Forecasting with Bayesian Global Vector Autoregressive Models

A Comparison of Priors

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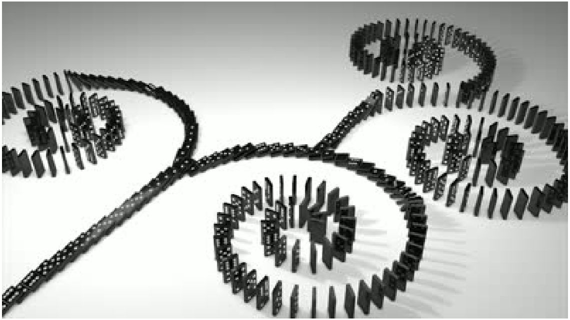
Overview

Research Objectives

- *Propose a framework that is flexible and can be used to do forecasting or structural analysis*
  - *Flexible prior distributions account for the heterogeneity observed in the world economy*
  - *Hierarchical modelling framework helps to automatically select appropriate country-specific models*
- *Do Bayesian methods help to cure the curse of dimensionality and increase the forecasting performance?*

This is achieved by combining the literature on Bayesian VARs with the literature on global VARs

The Global Vector AutoRegressive model



- The GVAR model is a compact representation of the world economy
- Estimation of an encompassing VAR is not feasible $\rightarrow$ even for moderate N
- GVAR short-cut: Estimate N country models in first stage, then stack them together to yield a global model.

Global VARs in a nutshell

- For each country $i \in 1, ..., N$, a VARX* model is estimated:

$$x_{it} = a_{i0} + a_{i1}t + \Phi_i x_{i,t-1} + \Lambda_{i0}x^*_{it} + \Lambda_{i1}x^*_{i,t-1} + \pi_{i0}d_t + \pi_{i1}d_{t-1} + \varepsilon_{it}$$

where $x^*_{it} := \sum_{j \neq i}^{N} \omega_{ij}x_{jt}$ and $\varepsilon_{it} \sim \mathcal{N}(0, \Sigma_i)$

- After some straightforward algebra it is possible to rewrite the GVAR in a standard VAR form

$$x_t \quad = \quad b_0 + b_1 t + F x_{t-1} + \Gamma_0 d_t + \Gamma_1 d_{t-1} + e_t,$$

$x_t = (x_{0,t}, x_{1,t}, ..., x_{N,t})$ denotes the global vector and b_0, b_1, Γ_1 contain the corresponding stacked vectors containing the parameter vectors of the country-specific specifications

Prior Specification

- For prior implementation, convenient to work with the parameter vector $\Psi_i = (a'_{i0} \;\; a'_{i1} \;\; \text{vec}(\Phi_i)' \;\; \text{vec}(\Lambda_{i0})' \;\; \text{vec}(\Lambda_{i1})' \;\; \text{vec}(\pi_{i0})' \;\; \text{vec}(\pi_{i1})')'$
- We assume the following conjugate prior setup on the coefficients of the local models:

Prior Setup

$$\Psi_i | \Sigma_i^{-1} \sim \mathcal{N}(\underline{\mu}_\Psi, \underline{V}_\Psi)$$
$$\Sigma_i^{-1} \sim \mathcal{W}(\underline{v}, \underline{S})$$

- Several choices for μ_Ψ and $\underline{V}_\Psi$ possible
- Several Choices implemented: Minnesota prior, Single Unit Root prior, simpler variants
- Note that the prior dependence of Ψ_i on Σ_i^{-1} could be dropped
- Finally, adding additional hierarchy leads to the Stochastic Search Variable Selection Prior

Stochastic Search Variable Selection Prior

Impose a mixture prior on the coefficients:

$$\Psi_{i,j}|\delta_{i,j} \sim (1-\delta_{i,j})\mathcal{N}(0,\tau_{0j}^2) + \delta_{i,j}\mathcal{N}(0,\tau_{1j}^2)$$

where $\delta_{i,j}$ is a dummy random variable which corresponds to coefficient j in country i. $\tau_{1j}^2 >> \tau_{0j}^2$ implies that if $\delta_{i,j} = 0$, the prior for $\Psi_{i,j}$ is centered around zero

- Estimation of the model using this prior setups requires MCMC $\rightarrow$ Gibbs sampling with data augmentation
- The SSVS prior tackles model uncertainty, which implies that coefficients on "unimportant" variables are shrunk towards zero
- This prior facilitates different individual country model structures $\rightarrow$ perfect for the present application

Computation of the Global Predictive Density

- Define $\Xi := (b_0', b_1', \text{vec}(F)', \text{vec}(\Gamma_0), \text{vec}(\Gamma_1))'$ and let $p(\Omega|\mathcal{D})$ be the posterior of the global VC-matrix
- Interest centers on $p(\Xi|\mathcal{D})$, not on the country individual $p(\Psi_i|\mathcal{D})$
- Interesting problem: How to estimate the global variance - covariance matrix Ω?
- Using the usual GVAR algebra, it is possible to transform draws from $p(\Psi_i|\mathcal{D})$ for all countries to get a valid draw from $p(\Xi|\mathcal{D})$
- The predictive density is given by

$$p(x_{T+n}|\mathcal{D}) = \int\int p(x_{T+n}|\mathcal{D}, \Xi, \Omega)p(\Xi, \Omega|\mathcal{D})d\Xi d\Omega$$

Data & Forecasting Design

Variable	Description, country coverage in brackets
y	real GDP (100%)
Δp	CPI inflation (100%)
e	Nominal exchange rate vis-à-vis the US dollar, deflated by national price levels (CPI) (98.10%)
i_S	Short-term interest rates (90.40%)
i_L	Long-term interest rates (32.70%)
p_{oil}	Price of oil (100%)

- *Weights* to construct foreign variables (and stack the model) based on average trade flows ($\omega_{ij,t}$)
- Together 45 countries + EA (in nominal terms, 92% of global output)
- Recursive one-quarter-ahead and one-year-ahead (four-quarters-ahead) predictions obtained by reestimating the models over a rolling window
- The initial estimation period ranges from 1995Q1 to 2009Q4, we use the period 2010Q1-2012Q4 as out-of-sample hold-out observations for the comparison of predictive ability across specifications
- Forecast comparison exercise based on root mean square error (RMSE) and the continuous rank probability score (CRPS)

Results: One Quarter Ahead

Relative Forecasting Performance, One-Quarter-Ahead: Root Mean Square Error and Continuous Rank Probability Score

	NC	M	SUR	IW	M-IW	SSVS	Diffuse	Pesaran	AR
y	1.3580	0.6075	0.8529	1.1874	0.6157	**0.5996**	0.7023	0.9059	0.7856
	(0.0390)	(0.0105)	(0.0182)	(0.0413)	(0.0345)	**(0.0075)**	(0.0148)	-	-
Δp	1.0236	0.8825	**0.7156**	1.0100	0.9172	0.8126	1.1100	1.2493	1.0139
	(0.0358)	(0.0116)	(0.0187)	(0.0395)	(0.0349)	**(0.0079)**	(0.0128)	-	-
e	0.6072	0.4870	0.5784	0.5869	0.7126	**0.4802**	0.8803	0.8519	0.7980
	(0.0502)	(0.0474)	(0.0772)	(0.0520)	(0.0542)	**(0.0301)**	(0.0544)	-	-
i_S	1.1084	0.7299	0.8296	1.0303	0.7659	0.6851	1.1312	0.8516	**0.5744**
	(0.0385)	(0.0115)	(0.0424)	(0.0417)	(0.0395)	**(0.0099)**	(0.0177)	-	-
i_L	1.1558	**0.4635**	0.4984	1.1127	0.7903	0.5696	0.7798	0.8450	0.6162
	(0.0355)	(0.0102)	(0.0112)	(0.0391)	(0.0336)	**(0.0060)**	(0.0105)	-	-

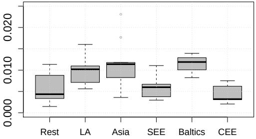
Results: One Year Ahead

Relative Forecasting Performance, Four-Quarters-Ahead: Root Mean Square Error and Continuous Rank Probability Score

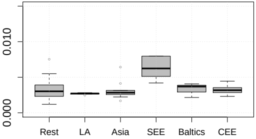
	NC	M	SUR	IW	M-IW	SSVS	Diffuse	Pesaran	AR
y	1.0090	0.4857	0.9999	0.8847	0.5799	**0.3651**	0.7192	0.9332	0.5198
	(0.0399)	(0.0187)	(0.0229)	(0.0423)	(0.0360)	**(0.0148)**	(0.0224)	-	-
Δp	0.9204	0.8538	1.2064	0.9126	0.9172	**0.7442**	1.2205	1.1408	1.0895
	(0.0356)	(0.0084)	(0.0178)	(0.0393)	(0.0346)	**(0.0081)**	(0.0118)	-	-
e	0.7658	0.6174	0.9803	0.7607	0.7126	**0.5328**	1.4179	0.5898	0.6679
	(0.0571)	(0.0566)	(0.0782)	(0.0585)	(0.0598)	**(0.0419)**	(0.0609)	-	-
i_S	0.8301	0.5984	1.0405	0.7876	0.7659	**0.4466**	0.9439	0.7686	0.5791
	(0.0383)	(0.0115)	(0.0398)	(0.0412)	(0.0392)	**(0.0103)**	(0.0169)	-	-
i_L	0.7541	0.7174	0.6917	0.7527	0.7903	**0.4277**	0.7448	0.6464	0.5012
	(0.0354)	(0.0056)	(0.0107)	(0.0390)	(0.0333)	**(0.0064)**	(0.0070)	-	-

Aggregate RMSE Distribution across country groups - 1-step ahead forecasts.

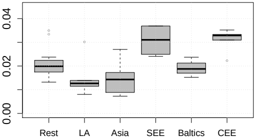
Real GDP



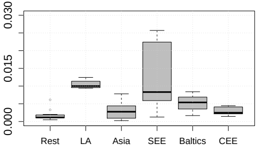
Inflation



Real Exchange Rate



Short-term interest rates



What about time varying Σ_i?

- To allow for time changing variance covariance structures at the country level we have two possibilities:
  1. Follow Primiceri (2005) and allow the variances and covariances in the different equations to change over time
  2. Take the simpler route and follow Clark, Carriero and Marcellino (2012) and use a scalar factor to drive the volatility for our macro aggregates
- From now on we assume that the individual country variance-covariance structure is time changing

$$\Sigma_{i,t} = \exp\left(h_{i,t}/2\right) \times \Sigma_i$$
$$h_{i,t} = \eta_i + \rho_i(h_{i,t-1} - \eta_i) + \sigma_i e_{i,t}$$
$$e_{i,t} \sim \mathcal{N}(0,1)$$

- Rescaling all observations by $\exp\left(h_{i,t}/2\right)$ allows us to use the same computations as in the standard case
- Sampling the latent log volatilities is done following Kastner & Fruehwirth-Schnatter (2013): Use **Ancillarity-sufficiency interweaving strategy (ASIS)**

Prior Setup for the SV Case

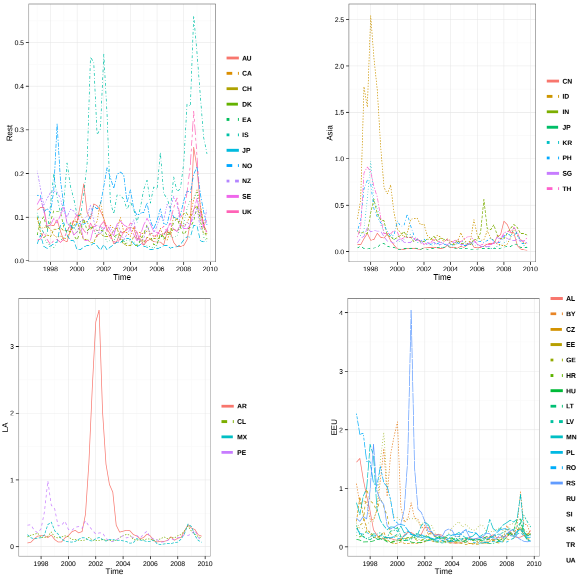
Prior Setup

$$\begin{aligned}
\Psi_i|\Sigma_i &\sim \mathcal{N}(\underline{\mu}_\Psi, \Sigma_i \otimes \underline{V}_\Psi) \\
\Sigma_i^{-1} &\sim \mathcal{W}(\underline{v}, \underline{S}^{-1}) \\
\eta_i &\sim \mathcal{N}(\underline{\mu}_\eta, \underline{V}_\eta) \\
\frac{\rho_i + 1}{2} &\sim \mathcal{B}(a_0, b_0) \\
\sigma_i &\sim \mathcal{G}(1/2, 1/2B_\sigma)
\end{aligned}$$

- $\underline{V}_\Psi$ is set to implement a symmetric Minnesota prior specification
- Choice of hyperparameters a_0 and b_0 for ρ_i can be quite influental, especially in our case
- We set $a_0 = 5$ and $b_0 = 1.5$, which translates into a prior mean of 0.54 and a prior standard deviation of 0.31 for ρ_i
- For all other components the hyperparameters are set such that the prior is effectively rendered non-influental

Filtered Volatilities for the World Economy

Posterior mean of $\exp(h_{i,t}/2)$



Results: One Quarter Ahead

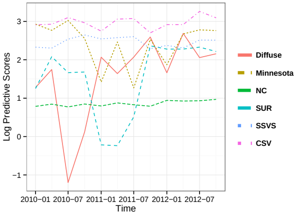
Forecasting Performance, One-Quarter-Ahead: Log Predictive Score

	Diffuse	M	NC	SUR	SSVS	CSV
y	18.8837	29.0401	10.3133	18.2054	29.5344	**35.6668**
Δp	24.1329	37.8102	10.7397	21.9634	31.8327	**39.8977**
e	-3.8832	**16.2315**	8.3951	-3.8468	13.7503	11.3237
i_S	17.3429	34.1630	9.7453	15.5587	26.8580	**38.2122**
i_L	28.6069	**42.3675**	10.5221	28.5819	29.9277	42.0357

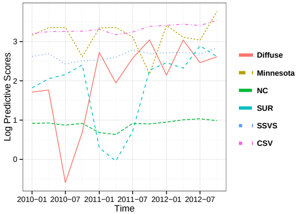
- Large outperformance of CSV specification for GDP, Inflation and short-term interest rates

Evolution of Log Predictive Scores over time - 1-step ahead predictive density.

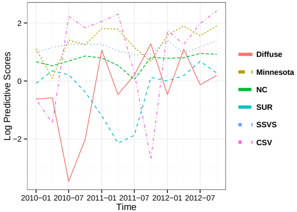
(a) Real GDP



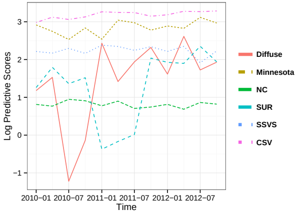
(b) Inflation



(c) Real Exchange Rate



(d) Short-term interest rates



Conclusions & Further Remarks

- Strong performance of Bayesian GVARs, both in terms of precise point forecasts and density forecasts
- SSVS and the simple Minnesota prior show the strongest performance in terms of density and point forecasting
- Performance of natural conjugate prior specifications could be increased by using hyperpriors (Giannone, Lenza & Primiceri, 2012). However, this would imply giving up flexibility as compared to the SSVS prior
- Preliminary results of a B-GVAR with SV indicate strong performance in terms of density forecasting (at little additional cost in terms of computing)

Selected Literature

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