## Discussion of

# "No arbitrage priors, drifting volatilities, and the term structure of interest rates"

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#### my specific interest in this paper

application of DSGE-VAR methodology of DS2004 => GATSM-VAR

This methodology provides an indication of how well the theoretical model fits the data and where to look for misspecification.

By optimizing the tightness of the prior the forecasting performance is also optimized.

estimation of the underlying stochastic volatility process

First step in measuring and accounting for time varying volatility.

Important to account for non-gaussian non-linear relations when integrating the macro-finance relations in our models: understanding relation between asset price, volatility, risk and macro-economic fundamentals.

- The JSZ-presentation allows to separate the estimation of the pricing factor model, a VAR(1) in the first three principal components, and the estimation of the affine measurement equations that link the yields to the pricing factors and that incorporate the no-arbitrage restrictions.
- ► OLS provides efficient forecasts for the pricing factors, and therefore, the number of structural parameters for the GATSM-VAR model is reduced to only 16 parameters in this application:  $\theta = (\lambda^Q, k_\infty^Q, \Sigma_P, \Sigma_y)$ .
- => this separation is very useful for this pure-yield-curve setting (where three factors and their VAR(1) dynamics is sufficient and estimated precisely by OLS), but it will become problematic if we want to include, spanned or unspanned, macro-variables or other 'hidden' variables that might be important for forecasting the pricing factors.
- => additional priors will be useful or required in such applications, or separation will no longer be valid with restrictions between yield and macrofactor dynamics.

- The GATSModel imposes three types of constraints on the overall model setup:
  - > the factor structure for the yields: six yields are reduce to three pricing factors;
  - > the no-arbitrage restrictions on the observation equations: 16 structural param;
  - > the dynamic structure for the pricing factors: VAR(1) with homoskedastic cov.
- → how useful is this GATSM prior for the VAR(3) model in six yields: how tight are these restrictions imposed ?  $\gamma = 0.48$  (st.e=0.056) (<=> appendix [0,25;3])
  - > the theoretical prior receives a relative small weight;
  - > more systematic information on the impact of gamma on the marginal likelihood would be informative;
  - > the pure-GATSM model: delivers substantial worse RMSFE, and bad pred.density.
- => important question: on which dimension is the model misspecified ?

- $\blacktriangleright$  the factor structure for the yields: six yields are reduce to three pricing factors;
  - > probably crucial restriction?
  - > illustrate this constraint by comparing the results for 4,5, 6 pricing factors;
  - > very similar to statistical priors: how many common factors in the dataset;
  - <=> claim in the paper that the direction of shrinkage is very important.

- the no-arbitrage restrictions on the observation equations:
  - > weak restriction: substituting these restricted parameters with completely unrestricted loadings generates very similar forecasting results;
  - > three factors do well in summarizing the cross-section yield curve info;
  - > the implied risk prices are very flexible functions: imposing additional restricting on the risk pricing functions is not evident in this JSZ-representation.

- $\blacktriangleright$  the dynamic structure for the pricing factors: VAR(1) with homoskedastic cov.
  - > homoskedastic covariance prior can be implemented easily but it can restrict the posterior too much?
  - comparing the GATSM-VAR with stochastic volatility to the restricted homoskedastic case: similar mean forecasts and better predictive score.
- => not obvious what explains the low tightness ( $\gamma$ ) for the prior ?

#### Some issues on the estimation of the stochastic volatility process

- the common stochastic volatility factor λ<sub>t</sub> is estimated with a very large uncertainty: this uncertainty seems larger than the typical standard error for Σ<sub>h</sub>-estimates for stochastic volatility processes:
  - > is there an interaction with gamma-prior estimate?
- ➤ why imposing the restriction of only one common stochastic factor?



Figure 2: Posterior distribution of the Common Stochastic Volatility process  $\lambda_t$ .

$$\begin{split} V|Y,\Lambda,\theta,\gamma \sim IW(\bar{S}(\theta),(\gamma+1)T-k),\\ \bar{S}(\theta) &= [(\gamma T \Gamma_{Y^{*\prime}Y^*}(\theta) + Y'\Lambda^{-1}Y) - (\gamma T \Gamma_{Y^{*\prime}X^*}(\theta) + Y'\Lambda^{-1}X)(\gamma T \Gamma_{X^{*\prime}X^*}(\theta) \\ &+ X'\Lambda^{-1}X)^{-1}(\gamma T \Gamma_{X^{*\prime}Y^*}(\theta) + X'\Lambda^{-1}Y)].\\ \Lambda &= diag(\lambda_1,...,\lambda_T). \end{split}$$

- > potential identification problem: does the level of  $\Lambda$  vary systematically with  $\gamma$ ?
- allow for different gamma's in the posterior for the mean parameters and for the covariance matrix of the VAR: is the optimal tightness for the prior on the mean parameters higher than on the homoskedastic covariance ?

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- TSModels with volatility as an additional pricing factor often fit worse because the volatility factor has conflicting roles in the time dimension and in the cross section dimension (Collin--Dufresne et al. 2009, Creal and Wu 2013 etc.). Only more flexible models (multiple spanned volatilities or unspanned volatility factor models) can overcome this problem (Creal and Wu 2014).
  - Is the GATSM-VAR approach with stoch.vol. not too flexible: is there a risk that stochastic volatility leads again to an overparameterisation that can deteriorate the point forecast precision ?
  - > the GATSM-VAR approach can not handle the more interesting TSM models with volatility factors: this limits the potential insights on the fundamental questions on the relation volatility-risk premia

- The paper develops an mcmc sampling approach to estimate the GATSM-VAR model including the stochastic volatility process:
- ➤ drawings for the stochastic volatility process are exploiting a backward-forward looking perspective:  $p(\Lambda|Y, \Phi, V, \phi) = \prod_{t=1}^{T} p(\lambda_t | \lambda_{t-1}, \lambda_{t+1}, \phi, w_t)$
- how does such a procedure perform for the end of the sample? does it affect the out-of-sample forecasting performance for the volatility process?
- is a particle filter method, which is a one-sided backward looking approach, not more appropriate for estimating the stochastic volatility process?

#### summary

- the GATSM-VAR model is an interesting application that delivers useful insights about the potential misspecification in the GATSM.
- ➤ the estimation of the stochastic volatility process could be refined.
- the next challenge is to apply this procedure to larger and more interesting TSModels: extensions do not seem straightforward ?