $\tilde{y}^i_t$, $\tilde{\theta}^i$, taking $\hat{y}^*_t$, and π^*_t as given. The path for aggregates $\{\hat{y}^*_t, \pi^*_t\}_{t\geq 0}$ affects the solution to this problem solely through linear terms in the objective function. The linear term $\frac{2\alpha}{1+\phi}\tilde{y}^i_t$ did not appear in the coordinated problem. It can be traced back to the fact that countries wish to manipulate their terms of trade. As a result, countries display a preference for lower output—a form of "deflationary bias". Because of this linear term, this approximation of the loss function for an individual country is only valid for small α.[21]

A central monetary authority can choose aggregates $\{\hat{y}^*_t, \pi^*_t\}$ by setting monetary policy subject to the following constraints. First, it must ensure that the solutions to the uncoordinated component planning problems satisfy $\int_0^1 \tilde{y}^i_t di = 0$ and $\int_0^1 \tilde{\pi}^i_{H,t} di = 0$. This amounts to verifying a fixed point, that aggregates are actually equal to their proposed path. Second, it must ensure that the aggregate Phillips curve is verified, $\dot{\pi}^*_t = \rho\pi^*_t - \hat{\kappa}\hat{y}^*_t$. Both requirements define a set $\mathcal{F}$ of feasible aggregate outcomes $\{\hat{y}^*_t, \pi^*_t\}_{t\geq 0}$. The set is a linear space and, as we will show below, includes $\hat{y}^*_t = \pi^*_t = 0$.

To determine the aggregate outcome we need to specify an objective for the central monetary authority. We suppose it seeks to maximize aggregate welfare. Thus, the problem is the same as (34) but where the constraint set is $\mathcal{F}$ instead of (33). Although the constraint sets differ, the solutions coincide and one obtains $\hat{y}^*_t = \pi^*_t = 0$. Indeed, the disaggregated variables also coincide with the coordinated outcome.

[20] We should note that the Cole-Obstfeld case may be somewhat special regarding the role of coordination—see for example Clarida et al. (2002) for a context with flexible exchange rates. However, given our results in the static model, which hold for any utility functions, this seems less likely to be a concern here for the issue of transfers in a currency union.

[21] A line by line derivation of the loss function for an individual country leads to a different coefficient on $(\tilde{\theta}^i)^2$ given by $\frac{\alpha(1-\alpha)}{1+\phi}\left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha\right)$, but the difference with α_θ is of order 1 in α, leading to a correction term of order 3 when multiplied by $(\tilde{\theta}^i)^2$ and can therefore be ignored.

Proposition 20 (Coordination vs. No Coordination)**.** *For small α, the coordinated and uncoordinated solutions are identical.*

The lack of need for coordination emphasized in Proposition 20 also confirms the message of Proposition 13 where we found that there were no benefits from coordination. The same caveats in terms of country size apply. Here our results require not only countries to be small, but also α to be small. Indeed when α is not small, even small countries might have an incentive to manipulate their terms of trade in one state vs. another, and have the ability to do so because each country is a monopolist producer of its varieties.

6 Conclusion

Even if private asset markets are perfect, we find that private insurance is imperfect within a currency union. A role emerges for governments to arrange for macro insurance. We think of this as proving one rationale for a fiscal union within a currency union.

Our model abstracted from liquidity or solvency problems in banks or sovereign governments. One possibility for future research is to explore how these considerations may interact with the role for fiscal unions we have focused on here, based on macroeconomic stabilization issues alone. We believe these issues are probably linked: problems in banks or sovereigns negatively impact the macroeconomy, and, vice versa, macroeconomic conditions due to nominal rigidities and the lack of independent monetary policy tailored to asymmetric shocks contributes to problems in banks and sovereigns. Thus, if we had to speculate, we would conjecture that our conclusions here would be relevant in a richer setting with these other features.

Another direction for future work is to consider the moral hazard or commitment problems that may limit the desirability of macroeconomic insurance. A cost-benefit appraisal of a fiscal union should take this into account. We view our paper, which abstracts from these problems, as contributing towards the benefits side of the ledger. But the cost side is equally important and more work needs to be done.

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A Appendix

A.1 Proof of Proposition 1

The necessity part was established in the text. The sufficiency part can be established by construction. Consider allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ and prices $\{P_T(s), P^i_{NT}\}$ satisfying equations (4), (9) and (10).

A.2 Price Setting with Constant Elasticity of Substitution

We have

$$1 - \frac{\int \tau^i(s) U^i_{C_{NT}}(s) C^i_{NT}(s) \pi(s) ds}{\int U^i_{C_{NT}}(s) C^i_{NT}(s) \pi(s) ds} = \frac{1}{1+\tau^i_L} \frac{\varepsilon - 1}{\varepsilon}.$$

We can rewrite the first order condition for P^i_{NT} as

$$\int \frac{\alpha^i_p(s)}{\alpha^i(s)} p^i(s)\, \alpha^i(s) C^i_T(s)\, \frac{1}{p^i(s)} U^i_{C_T}(s)\, \tau^i(s)\, \pi(s)\, ds = 0.$$

If $\frac{\alpha^i_p(s)}{\alpha^i(s)} p^i(s)$ is constant then this implies that

$$\int C^i_{NT}(s)\, U^i_{C_{NT}}(s)\, \tau^i(s)\, \pi(s)\, ds = 0.$$

Thus in this case $\frac{1}{1+\tau^i_L} \frac{\varepsilon-1}{\varepsilon} = 1$ or $\tau^i_L = -1/\varepsilon$.

A.3 Nonlinear Calvo Price Setting Equations

The equilibrium conditions for the Calvo price setting model can be expressed as follows

$$\frac{1-\delta\Pi_{H,t}^{\epsilon-1}}{1-\delta} = \left(\frac{F_t}{K_t}\right)^{\epsilon-1},$$

$$K_t = \frac{\varepsilon}{\varepsilon-1}\frac{1+\tau^L}{A_{H,t}}Y_t N_t^{\phi} + \delta\beta\Pi_{H,t+1}^{\epsilon}K_{t+1},$$
$$F_t = Y_t C_t^{-\sigma} S_t^{-1}\mathcal{Q}_t + \delta\beta\Pi_{H,t+1}^{\epsilon-1}F_{t+1},$$

together with an equation determining the evolution of price dispersion

$$\Delta_t = h(\Delta_{t-1}, \Pi_{H,t}),$$

where $h(\Delta, \Pi) = \delta\Delta\Pi^{\epsilon} + (1-\delta)\left(\frac{1-\delta\Pi^{\epsilon-1}}{1-\delta}\right)^{\frac{\epsilon}{\epsilon-1}}$.

A.4 Proof of Proposition 15

We first solve the behavior of an economy for a given transfer $\tilde{\theta}^i$:

$$\dot{\tilde{\pi}}_{H,t}^i = \rho\tilde{\pi}_{H,t}^i - \hat{\kappa}\tilde{y}_t^i - \lambda\alpha\tilde{\theta}^i,$$
$$\dot{\tilde{y}}_t^i = -\tilde{\pi}_{H,t}^i - \dot{\bar{\bar{s}}}_t^i,$$

$$\tilde{y}_0^i = (1-\alpha)\tilde{\theta}^i - \bar{\bar{s}}_0^i.$$

Define $E_1 = [1,0]'$ and $E_2 = [0,1]'$. Let $X_t^i = [\tilde{\pi}_{H,t}^i, \tilde{y}_t^i]'$, $B_t^i = [-\lambda\alpha\tilde{\theta}^i, -\dot{\bar{\bar{s}}}_t^i]' = -\lambda\alpha\tilde{\theta}^i E_1 - \dot{\bar{\bar{s}}}_t^i E_2$. Define $A = \begin{bmatrix} \rho & -\hat{\kappa} \\ -1 & 0 \end{bmatrix}$. Let $\nu = \frac{\rho - \sqrt{\rho^2+4\hat{\kappa}}}{2} < 0$ be the (only) negative eigenvalue of A, and $X_\nu = [-\nu, 1]'$ and be an eigenvector associated with the negative eigenvalue of A. The solution is given by

$$X_t^i = e^{\nu t}\alpha_\nu^i X_\nu - \int_t^\infty e^{A(t-s)}B_s^i ds = e^{\nu t}\alpha_\nu^i X_\nu + \lambda\alpha\tilde{\theta}^i A^{-1}E_1 + \int_t^\infty \dot{\bar{\bar{s}}}_u^i e^{A(t-u)}E_2 du,$$

where

$$X_0^i + \int_0^\infty e^{-As}B_s^i ds = \alpha_\nu^i X_\nu,$$

$$E_2' X_0^i = (1-\alpha)\tilde{\theta}^i - \bar{\bar{s}}_0^i.$$

We find

$$\alpha_\nu^i = \left[(1-\alpha) - \lambda\alpha E_2' A^{-1} E_1\right] \tilde{\theta}^i - \bar{\bar{s}}_0^i - \int_0^\infty \dot{\bar{\bar{s}}}_t^i E_2' e^{-At} E_2 dt.$$

from which we can infer the path for output

$$\tilde{y}_t^i = e^{\nu t}\alpha_\nu^i + \lambda\alpha\tilde{\theta}^i E_2' A^{-1} E_1 + \int_t^\infty \dot{\bar{\bar{s}}}_u^i E_2' e^{A(t-u)} E_2 du,$$

and inflation

$$\tilde{\pi}_{H,t}^i = -\nu e^{\nu t}\alpha_\nu^i + \lambda\alpha\tilde{\theta}^i E_1' A^{-1} E_1 + \int_t^\infty \dot{\bar{\bar{s}}}_u^i E_1' e^{A(t-u)} E_2 du,$$

Using $E_2' A^{-1} E_1 = -\hat{\kappa}^{-1}$, and $E_1' A^{-1} E_1 = 0$, we can then compute the transfer multipliers.

A.5 Derivation of the Optimum in Section 5.3

In Appendix A.4, we solved for the behavior of the disaggregated variables $X_t^i = [\tilde{\pi}_{H,t}^i, \tilde{y}_t^i]'$ for a given $\tilde{\theta}^i$. In the particular case where $\bar{\bar{s}}_t^i = \bar{\bar{s}}_0^i e^{-\psi t}$, we get

$$X_t^i = e^{\nu t}\alpha_\nu^i X_\nu + \lambda\alpha\tilde{\theta}^i A^{-1} E_1 - \psi e^{-\psi t}\bar{\bar{s}}_0^i (A + \psi I)^{-1} E_2, \tag{37}$$

where

$$\alpha_\nu^i = \left[(1-\alpha) - \lambda\alpha E_2' A^{-1} E_1\right] \tilde{\theta}^i - \bar{\bar{s}}_0^i + \psi\bar{\bar{s}}_0^i E_2' (A + \psi I)^{-1} E_2,$$

$E_1 = [1, 0]'$, $E_2 = [0, 1]'$, $A = \begin{bmatrix} \rho & -\hat{\kappa} \\ -1 & 0 \end{bmatrix}$, $\nu = \frac{\rho - \sqrt{\rho^2 + 4\hat{\kappa}}}{2} < 0$ is the negative eigenvalue of A, and $X_\nu = [-\nu, 1]'$ is an eigenvector associated with the negative eigenvalue of A.

We need to solve

$$\min_{\tilde{\theta}^i} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[(X_t^i)' \Omega (X_t^i) + (1-\alpha)\alpha_\theta (\tilde{\theta}^i)^2\right] dt,$$

where

$$\Omega \equiv \begin{bmatrix} \alpha_\pi & 0 \\ 0 & 1 \end{bmatrix}.$$

Replacing the X_t^i by its expression as a function of $\tilde{\theta}^i$ given in (37), we find that $\tilde{\theta}^i$ minimizes the following quadratic form:

$$\frac{1}{2}\frac{1}{\rho}(1-\alpha)\alpha_\theta(\tilde{\theta}^i)^2 + \frac{1}{2}(\alpha_\nu^i)^2 \frac{1}{\rho - 2\nu}(X_\nu' \Omega X_\nu) + \frac{1}{2}(\tilde{\theta}^i)^2 (\lambda\alpha)^2 \frac{1}{\rho}(E_1' (A')^{-1} \Omega A^{-1} E_1)$$

$$+\frac{1}{2}(\bar{\bar{s}}_0^i)^2(\psi)^2\frac{1}{\rho+2\psi}(E_2'(A'+\psi I)^{-1}\Omega(A+\psi I)E_2)+\alpha_\nu^i\tilde{\theta}^i\lambda\alpha\frac{1}{\rho-\nu}(X_\nu'\Omega A^{-1}E_1)$$
$$-\alpha_\nu^i\bar{\bar{s}}_0^i\psi\frac{1}{\rho+\psi-\nu}(X_\nu'\Omega(A+\psi I)^{-1}E_2)-\tilde{\theta}^i\bar{\bar{s}}_0^i\psi\lambda\alpha\frac{1}{\rho+\psi}(E_1'(A')^{-1}\Omega(A+\psi I)^{-1}E_2),$$

where α_ν^i is the linear function of $\tilde{\theta}^i$ and $\bar{\bar{s}}_0^i$ derived above. Solving the corresponding FOC gives us the solution.

A.6 Proof of Proposition 18

The solution for the closed economy limit can be obtained as a particular case of the analysis in Appendix A.5. When $\bar{\bar{s}}_t^i=\bar{\bar{s}}_0^i e^{-\psi t}$, for a given $\tilde{\theta}^i$, we have that $X_t^i=[\tilde{\pi}_{H,t}^i,\tilde{y}_t^i]'$ is given by

$$X_t^i=e^{\nu t}\alpha_\nu^i X_\nu-\psi e^{-\psi t}\bar{\bar{s}}_0^i(A+\psi I)^{-1}E_2,$$

where

$$\alpha_\nu^i=\tilde{\theta}^i-\bar{\bar{s}}_0^i+\psi\bar{\bar{s}}_0^i E_2'(A+\psi I)^{-1}E_2.$$

We find that $\tilde{\theta}^i$ minimizes the following quadratic form:

$$\frac{1}{2}(\alpha_\nu^i)^2\frac{1}{\rho-2\nu}(X_\nu'\Omega X_\nu)-\alpha_\nu^i\bar{\bar{s}}_0^i\psi\frac{1}{\rho+\psi-\nu}(X_\nu'\Omega(A+\psi I)^{-1}E_2)$$
$$+\frac{1}{2}(\bar{\bar{s}}_0^i)^2(\psi)^2\frac{1}{\rho+2\psi}(E_2'(A'+\psi I)^{-1}\Omega(A+\psi I)E_2).$$

The solution is

$$\tilde{\theta}^i=\bar{\bar{s}}_0^i\left[1-\psi E_2'(A+\psi I)^{-1}E_2+\psi\frac{\rho-2\nu}{\rho+\psi-\nu}\frac{X_\nu'\Omega(A+\psi I)^{-1}E_2}{X_\nu'\Omega X_\nu}\right].$$

Using $E_2'(A+\psi I)^{-1}E_2=\frac{\psi}{(\psi+\nu)(\psi+\rho-\nu)}$, $X_\nu'\Omega(A+\psi I)^{-1}E_2=\frac{\nu\alpha_\pi\hat{\kappa}+\psi}{(\psi+\nu)(\psi+\rho-\nu)}$ and $X_\nu'\Omega X_\nu=\alpha_\pi\nu^2+1$, we get the proposition.

A.7 Proof of Proposition 20

The planning problem of each country is linear quadratic. This has two important consequences that we exploit for our proof.

First, In order for aggregates $(\hat{y}_t^*,\pi_t^*)$ to be feasible, it must be the case that the solution of the following problem is $\tilde{\theta}^i=0$:

$$\min \frac{1}{2}\mathbb{E}\int_0^\infty e^{-\rho t}\left[\alpha_\pi(\tilde{\pi}^i_{H,t})^2 + 2\alpha_\pi\pi^*_t\tilde{\pi}^i_{H,t} + (\tilde{y}^i_t)^2 + 2\hat{y}^*_t\tilde{y}^i_t + \frac{2\alpha}{1+\phi}\tilde{y}^i_t + (1-\alpha)\alpha_\theta(\tilde{\theta}^i)^2\right]dt$$

subject to

$$\begin{aligned}\dot{\tilde{\pi}}^i_{H,t} &= \rho\tilde{\pi}^i_{H,t} - \hat{\kappa}\tilde{y}^i_t - \lambda\alpha\tilde{\theta}^i,\\ \dot{\tilde{y}}^i_t &= -\tilde{\pi}^i_{H,t},\end{aligned}$$

$$\tilde{y}^i_0 = (1-\alpha)\tilde{\theta}^i,$$

$$\mathbb{E}[\tilde{\theta}^i] = 0,$$

where the minimization is over the (random) variables $\tilde{\pi}^i_{H,t}$, $\tilde{y}^i_t$, $\tilde{\theta}^i$, taking $\hat{y}^*_t$, and π^*_t as given. Clearly $\hat{y}^*_t = \pi^*_t = 0$ is feasible.

Second, for any feasible aggregates $(\hat{y}^*_t, \pi^*_t)$, the solution of the planning problem of each country country coincides with the disaggregated solution of the coordinated planning problem.

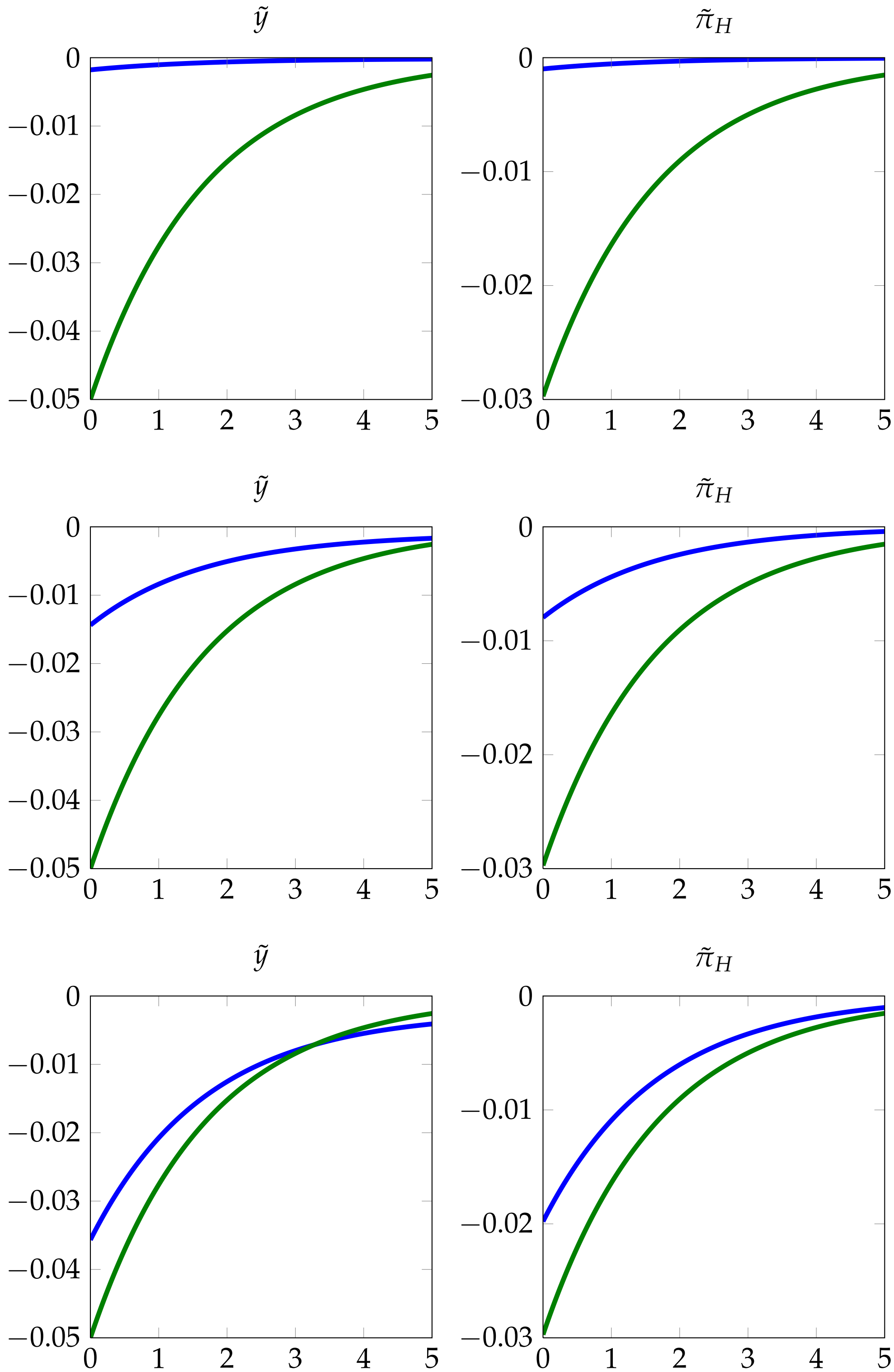

Figure 2: Allocations with optimal transfers (blue) and no transfers (green). The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. Time is measure in years and inflation is annualized.

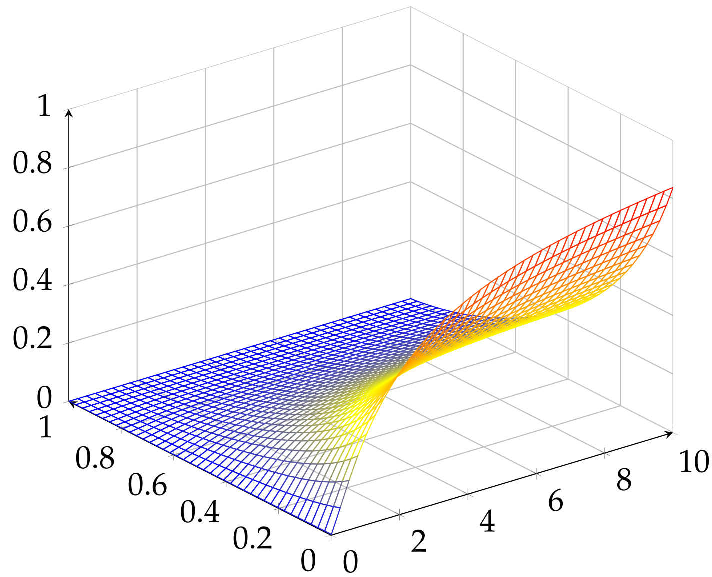

Figure 3: Optimal initial output gap mitigation at impact as a function of openness $\alpha \in (0, 1)$ and persistence (half-life of the shock) $-\frac{\log(0.5)}{\psi} \in (0, 10)$.

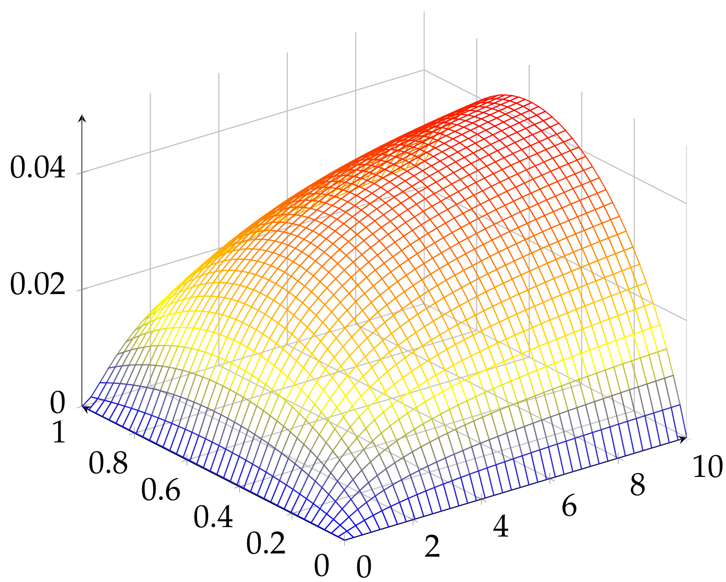

Figure 4: Transfers (as fraction of GDP) for a 1% shock to the terms of trade as a function of openness $\alpha \in (0, 1)$ and persistence (half-life of the shock) $-\frac{\log(0.5)}{\psi} \in (0, 10)$.